Multistate Transition Models in Demography

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Abstract

Multistate transition models provide an analytical framework for situations in which there exists a certain number of 'states' of interest that individuals can occupy, and between which individuals can move. Multistate models allow one to describe these situations by examining patterns of transitions across states and estimating state-specific life expectancies. This article emphasizes the basic properties of multistate transition models, as well as practical strategies for estimating them.

Introduction

In demography, multistate transition models refer to situations in which there exists a certain number of 'states' of interest that individuals can occupy, and between which individuals can move (Rogers, 1975; Land et al., 1982; Schoen, 1988a,b). Examples of 'states' include the following:

- 'Residing in urban areas' versus 'residing in rural areas.' In this example, transitions between states refer to migrations from urban to rural areas and vice versa.
- 'Having no disability' versus 'being disabled.' In this example, transitions between states refer to the incidence of disability (transitions from having no disability to being disabled) and to recovery from disability (transitions from being disabled to having no disability).
- 'Being never married'; 'being married'; 'being divorced'; 'being widowed.' In this example, transitions between states refer to various changes of marital status, including the incidence of first marriage (transitions from 'never married' to 'married'), the incidence of divorce (transitions from 'married' to 'divorced'), etc.

In all the above examples, there necessarily exists an additional state, namely, the state of 'being dead.' Transitions to that state represent mortality. A multistate transition model contains a minimum of two 'living' states, in which individuals can spend a certain amount of time, as well as the state of being dead.

An important feature of multistate transition models is that they allow the study of situations in which individuals can move back and forth between the different 'living' states of interest. This contrasts with single state models, like the classic life table, in which only one transition is possible with no possible return to the state of origin, that is, from 'being alive' to 'being dead.'

Multistate transition models in demography allow one to quantify patterns of transitions between states and study how they vary over age and time. These models also allow one to calculate a number of interesting quantities that result from given transition patterns. For example, if we take the marriage formation model described above, the multistate framework allows one to calculate the following quantities of interest:

- How many years can a newborn expect to live in the 'married' state?
- How many additional years can a person in the 'divorced' state at age 50 expect to live in the 'married' state?
- What is the probability that a marriage will end in a divorce?
- What is the probability of remarriage from divorce or widowhood?
- What is the probability that a newborn will die in the married state?

This article is organized as follows. First, I discuss the simple case of transition models without age. The next section introduces age with a discussion of multistate life tables. The last section discusses multistate population models, i.e., models in which patterns of entry into the multistate system are integrated (for example, via births), generating populations with interesting properties.

Multistate Transition Models without Age

In their simplest form, multistate transition models do not include age: the population of interest is broken down by state but not by age. For example, let us consider a population broken down into two states: urban population versus rural population. A simple multistate model allows one to examine patterns of migration between urban and rural populations and how these populations grow or decline over time as a result of these transitions. Assuming that the overall population is closed to international migration, the following forces are affecting the size of the urban and rural populations:

- For the urban population: the crude rate of natural increase (defined as the difference between the crude birth rate and the crude death rate for the urban population) and the crude rate of out-migration (from urban to rural areas).
- For the rural population: the crude rate of natural increase (here defined for the rural population) and the crude rate of out-migration (here from rural to urban areas).

An interesting feature of this model is the interdependence between the two states: the size of the urban population affects the size of the rural population through out-migration from urban to rural areas, and the size of the rural population affects the size of the urban population through out-migration from rural to urban areas. This is an important feature shared by all multistate transition models.

Another important feature of this model is that if rates of natural increase and out-migration remain constant over time for both urban and rural populations, eventually urban and rural populations will be growing at the same rate, and the proportionate distribution of the population by urban/rural residence will become constant. This eventual state of equilibrium is independent of the initial distribution of the population by urban/rural residence. This is illustrated in Figure 1, which shows trajectories toward equilibrium in populations with proportion urban ranging from 10% to 90% at baseline. Assuming a crude rate of natural increase of 20 per 1000 in urban areas and of 30 per 1000 in rural areas, and a crude rate of out-migration of 10 per 1000 in urban areas and of 30 per 1000 in rural areas, populations will all reach an equilibrium proportion urban of 69.7%, regardless of the initial proportion urban. This equilibrium proportion is entirely the product of the assumed constant rates of natural increase and out-migration.

Multistate (Increment–Decrement) Life Tables

Multistate life tables are a different but related kind of multistate transition model. Instead of following a population over time, like in the above example, a multistate life table follows a cohort through different ages. Members of a cohort are born in a given state, and as they age they may move through different states, until they eventually die. A multistate life table describes and quantifies this process.

Multistate life tables are also called increment–decrement life tables because the number of individuals in one state can increase or decrease with age, depending on the balance of flows in-and-out of that state. This contrast with the classic life table model where the number of people in the state of 'being alive' can only decrease with age.

Multistate Life Tables for a Cohort

Multistate life tables are usually calculated for periods (using synthetic cohorts), but are best understood by looking at a real cohort and tracking cohort survivors over time as they move between different states. Therefore, we first discuss features of a multistate life table by looking at the case of a cohort.

For example, let us examine the simple case of a multistate life table model representing marriage formation, in which there are three states: (1) unmarried; (2) married; and (3) dead. Figure 2 represents the three states and the arrows represent possible transitions between the states.

It can be assumed that all individuals are born in the 'unmarried' state. As individuals age, they may transition between states: they may become married and then return to the 'unmarried' state through divorce or widowhood; they may move back to the state of being married through remarriage. Eventually, due to mortality, all individuals transition to the 'dead' state, either from the 'unmarried' state or from the 'married' state, depending on their marital status at the time of death.

A multistate life table summarizes the history of the cohort with the following columns:

- x = exact age;
- l_x^i = number of individuals in the cohort who are in state i at age x.

For example, in the marriage model described above, we have

l_x^U = number of individuals in the cohort who are *unmarried* at age x;
l_x^M = number of individuals in the cohort who are *married* at age x.

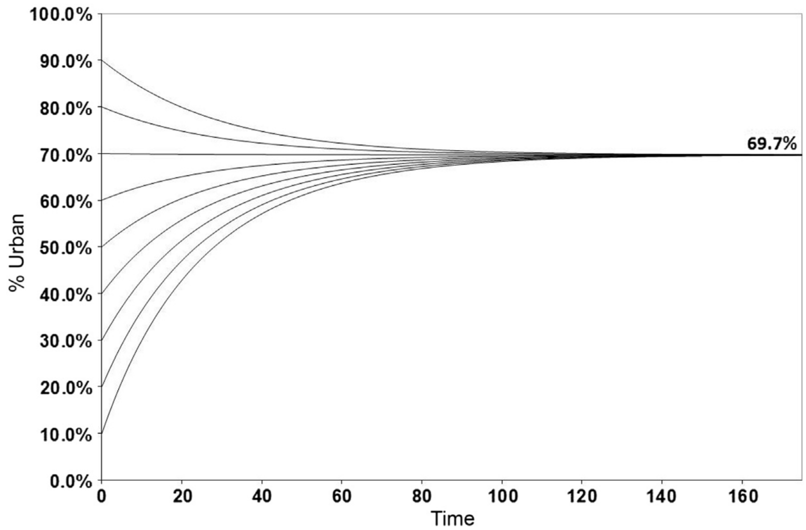

Figure 1 Proportion of the population living in urban areas for populations in which rates of natural increase and rates of out-migration are set to specific values and constant over time. In this example, all populations converge to a proportion urban of 69.7%.

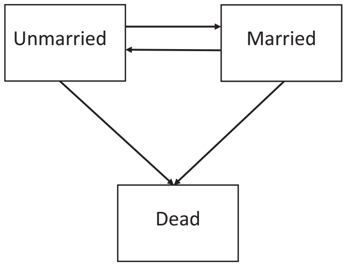

Figure 2 Possible transitions among states in a simple marriage formation multistate model.

Note that l_x^U or l_x^M may increase or decrease with age (which is why, as said earlier, multistate life tables are also called increment–decrement life tables). Note also that $l_x^U + l_x^M = l_x$, the number of cohort survivors at age x. l_0 is the initial size of the cohort (at birth). In this example, all individuals are unmarried at birth, so $l_0^M = 0$ and $l_0^U = l_0$. In other examples, individuals may be distributed across more than one state at birth. For example, in an urban/rural residence transition model, individuals may be born either in urban or rural areas.

- ${}_nd_x^{ij}$ = number of transitions from state i to state j occurring between ages x and $x+n$ among cohort members

For example, in the marriage model described above, we have four different types of transitions:

${}_nd_x^{UM}$ = number of transitions from unmarried to married (i.e., number of marriages) among cohort members aged x to $x+n$;
${}_nd_x^{UD}$ = number of transitions from 'unmarried' to 'dead' (i.e., number of deaths) among unmarried cohort members aged x to $x+n$;
${}_nd_x^{MU}$ = number of transitions from 'married' to 'unmarried' (i.e., through divorce or widowhood) among cohort members aged x to $x+n$; and
${}_nd_x^{MD}$ = number of transitions from married to dead (i.e., number of deaths) among married cohort members aged x to $x+n$.

In a multistate life table, survivors and transitions are related with one another in the following fashion (using the marriage model as an example):

$$l_{x+n}{}^U = l_x{}^U - {}_nd_x{}^{UM} - {}_nd_x{}^{UD} + {}_nd_x{}^{MU}$$

$$l_{x+n}{}^M = l_x{}^M - {}_nd_x{}^{MU} - {}_nd_x{}^{MD} + {}_nd_x{}^{UM}$$

- ${}_nq_x^{ij}$ = probability that an individual who is in state i at age x will be in state j at age $x+n$.

If it can be assumed that only one transition is possible between x and $x+n$, this probability is related to other life table quantities through the following equation:

$${}_nq_x{}^{ij} = {}_nd_x{}^{ij}/l_x{}^i$$

In the marriage example, there are four possible transition probabilities:

${}_nq_x^{UM}$ = probability that an *unmarried* individual aged x will be *married* at age $x+n$;
${}_nq_x^{UD}$ = probability that an *unmarried* individual aged x will *die* by age $x+n$;
${}_nq_x^{MU}$ = probability that a *married* individual aged x will be *unmarried* at age $x+n$; and
${}_nq_x^{MD}$ = probability that a *married* individual aged x will *die* by age $x+n$.

- ${}_nL_x^i$ = number of person-years lived in state i between age x and $x+n$ by individuals in the cohort

Obviously, person-years can only be lived in a 'living' state, so in the marriage example, there are only two types of person-years:

${}_nL_x^U$ = person-years lived in the unmarried state between age x and $x+n$ by individuals in the cohort; and
${}_nL_x^M$ = person-years lived in the married state between age x and $x+n$ by individuals in the cohort.

Note that ${}_nL_x^U + {}_nL_x^M = {}_nL_x$, i.e., the number of person-years lived between x and $x+n$ by all members of the cohort, regardless of marital status.

- ${}_nm_x^{ij}$ = transition rate from state i to state j between age x and $x+n$

This transition rate is related to other life table functions through the following equation:

$${}_nm_x{}^{ij} = {}_nd_x{}^{ij}/{}_nL_x{}^i$$

This is a classic occurrence/exposure rate with events (transitions) in the numerator and person-years of exposure in the denominator. Since only individuals in state i are at risk of experiencing a transition from state i to state j, the exposure term in the denominator is limited to person-years lived in state i. In the marriage example, as in the case of transition probabilities, each age group has four possible transition rates: ${}_nm_x^{UM}$; ${}_nm_x^{UD}$; ${}_nm_x^{MU}$; and ${}_nm_x^{MD}$.

- T_x^i = number of person-years lived in state i above age x by individuals in the cohort

This quantity corresponds to the sum of ${}_nL_x^i$ for ages x and higher. In the marriage example, there will be two types of T_x^i values:

T_x^U = number of person-years lived above age x in the *unmarried* state; and
T_x^M = number of person-years lived above age x in the *married* state.

Note that $T_x^U + T_x^M = T_x$, the number of person-years lived above age x by all members of the cohort.

- e_x^i = life expectancy at age x in state i = number of years that an individual aged x can expect to live in state i

These life expectancies are related to other life table functions through the following equation:

$$e_x{}^i = T_x{}^i/l_x$$

These life expectancies are called 'unconditional' life expectancies, because the denominator includes all survivors at age x (l_x), regardless of state occupancy at age x.

In the marriage example, there will be two types of unconditional life expectancies:

e_x^U = number of years that an individual aged x can expect to live in the *unmarried* state; and
e_x^M = number of years that an individual aged x can expect to live in the *married* state.

Note that $e_x^U + e_x^M = e_x$. That is, a multistate life table apportions the classic life expectancy, e_x, among different states. In turn, state-specific unconditional life expectancies add up to the classic life expectancy.

- T_x^{ij} = number of person-years lived in state j above age x by cohort members who are in state i at age x

Typically, in a multistate transition model, future state occupancy depends highly on the state that an individual occupies at age x. For example, an individual who is married at age 50 cannot expect to spend the same number of years above 50 in the married state as an individual who is not married at age 50. When calculating person-years lived above age x, it is thus useful to calculate person-years lived above age x in a given state conditional on state occupancy at age x. In the marriage example, there will be four types of conditional T_x^{ij} values:

T_x^{UU} = number of person-years lived above age x in the *unmarried* state by individuals who are *unmarried* at age x;
T_x^{UM} = number of person-years lived above age x in the *married* state by individuals who are *unmarried* at age x;
T_x^{MU} = number of person-years lived above age x in the *unmarried* state by individuals who are *married* at age x; and
T_x^{MM} = number of person-years lived above age x in the *married* state by individuals who are *married* at age x.

Note that $T_x^{UU} + T_x^{MU} = T_x^U$ defined earlier. Likewise, $T_x^{UM} + T_x^{MM} = T_x^M$.

These person-years conditional on state occupancy at age x allow one to calculate a second kind of life expectancies, called 'conditional' life expectancies:

- e_x^{ij} = life expectancy at age x in state j, for individuals who are in state i at age x.

These life expectancies are related to other life table functions through the following equation:

$$e_x^{ij} = T_x^{ij} / l_x^i$$

In the marriage example, there are four types of conditional life expectancies:

e_x^{UU} = number of years that an individual who is *unmarried* at age x can expect to live in the *unmarried* state;
e_x^{UM} = number of years that an individual who is *unmarried* at age x can expect to live in the *married* state;
e_x^{MU} = number of years that an individual who is *married* at age x can expect to live in the *unmarried* state; and
e_x^{MM} = number of years that an individual who is *married* at age x can expect to live in the *married* state.

Note that unconditional life expectancies are weighted averages of conditional life expectancies, with the weights being the proportion of survivors by state at age x. For example, in the marriage example

$$e_x^U = l_x^U / l_x \cdot e_x^{UU} + l_x^M / l_x \cdot e_x^{MU}$$

In a multistate life table, the most important quantities are ${}_nq_x^{ij}$ or ${}_nm_x^{ij}$, which summarize the pattern of transitions between states, as well as e_x^i or e_x^{ij}, which summarize mean duration of state occupancy above age x.

Multistate Life Tables in a Period

If individual-level cohort information is available, life expectancies can be readily observed without resorting to any kind of complex computational approach. Person-years spent in each state by cohort members can be directly observed and summed across individuals. Life expectancies in a given state are simply mean person-years lived in a given state, conditional or not on earlier state occupancy.

As a simple numerical example, let us assume that the following information is available on two individuals born the same day: individual #1 died at age 80 with 30 years spent in the unmarried and 50 years spent in the married state, while individual #2 died at age 60 with 46 years spent in the unmarried state and 14 years spent in the married state. It can then be readily estimated that the life expectancy at birth for this cohort of two individuals is $(80 + 60)/2 = 70$ years, with an unconditional life expectancy at birth in the unmarried state of $(30 + 46)/2 = 38$ years and an unconditional life expectancy at birth in the married state of $(50 + 14)/2 = 32$ years. Since in this model all individuals are born in the unmarried state, life expectancies at birth conditional on being unmarried at birth (e_0^{UU} and e_0^{UM}) are equal to the unconditional life expectancies (e_0^U and e_0^M, respectively) and the life expectancies at birth conditional on being married at birth (e_0^{MU} and e_0^{MM}) are undefined. With additional information on ages at transition for cohort members, life expectancies at ages above 0, conditional or unconditional, can also be readily observed without using information on transition rates or probabilities.

Most often, however, analysts are interested in estimating multistate life tables for a period. In a period multistate life table, age-specific transition rates are observed for a given year or period, and the analyst is interested in examining what would happen to a cohort of individuals if they were exposed at each age to the transition rates observed during the given period. This is the classic synthetic cohort approach that is used whenever constructing a classic period life table.

When constructing a period life table, however, person-years and life expectancies by state cannot be readily observed, because the synthetic cohort is not a real cohort and thus cannot be tracked over time. One classic strategy for calculating a period multistate life table starts from calculating transition rates, which can be estimated from period data if information on transitions and population at risk is available:

$${}_nm_x^{ij} \approx {}_nD_x^{ij} / {}_nN_x^i$$

where ${}_nD_x^{ij}$ are observed transitions from state i to state j between age x and $x + n$ during a given year, and where ${}_nN_x^i$ is the midyear population in state i between ages x and $x + n$, used as an estimate of the number of person-years of exposure in state i between ages x and $x + n$ during a given year.

Using these transition rates, a synthetic cohort can then be simulated and corresponding person-years and life expectancies can be estimated. Like in the case of a classic life table construction, a key aspect of period multistate life table construction consists of performing what amounts to a rate-to-probability conversion, so that the synthetic cohort can be simulated. This involves solving a system of equations, with assumptions about how transitions are distributed within the age interval. One classic assumption is to assume that the number of state-specific survivors varies linearly between age x and $x+n$. In the marriage example, this assumption would produce the following system of equations (one system for each age group):

1. $l_{x+n}^{U} = l_x^{U} - {}_nd_x^{UM} - {}_nd_x^{UD} + {}_nd_x^{MU}$
2. $l_{x+n}^{M} = l_x^{M} - {}_nd_x^{MU} - {}_nd_x^{MD} + {}_nd_x^{UM}$
3. ${}_nd_x^{UM} = {}_nm_x^{UM} \cdot {}_nL_x^{U}$
4. ${}_nd_x^{UD} = {}_nm_x^{UD} \cdot {}_nL_x^{U}$
5. ${}_nd_x^{MU} = {}_nm_x^{MU} \cdot {}_nL_x^{M}$
6. ${}_nd_x^{MD} = {}_nm_x^{MD} \cdot {}_nL_x^{M}$
7. ${}_nL_x^{U} = n/2 \cdot (l_x^{U} + l_{x+n}^{U})$
8. ${}_nL_x^{M} = n/2 \cdot (l_x^{M} + l_{x+n}^{M})$

In this system of eight equations, the only known quantities are the following: ${}_nm_x^{UM}$, ${}_nm_x^{UD}$, ${}_nm_x^{MU}$, and ${}_nm_x^{MD}$. In total, there are eight equations and eight unknowns, which is solvable. The algebraic solution for the case of two living states can be found in Schoen (1988a). If there are more than two living states, the system of equations will be more complex and can be solved using matrix algebra (Palloni, 2001).

Like in any life table construction, the rate-to-probability conversion involves certain assumptions. In the example above, the assumption is that the number of survivors in state i (l_x^i) varies linearly between ages x and $x+n$. The most common alternative is to assume that transition rates, in continuous terms, are constant between ages x and $x+n$ (Schoen, 1988a).

Once the system of equations has been solved, the synthetic cohort can be simulated. One simply needs to choose an arbitrary radix for the life table (l_0), and apportion this radix by state (l_0^i). In many cases, the distribution by state of individuals aged 0 is theoretically implicit. For example, in the marriage model, it is assumed that $l_0^U = l_0$. In some cases, however, the analyst needs to decide how to distribute individuals aged 0 by state. A common option is to use actual distributions of births by state as recently observed in the population. We will see later that in multistate models that involve reproduction, the distribution of births by state is entirely determined by the model, so no arbitrary choice needs to be made. Once values of l_0^{i} are chosen, state-specific survivors can be simulated as if they were exposed to the observed period transition rates, and the different life expectancies described above can be calculated.

Note that the construction of a multistate life table typically makes an assumption of homogeneity: all the individuals in state i at age x are assumed to have the same probability of experiencing a transition. That is, transition probabilities depend only on age and current state (a setup also known as a first-order Markov chain in the statistics literature). The past history of state occupancy is not taken into account. This assumption is potentially problematic for period life tables if indeed duration in a state influences transition rates in addition to age.

Statistical Approaches to Estimating Multistate Life Tables

The life table approach discussed above assumes that transition rates can be estimated with precision for well-defined age groups and time periods using survey data or exhaustive population information. In many cases, however, this goal is difficult to attain, especially when dealing with survey data. Transitions of interest to demographers (such as death, marriage, disability, etc.) tend to be relatively rare events. Sample sizes in available surveys are often too small to produce estimates of transition rates that are precise enough for direct use in life table construction. Moreover, longitudinal surveys typically have varying interview dates and individuals are observed at irregular intervals, which create additional difficulties for the calculation of well-defined rates or probabilities. Finally, the approach discussed above is deterministic and does not readily allow statistical inference for transition rates or life expectancies.

The typical data configuration for the statistical estimation of transition rates on the basis of longitudinal data is as follows: a sampled individual is observed at a first interview date, during which information is gathered about age and current state. Later, the individual is reinterviewed, and current state is updated. If the individual has changed states by the time of the second interview, he/she may be asked about the date at which his/her status changed. If the individual died between the two interview dates, information about the exact date of death is sometimes recorded. Sometimes information at a third interview may be available, but some individuals may be missing information at one of the three interviews. Statistical approaches for the estimation of a multistate transition model will use information about sampled individuals with a range of ages at a first interview (for which the exact date may vary across individuals), and then estimate a set of period age-specific transition rates that best agree with the observed patterns of transitions among sampled individuals.

There are many different statistical approaches to estimating period age-specific transitions rates from longitudinal survey data (Willekens and Putter, 2014). One commonly used approach is called Interpolated Markov Chains (IMaCh) and is available as a software (Lièvre et al., 2003; Brouard and Lièvre, 2009). IMaCh is tailored to the estimation of health expectancies (with a transition model involving three states: healthy; disabled; and dead) and makes parametric assumptions about how transition rates vary with age. Specifically, IMaCh assumes that the partial odds of monthly transition probabilities follow a log-linear function of age (Laditka and Wolf, 1998). Using a maximum likelihood approach, the software produces a set of parameters for the log-linear functions that best agrees with the set of observed individual transitions. The estimated transition probabilities are then used to estimate of complete multistate life table, including state-specific life expectancies. Confidence intervals around life expectancies are calculated using the Delta method.

Another software, called SPACE, was published in 2010 by Cai et al. (Cai et al., 2010). One important difference with IMaCh is that life table outputs are calculated by resorting to micro-simulation. Once transition probabilities are estimated, individual life histories are simulated and then aggregated to produce mean values of the output quantities. Confidence intervals around mean output values are calculated using bootstrapping. The SPACE approach is being increasingly used for the estimation of health expectancies (Payne et al., 2013).

Multistate Population Models

Multistate Stationary Populations

If it can be assumed that there is a constant annual number of births by state (B^i), and that age-specific transition rates are constant over time, then the corresponding population is a multistate stationary population. Like a classic stationary population, this population will have a constant size and constant state and age distribution. Moreover, the population distribution by state will be proportional to the ${}_nL_x{}^i$ column of the corresponding multistate life table:

$$ {}_nN_x^i/N^i = {}_nL_x^i/T_0{}^i, $$

where ${}_nN_x{}^i$ is the population aged x to $x+n$ in state i, and where N^i is the total population in state i; and

$$ N^i/N = T_x{}^i/T_0, $$

where N is the total population size for all states combined.

Thus, in a stationary multistate population, unconditional life expectancies (e_0^i) can be readily calculated by applying the proportion of the total population that is in state i to the overall life expectancy: $e_0^i = e_0 \cdot (N^i/N)$. In the marriage example, if the multistate model is stationary with an overall life expectancy of 75 years and with 40% of the total population in the 'married' state, then the number of years that a newborn can expect to live in the married state if $75 \cdot 0.4 = 30$ years.

This stationary model is a special case of a more general population model in which transition rates are constant over time, and in which the number of births, while varying over time, has a constant distribution by state. That is, $B^i(t)/B(t)$ is constant over time t. This is a useful model, because in many cases, $B^i(t)/B(t)$ is constant over time due to the very nature of the multistate process. In the marriage model, for example, regardless of variations in the number of births each year, 100% of births occur in the unmarried state every year.

In this less restrictive population model, N^i/N is not equal to $T_0{}^i/T_0$, like in the case of a stationary population. However, ${}_nN_x^i/{}_nN_x = {}_nL_x^i/{}_nL_x$. This provides a useful shortcut for calculating a multistate life table in the absence of information on transitions. One simply needs to first calculate an ${}_nL_x$ column for all states combined (i.e., as in a classic life table). ${}_nL_x{}^i$ can then be calculated by applying at each age the observed proportion of the age-specific population that is in state i, using for example, information from a census or a cross-sectional survey:

$$ {}_nL_x^i = {}_nL_x \cdot {}_nN_x^i/{}_nN_x $$

This approach is commonly used for estimating disability-free life expectancy in the absence of information on transitions in-and-out of disability (Cambois et al., 1999). This method, called the Sullivan method (Sullivan, 1971), assumes that: (1) age-specific transition rates in the multistate system have been constant over time; and (2) the proportion of births by disability status has been constant over time. While the second assumption is not problematic (the proportion of individuals who are free of disability at birth is close to 100% and can be assumed to be constant over time), the assumption of constant transition rates can be problematic in populations that have experienced fast mortality declines and changes in the incidence of disability (Rogers et al., 1990; Barendregt et al., 1994, 1997; Lièvre et al., 2003).

Multistate Stable Populations

The multistate stable population model is a model which includes entry into the population through reproduction, in addition to attrition out of the population through mortality (Schoen, 1988a,b). This model makes the following assumptions:

- Age-specific transition rates are constant over time;
- Age-specific fertility rates by state are constant over time.

If these assumptions hold, then a multistate stable population emerges with the following features:

- The proportionate distribution of the population by age and state is constant over time;
- The overall growth rate of the population is constant over time.

Multistate stable populations have the same property of ergodicity as classic stable populations. This implies that any population with multiple states has an underlying 'stable-equivalent' multistate population with an intrinsic growth rate and an intrinsic distribution of the population by age and state. Although it does not include age, the urban/rural residence model presented in **Figure 1** illustrates this property of ergodicity and the concept of intrinsic parameters. In this example, the observed set of rates of natural increase and out-migration produces an 'intrinsic' proportion of the population living in urban areas of 69.7%.

Multistate stable populations have been used in a number of applications. One classic application shows that fertility differentials by IQ levels (with low-IQ persons having above-average fertility), rather than generating decreases in the population's mean IQ level in the long term, generate constant mean IQ levels in the long term that may or may not be lower than the population's current mean IQ level (Preston and Campbell, 1993).

See also: Aging and Health in Old Age; Demographic Models; Families and Households, Formal Demography of; Life Table; Microsimulation in Demographic Research; Period and Cohort Analysis in Demography; Population Dynamics: Classical Applications of Stable Population Theory; Population Dynamics: Mathematic Models of Population, Development, and Natural Resources; Population Dynamics: Momentum of Population Growth.

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