Population Dynamics: Momentum of Population Growth

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Abstract

This article summarizes the recent literature on population momentum, usually defined as the subsequent population growth or decline after replacement conditions have been achieved. We review total momentum, its constituent parts (nonstable and stable momentum), trends over time, and how momentum is affected by immigration and emigration. Numerical examples are provided. The unifying concept is that momentum is created by a deviation between a population's observed age distribution and its underlying stationary (or sometimes stable) age distribution, especially at younger ages.

Introduction

It is natural to assume that, in a closed population, the rate of population growth is determined by the level of fertility and mortality. After all, the observed growth rate at time t, $r(t)$, is simply the difference between the crude birth rate, $b(t)$, and the crude death rate, $d(t)$, or

$$r(t) = b(t) - d(t)$$

It is true that in a long run stable equilibrium population the intrinsic growth rate, r, depends only on the fertility schedule, $m(a)$ – the annual rate of bearing daughters for women at exact age a; and on the survival function, $p(a)$ – the probability of surviving from birth to exact age a, given the underlying force of mortality schedule, $\mu(a)$ – the annual rate of dying for women at exact age a. In the short run, however, the shape of the current age distribution also matters.

If the proportion of women in the female population at age a and time t is given by $c(a, t)$, then

$$b(t) = \int_0^{\omega} c(a,t)m(a)\mathrm{d}a$$

where ω is the oldest age attained, and

$$d(t) = \int_0^{\omega} c(a,t)\mu(a)\mathrm{d}a$$

so that

$$\begin{aligned} r(t) &= \int_0^{\omega} c(a,t)m(a)\mathrm{d}a - \int_0^{\omega} c(a,t)\mu(a)\mathrm{d}a \\ &= \int_0^{\omega} c(a,t)[m(a) - \mu(a)]\mathrm{d}a \qquad [1] \end{aligned}$$

The term in square brackets in eqn [1] is negative below the age when childbearing begins (α) and above the age when it ends (β), and is usually positive in between. This means that all other things being equal, the so-called middle-heavy age distributions that have relatively large numbers of women at or near the peak ages of childbearing tend to maximize population growth rates. Even when a schedule of fertility at replacement, $m_0(a)$, is substituted for $m(a)$ in eqn [1], the rate of population growth at time t will not necessarily be zero, unless $c(a, t)$ is the age distribution of the stationary population.

Here is where the idea of population momentum enters. Equation [1], with $m_0(a)$ substituted for $m(a)$, shows the contribution of age composition to population growth in a given year t when fertility is at replacement. Population momentum measures the cumulative contribution of age composition to population growth (or decline) in all future years after fertility has been set at replacement.

Total Population Momentum

An Empirical Approach

A population typically does not stop growing or declining the instant its fertility reaches replacement level. Instead, in a closed population, growth or decline slows gradually until a zero-growth stationary population is reached, in much the same way that a car gradually comes to a complete stop after the driver's foot is lifted from the accelerator (Schoen and Kim, 1991). The amount of momentum contained in a population's age distribution is usually measured in a relative sense by the ratio of the size of the long-run stationary population to that of the observed population when replacement fertility is first attained. Momentum coefficients in AD 1935 ranged from 1.08 in Austria, Belgium, and France to 1.60 in Puerto Rico and Honduras (Vincent, 1945). In AD 2005, momentum coefficients among United Nations countries fell between a low of 0.81 for Bulgaria and a high of 1.76 in Oman (Espenshade et al., 2011). Coefficients of momentum can be greater than 1.0 (indicating positive momentum) or less than 1.0 (when momentum is negative). Momentum can play a large or a small role in population dynamics (Bongaarts, 1994, 2007).

It may seem paradoxical that population growth (or decline) does not necessarily cease the moment replacement fertility is reached. Consider two examples. Suppose, first, the case of a rapidly growing population in which the total fertility rate (TFR) has been above five for the past several generations. In this situation, young daughters are numerous relative to the number of mothers, and mothers are in plentiful supply relative to the number of grandmothers. Now suppose that fertility is suddenly reduced to replacement and the net reproduction rate (NRR) is 1.0. Going forward, girls in the population who have not yet entered childbearing will have only enough daughters to replace themselves, so that eventually there will be roughly equal numbers of young daughters and mothers (allowing for mortality). But the population of grandmothers swells in the short run as their relatively more numerous daughters age into their later years (Preston, 1986). The total population continues to grow for a while after fertility has been lowered, and the growth is largely concentrated at the older ages.

Alternatively, consider a population that has experienced a sharp fertility decline to below replacement levels, or one where, hypothetically, fertility has been below replacement for a very long time. Children in this population are likely to be relatively few compared to the number of mothers. If fertility is raised instantaneously to replacement, then going forward these children will replace themselves as they move through their childbearing years. But when they age into their later years, they will succeed larger cohorts of their mothers. The ensuing population decline will be largely concentrated at the older ages.

These two situations are illustrated in Table 1, which contains data for the female populations of Nigeria and Bulgaria. Nigeria's female population in the mid 2010s totaled 78.2 million. Its TFR for the previous 5 years was 5.61; and this high level of fertility had been sustained for many years, with only small declines from above 6.0 in the past decade (United Nations, 2011). The population is approximately stable, as indicated by the similarity of the observed annual rate of population growth (2.56%) compared to the intrinsic growth rate (2.49%). Nigeria's coefficient of population momentum for AD 2010 was calculated by projecting the AD 2010 population forward 300 years by which time a stationary population had been achieved, using observed rates for AD 2005–10 and then setting fertility to replacement by normalizing the maternity schedule by the NRR. A stationary population with 114.0 million women implies a momentum coefficient of $114.0/78.2 = 1.457$. The impact of age composition alone is sufficient to propel population growth, resulting in an eventual stationary population nearly 46% larger than the observed population in 2010. It is clear from Table 1 that this growth is not shared equally by all age groups, but is concentrated above about age 30.

Bulgaria, also shown in Table 1, represents a country with a dramatic fertility collapse. As recently as AD 1985–90, Bulgaria's TFR was 1.92, just below replacement, and it had been comfortably above replacement for most prior years dating back to AD 1950. But the TFR fell to 1.51 in AD 1990–95 and further to between 1.2 and 1.3 for the period AD 1995–2005. By AD 2005–10 it had rebounded slightly to 1.46 (United Nations, 2011). This fertility decline is reflected in Bulgaria's female age distribution for AD 2010. Just 12.9% of its population is under age 15 years, compared to 42.4% in Nigeria. The observed annual growth rate was −0.59% and the intrinsic rate was −1.36%. The momentum coefficient for Bulgaria shows that, even if fertility were raised immediately to replacement and remained fixed, the female population would nevertheless decline from 3.9 million in AD 2010 to 3.0 million – a decline of more than 22%. The overall momentum coefficient is 0.777, but the values for individual age groups indicate that, as anticipated, the shrinkage is confined to ages 20 years and above.

An Analytic Approach

The previous discussion has suggested that what determines total population momentum is the shape of a population's observed

Table 1 Population momentum values for Nigerian and Bulgarian females, 2010[a]

	Nigeria			*Bulgaria*		
Age	*Observed population*	*Stationary population*	*Stationary ÷ observed*	*Observed population*	*Stationary population*	*Stationary ÷ observed*
0–4	12 974.3	9479.8	0.731	181.2	194.4	1.073
5–9	10 913.5	9187.8	0.842	160.3	194.2	1.211
10–14	9276.4	8983.1	0.968	157.6	194.0	1.231
15–19	8031.5	8819.3	1.098	194.4	193.7	0.996
20–24	7142.1	8633.2	1.209	245.6	193.3	0.787
25–29	6219.1	8365.7	1.345	256.3	192.8	0.752
30–34	5158.9	8002.5	1.551	279.3	192.2	0.688
35–39	4016.9	7562.8	1.883	273.1	191.4	0.701
40–44	3152.4	7103.7	2.253	258.1	190.0	0.736
45–49	2688.9	6682.5	2.485	258.8	187.6	0.725
50–54	2303.9	6295.9	2.733	269.4	184.0	0.683
55–59	1925.6	5881.9	3.055	276.4	178.7	0.647
60–64	1543.5	5358.5	3.472	280.3	171.4	0.611
65–69	1205.2	4658.2	3.865	222.6	160.0	0.719
70–74	832.4	3751.8	4.507	197.8	140.6	0.711
75–79	495.1	2679.3	5.412	178.5	111.8	0.626
80–84	237.8	1591.4	6.692	115.5	75.1	0.650
85+	104.5	970.4	9.283	66.3	62.0	0.935
Total	78 222.0	114 008.0	1.457	3 871.5	3 007.1	0.777

[a]Female population in thousands.
United Nations, 2011. World Population Prospects: The 2010 Revision. Department of Economics and Social Affairs, Population Division. Retrieved from: http://esa.un.org/wpp/index.htm (on 22.05.13.) and authors' calculations.

age distribution in relation to the age distribution of the long-run stationary population. Young populations, especially those with high proportions below age 15 years, tend to have a substantial amount of built-in positive momentum, whereas older populations with relatively few children typically have latent population decline even after fertility is raised to replacement.

These notions are formalized in the following analytic expression for total population momentum (Espenshade et al., 2011).

$$\text{Total Momentum} = \int_0^\beta \frac{c(x)}{c_0(x)} \int_x^\beta p(a)m_0(a)\mathrm{d}a\mathrm{d}x/A_0 \qquad [2]$$

where $c(x)$ is the observed proportionate age distribution, A_0 is the mean age of childbearing in the long-run stationary population, and $c_0(x)$ and $m_0(a)$ are the stationary population counterparts to $c(x)$ and $m(a)$. The subscript 0 is used here to indicate replacement values in the stationary population where $r = 0$ by definition. An identical formula for total momentum appears in Preston and Guillot (1997: pp. 20–21), and it was anticipated in somewhat different form by Keyfitz (1985: pp. 155–157).

Equation [2] shows that relative deviations between a population's observed age distribution, $c(x)$, and its stationary population age distribution, $c_0(x)$, below the oldest age of childbearing, β, largely determine population momentum. Young populations (in relation to their stationary forms) will contain large amounts of positive momentum; whereas populations with relatively few young people in relation to the stationary population will have negative momentum. In addition, the second integral in eqn [2] is a weighting function. These weights sum to unity and are greatest before the onset of childbearing. Therefore, the deviations between $c(x)$ and $c_0(x)$ that matter most are those that occur at the youngest ages. Finally, it is apparent from eqn [2] that if the observed population is already stationary so that $c(x)/c_0(x) = 1.0$ everywhere, then the coefficient of momentum is identically 1.0 because all momentum has been wrung out of the age distribution (Espenshade et al., 2011; Preston and Guillot, 1997).

Disaggregating Population Momentum

Nonstable Momentum

Total population momentum, or the cumulative contribution of age composition to the change in future population size, has two principal parts. The first part involves the transition to a stable equilibrium population. Imagine an initial arbitrary female population with size P and proportionate age distribution $c(x)$. We know from eqn [1] that the observed rate of population growth is influenced by the shape of $c(x)$ along with patterns of fertility and mortality. Let the contribution of fertility and mortality to $r(t)$ be captured by the intrinsic growth rate, r. Then the residual $r(t) - r$ measures the contribution of age composition at time t. If fertility and mortality are held constant, $r(t)$ will converge to r. But in the meantime, the population grows or declines due to age composition alone at rate $r(t) - r$. By the time $r(t) = r$ so that $r(t) - r = 0$, the population will have reached size Q due solely to the effects of age composition.

Q will be a stable population. In fact, it is called the stable equivalent population, because if populations P and Q are projected forward using fertility and mortality observed at time $t = 0$, they will eventually become indistinguishable in terms of size, age composition, and rate of growth (or decline). Another way to see how Q is determined is to imagine projecting population P forward using constant fertility and mortality until it becomes a stable population, and then reverse projecting the resulting stable population back to time $t = 0$ using the intrinsic growth rate, r.

The first part of total population momentum, then, is the ratio Q/P. It is called 'nonstable' momentum. It is the transient or ephemeral part of total momentum, because it becomes smaller and smaller (and eventually disappears) as the initial population P gets closer to a stable population. If P were stable to begin with, then $P = Q$, and there would be no nonstable momentum.

Formally, nonstable momentum can be written as

$$\begin{aligned}\text{Nonstable Momentum} &= \frac{Q}{P} \\ &= \int_0^\beta \frac{c(x)}{c_r(x)} \int_x^\beta e^{-ra} p(a)m(a)\mathrm{d}a\mathrm{d}x/A_r\end{aligned} \qquad [3]$$

where $c_r(x)$ is the stable age distribution corresponding to intrinsic growth rate r, and A_r is the mean age of childbearing in the stable population (Espenshade et al., 2011). Nonstable momentum is a weighted average of relative deviations between the observed age distribution and the implied stable age distribution. Because the weights (in the second integral) are nonincreasing, the deviations that matter most are those in the early years of life before childbearing begins. The importance of nonstable momentum has been recognized by Bourgeois-Pichat (1971), Espenshade and Campbell (1977), Feeney (2003), and Schoen and Kim (1991).

Stable Momentum

Instead of starting with an arbitrary female population at time $t = 0$, suppose we begin with a stable population (for example, population Q). Now find momentum in the usual way by setting fertility at replacement, holding fertility and mortality constant, and projecting the population until it becomes stationary with eventual constant size S_2. Then stable momentum is the ratio S_2/Q, or,

$$\begin{aligned}\text{Stable Momentum} &= \frac{S_2}{Q} \\ &= \int_0^\beta \frac{c_r(x)}{c_0(x)} \int_x^\beta p(a)m_0(a)\mathrm{d}a\mathrm{d}x/A_0\end{aligned} \qquad [4]$$

It is called 'stable' momentum because, just like other stable population concepts, it depends only on fertility and mortality schedules and not on the current age distribution (Espenshade et al., 2011). The value for stable momentum will depend on relative deviations between the stable and stationary age distributions, with the youngest ages carrying the most weight.

Keyfitz (1971) popularized the concept of population momentum in a stable population. He showed that, starting with a stable population, total momentum is the ratio

$$\frac{S_2}{Q} = \frac{b\mathring{e}_0}{rA_0R_0}(R_0 - 1) \quad [5]$$

where b is the stable birth rate, R_0 is the initial NRR, and $\mathring{e}_0$ is life expectancy at birth. It is straightforward to see that eqns [4] and [5] are identical. Alternative formulations for momentum in initially stable populations have been proposed by Goldstein and Stecklov (2002), Kim and Schoen (1993, 1997), and Schoen and Jonsson (2003). Some of this work assumes that fertility moves gradually (and not instantaneously) to replacement.

Fitting the Pieces Together

Return to the initial, arbitrary population P and recall how total momentum is calculated. Put fertility at replacement, hold fertility and mortality constant, and project P until it becomes stationary with size S_1. Total momentum is then S_1/P. The identity

$$S_1/P \equiv Q/P \times S_2/Q \times S_1/S_2$$

tells us that total momentum can be factored into the product

$$\text{Total Momentum} \equiv \text{Nonstable Momentum} \times \text{Stable Momentum} \times S_1/S_2 \quad [6]$$

where S_1/S_2 is an offset factor. But if $S_1 = S_2$ we would have achieved an exact factorization of total momentum into the product of nonstable and stable momentum.

On the basis of an analysis of 176 UN countries, Espenshade et al. (2011) concluded that S_1 and S_2 are almost identical. In roughly 40% of the cases, the deviation was less than $\pm 0.1\%$. In two-thirds of all cases, the deviations were contained within $\pm 0.2\%$. And in three-fourths of all cases, the relative difference between S_1 and S_2 lay within $\pm 0.3\%$. One implication is that the product of nonstable and stable momentum is an extremely good approximation to total momentum. More important, the identity in eqn [6] provides a unifying framework within which the types of momentum that various authors have considered can be situated.

There are several special cases in which S_1 is identically equal to S_2 – if the initial population is already stable or if current fertility is already at replacement. Finally, there is the degenerate case where fertility is at replacement and the population is stationary. In this instance, there is no momentum of any kind: neither total, nor nonstable, nor stable (Espenshade et al., 2011).

There are distinct regional patterns for total momentum and its constituent parts. Total momentum in AD 2005 for more-developed countries (MDCs) was typically near 1.0 and frequently below it, whereas total momentum for the least-developed countries (LDCs) clustered around 1.5. Stable momentum was usually below 1.0 and often near 0.5 for MDCs, but close to 1.5 for LDCs. Nonstable momentum, on the hand, was near 1.5 for MDCs but closer to 1.0 for LDCs. Other developing countries had momentum values intermediate between those for MDCs and LDCs. This analysis shows that as the level of development increases, overall momentum values fall, nonstable momentum becomes stronger, and stable momentum grows weaker (Espenshade et al., 2011).

Population Momentum Over Time

Population momentum is usually considered as a static concept, calculated for a given point in time. Blue and Espenshade (2011) describe the arc of momentum over time in 16 populations, selected to represent different stages of the demographic transition. These stages are divided into five categories: (1) pretransition, when birth and dates rates are uniformly high, (2) early transition, with falling mortality and still relatively high fertility, (3) mid-transition, characterized by a robust rate of natural increase and the maximum difference between birth and death rates, (4) late transition, when falling fertility approaches low mortality, and (5) the second demographic transition, with low birth and death rates, and subreplacement fertility.

It is difficult to identify today any national population that falls into Stage 1 of the demographic transition, because no country remains untouched by improvements to mortality. There are, however, numerous examples of countries in each of the four later stages. Blue and Espenshade (2011) rely for their examples on Ethiopia, Haiti, Kenya, and Nigeria for Stage 2, on Egypt, India, and South Africa for Stage 3, on Argentina, Brazil, Indonesia, and Turkey for Stage 4, and on Belgium, Finland, France, Japan, and Sweden for Stage 5.

Measures of total population momentum, calculated at 5-year intervals starting as early as reliable data permit and continuing with projected data to AD 2050, support the following conclusions. Especially for the examples from developing countries, total momentum tends to rise at the beginning of the period of observation as mortality decline (with nearly constant fertility) creates a younger observed population and an older stationary population, thereby widening relative deviations between $c(x)$ and $c_0(x)$. Later, as fertility falls, total momentum also falls. Toward the end of the transition, if fertility appears to stabilize at below-replacement levels, then total momentum values stabilize below 1.0.

In particular, results for all 11 developing countries that comprise the examples for transition Stages 2, 3, and 4 are quite similar. In each case, total momentum tends to peak in an interval between 1.4 and 1.6. The main difference among the stages is in the timing of transition. The peak occurs about AD 1980 for countries in Stage 4, about AD 1995 for Stage 3 countries, and is projected to arrive about AD 2015 for countries now in the earliest stages of the demographic transition. For the five developed countries in Stage 5, the patterns are less uniform. The transition occurred more gradually, and therefore the rise and fall in total momentum also occurred more slowly. In addition, the peak momentum value for Stage 5 countries tends to be no higher than about 1.2. Japan appears to be a conspicuous outlier. Total population momentum was about 1.4 for Japan in AD 1950. But because of Japan's sudden fertility collapse, total momentum fell to roughly 0.8 by AD 2000 and is projected to decline further to close to 0.6 by AD 2045.

Blue and Espenshade (2011) also traced the evolution of stable and nonstable momentum in these 16 countries. If we imagine values for stable momentum graphed along a horizontal axis and those for nonstable momentum along a vertical axis, then demographic transition theory predicts that coordinates for stable and nonstable momentum will emanate from the origin (a value of 1.0 for both stable and nonstable momentum), form a counter-clockwise swirl pattern, and eventually return back to the origin (if fertility at the end of the demographic transition returns to replacement level) or to a point on the horizontal axis to the left of the origin if end-of-transition fertility stabilizes below replacement. Developing countries may exhibit larger swirls than those in developed countries because the former typically pass through the demographic transition more quickly.

The evidence we have supports the counter-clockwise swirl pattern for developing countries (especially those not hit hard by HIV), because these countries have largely avoided major fertility reversals. For example, we pick up the data for India in AD 1950 when stable momentum is 1.24 and nonstable momentum is 0.96. The curve then heads off to the Northeast until AD 1975, when total momentum reaches 1.5, stable momentum equals 1.43, and nonstable momentum is 1.05. From that point, the curve changes direction heading in a Northwesterly direction until AD 2030 when stable momentum will have fallen back to 0.82 and nonstable momentum is at its maximum (for India) of 1.44. Beyond that point, the momentum swirl heads southward. There is little further change in stable momentum, though nonstable momentum declines to 1.21 by AD 2050. Developed countries tend to follow this pattern, too, but only up to about AD 1950. The ensuing fertility swings from the baby boom inject some jagged fluctuations into these patterns along a vector running from the Northwest to the Southeast.

Population Momentum Incorporating Migration

This section draws on recent work by Espenshade and Tannen (2013). Discussions of population momentum typically assume a closed population. When these models are extended to include immigration and emigration, however, new possibilities emerge. In this expanded context, a given population can be associated with a range of momentum values depending on how the task of reaching replacement conditions is shared between fertility change and immigration change.

In the expanded framework, the survival function is no longer the survival function, $p(a)$, described above. Instead, one can exit a population by dying or by emigrating, and one can enter by immigrating above age 0. To accommodate these possibilities define a migration-inclusive survival function $\pi(a)$ as

$$\pi(a) = e^{-\int_0^a (\mu(x)+\nu(x)-\iota(x))\mathrm{d}x} \qquad [7]$$

with mortality rates $\mu(x)$, in-migration rates $\iota(x)$, and out-migration rates $\nu(x)$. Notice that, unlike $p(a)$, $\pi(a)$ can increase over some age intervals if the immigration rate exceeds the sum of mortality and out-migration rates, and it can even exceed 1.0. But eventually $\pi(a)$ must return to zero. If we define maternity rates $m(a)$ as before, then a population of size P with proportionate age structure $c(x)$ has an intrinsic growth r^* that satisfies

$$\int_0^\beta e^{-r^*a}\pi(a)m(a)\mathrm{d}a = 1 \qquad [8]$$

In closed-population scenarios, replacement fertility is generally achieved by dividing the maternity schedule by the NRR, effecting the same proportionate fertility change (higher or lower) at all ages. Here, if replacement conditions are attained entirely through adjustments to fertility, suppose fertility is modified according to

$$m_0(a) = e^{-r^*a}m(a)$$

Alternatively, suppose fertility is left unchanged and the immigration rate schedule is moved up or down by a constant amount r^* at all ages to achieve replacement. In particular, let

$$\begin{aligned}\pi_0(a) &= e^{-r^*a}\pi(a)\\ &= e^{-\int_0^a (\mu(x)+\nu(x)-[\iota(x)-r^*])\mathrm{d}x}\end{aligned}$$

These two extreme cases can be combined into a set of fertility and migration-inclusive survival functions that together achieve replacement. There are, in fact, many such combinations that jointly satisfy replacement conditions. Define new replacement-level fertility and migration-inclusive survival functions as

$$m_0(a|\gamma) = e^{-(1-\gamma)r^*a}m(a) \qquad [9]$$

$$\pi_0(a|\gamma) = e^{-\gamma r^*a}\pi(a) \qquad [10]$$

where γ is a parameter that ranges between 0 and 1 and represents the proportion of the overall rate change absorbed by immigration. If $\gamma = 0$, then all of the work of reaching replacement is being done by fertility. If $\gamma = 1$, then all of the adjustment to replacement falls on immigration. When these two new schedules are used in tandem, they imply a migration-inclusive intrinsic growth rate r^* equal to zero.

In the context of open populations that incorporate migration, two aspects of the long-run stationary population need to be examined: the shape of its age distribution and its size. Both will depend on the particular value assigned to γ. The age distribution is

$$\begin{aligned}c_0(a|\gamma) &= b_0(\gamma)\pi_0(a|\gamma)\\ &= b_0(\gamma)e^{-\gamma r^*a}\pi(a)\end{aligned} \qquad [11]$$

where $b_0(\gamma)$ is the birthrate in the migration-inclusive stationary population and equal to $1/\int_0^\omega \pi_0(a|\gamma)\mathrm{d}a$.

How the age distribution responds to changes in γ will depend on the value of r^*. The age distribution becomes younger as γ varies from 0 to 1 whenever $r^* > 0$. For example, Iceland in AD 2008 had a value for $r^* = 0.015$. As γ increases from 0 to 1, the share of Iceland's migration-inclusive stationary population that is under age 15 years rises from 11.7 to 20.2%, while the share over age 75 years falls from 16.2 to 8.7%. On the other hand, if $r^* < 0$, then the stationary population becomes older with increases in γ. Germany had a negative r^*-value in AD 2009 of -0.011. As γ ranges from 0 to 1, the share of Germany's stationary population over age 75 years rises from 13.5 to 20.9%, while the percentage under age 15 years falls from 16.9 to 10.6. In the special case where $r^* = 0$, the shape of the stationary age distribution is invariant with respect to changes in γ.

Extending work by Preston and Guillot (1997) and Espenshade et al. (2011), we may define migration-inclusive momentum for a population with size P and age structure $c(x)$ as

$$M(\gamma) = \frac{S(\gamma)}{P} = \int_0^{\beta} \frac{c(x)}{c_0(x|\gamma)} \int_x^{\beta} e^{-r^* a} \pi(a) m(a) \mathrm{d}a \mathrm{d}x / A_0 \qquad [12]$$

where $S(\gamma)$ is the size of an eventual stationary population that is achieved after converting fertility and immigration instantaneously to γ-dependent replacement, and then holding fertility, mortality, and both in-migration and out-migration rates constant until an equilibrium stationary population is reached. A_0 is the mean age of childbearing in the long-run stationary population. Equation [12] indicates that γ influences total momentum only through the shape of the stationary population age distribution. The form of eqn [12] is much like eqns [2]–[4]. Total momentum in the migration-inclusive case is once again a weighted average of age-specific relative deviations between the observed age distribution and the long-run migration-inclusive stationary population age distribution, with weights that are largest below the beginning of childbearing and decline toward zero with advancing age.

Equation [12] also shows that, unlike the closed-population example, a given open population will typically have many different coefficients of momentum associated with it, depending upon the particular value assigned to γ. Moreover, when $r^* > 0$, $c_0(x|\gamma)$ becomes younger as γ increases, and, consequently, total momentum falls. To take the case again of Iceland in AD 2008 with an r^*-value of 0.015, total momentum declines from 1.66 to 1.08 as γ increases from 0 to 1. On the other hand, for negative values of r^*, total momentum rises as γ increases. In the case of Germany in AD 2009, for instance, migration-inclusive momentum goes up from 0.87 to 1.25 when γ ranges from 0 to 1. Finally, if $r^* = 0$, then the stationary population age distribution is invariant with respect to changes in γ, and total momentum will not change.

Espenshade and Tannen (2013) analyzed data for all European Union countries having complete fertility, mortality, immigration, and emigration data. Two important policy lessons can be drawn from their analysis. First, depending on how replacement conditions are achieved and, therefore, on the value of γ that is chosen, in roughly two out of every five of their 58 country-year observations, the long-run stationary population size can be larger or smaller than the current population size. There are 34 instances in which population momentum was consistently positive, that is, in which momentum coefficients were always greater than 1.0. In some cases, the variation in momentum was relatively small, as in the case of Denmark in AD 2010 (1.055, 1.059) or the Netherlands in AD 2009 (1.087, 1.105). But in others, as γ ranges between 0 and 1, there can be wide swings in momentum, as in Iceland AD 2008 noted above (1.082, 1.663) or in Luxembourg AD 2008 (1.096, 1.816). However, there are 24 country-year observations – 41% of the total – for which momentum can be positive or negative depending on the choice of γ. In Lithuania in AD 2010, for instance, migration-inclusive momentum is 0.57 when $\gamma = 0$, but 1.39 when $\gamma = 1$. With such wide variation, policymakers have considerable leeway when developing judgments about future population size and age composition.

Second, migration-inclusive population momentum is often grossly underestimated by total momentum calculated for a closed population, at least for these European Union countries. In almost 60% (34 out of 58 cases) of the country-years for which the Espenshade and Tannen (2013) study provides data, the value for closed-population momentum falls short of the open-population momentum range, and it does so by a substantial margin two-thirds of the time. Moreover, in slightly more than half of these cases (18 out of 34), both high and low open-population estimates suggest that momentum is positive, whereas the closed-population estimate points in a negative direction. Therefore, in 31% of all cases, both the magnitude and the direction of the closed-population momentum estimate give misleading information.

Future Research

Several new research directions are suggested by the current state of our knowledge. First, typically in closed populations, momentum estimates are produced by reducing fertility immediately to replacement by dividing the maternity schedule by the NRR. Mitra (1976) proposed moving to replacement by an abrupt, but exponential, fertility decline. Other authors have examined the effects of a gradual shift to replacement fertility (Frauenthal, 1975; Goldstein, 2002; Goldstein and Stecklov, 2002; Li and Tuljapurkar, 1999, 2000; O'Neill et al., 1999; Schoen, 2005; Schoen and Jonsson, 2003; Schoen and Kim, 1998). It may be useful to investigate gradual transitions to replacement in fertility–migration combinations.

Second, Rogers (1975) outlined a framework for thinking about population momentum in a multiregional setting. Rogers (1995), Rogers and Willekens (1978), and Schoen (2002, 2006) applied this framework to investigate momentum in a two-region, urban–rural context. Typically in this work, the fertility schedule bears the full responsibility for getting to replacement. It is worth extending the analysis to let fertility and migration rates share this responsibility in some proportion. In addition, Raymer et al. (2012) propose three alternatives for incorporating flows of international migrants into multiregional models. One alternative that they do not consider, but that may be worth examining, is simply to expand an N-dimensional multiregional system to $N + 1$ dimensions to incorporate the 'rest of the world'.

Third, analyses of stable and nonstable momentum have so far been limited to single-region, closed populations. It would be productive to study the behavior of these two constituent parts of total momentum in a multiregional setting.

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See also: Fertility Change: Quantum and Tempo; Population Dynamics: Probabilistic Extinction, Stability, and Explosion Theorems; Population Dynamics: Theory of Nonstable Populations; Population Dynamics: Theory of Stable Populations; Second Demographic Transition.

Bibliography

Blue, L., Espenshade, T.J., 2011. Population momentum across the demographic transition. Population and Development Review 37 (4), 721–747.

Bongaarts, J., 1994. Population policy options in the developing world. Science 263, 771–776.

Bongaarts, J., 2007. Population Growth and Policy Options in the Developing World. The Population Council, New York (Paper presented at the Beijing Forum, Beijing, November 1–4.).

Bourgeois-Pichat, J., 1971. Stable, semi-stable populations and growth potential. Population Studies 25 (2), 235–254.

Espenshade, T.J., Campbell, G., 1977. The stable equivalent population, age composition, and Fisher's reproductive value function. Demography 14 (1), 77–86.

Espenshade, T.J., Olgiati, A.S., Levin, S.A., 2011. On nonstable and stable population momentum. Demography 48 (4), 1581–1599.

Espenshade, T.J., Tannen, J.B.C., 2013. Population Momentum with Migration. Unpublished manuscript. Office of Population Research, Princeton University, Princeton, NJ.

Feeney, G., 2003. Momentum of population growth. In: Demeny, P., McNicoll, G. (Eds.), Encyclopedia of Population, vol. 2. Macmillan Reference USA, New York, pp. 646–649.

Frauenthal, J., 1975. Birth trajectory under changing fertility conditions. Demography 12 (3), 447–454.

Goldstein, J.R., 2002. Population momentum for gradual demographic transitions: an alternative approach. Demography 39 (1), 65–73.

Goldstein, J.R., Stecklov, G., 2002. Long-range population projections made simple. Population and Development Review 28 (1), 121–141.

Keyfitz, N., 1971. On the momentum of population growth. Demography 8 (1), 71–80.

Keyfitz, N., 1985. Applied Mathematical Demography, second ed. Springer-Verlag, New York.

Kim, Y.J., Schoen, R., 1993. Crossovers that link populations with the same vital rates. Mathematical Population Studies 4, 1–19.

Kim, Y.J., Schoen, R., 1997. Population momentum expresses population aging. Demography 34 (3), 421–427.

Li, N., Tuljapurkar, S., 1999. Population momentum for gradual demographic transitions. Population Studies 53 (2), 255–262.

Li, N., Tuljapurkar, S., 2000. The solution of time-dependent population models. Mathematical Population Studies 7 (4), 311–329.

Mitra, S., 1976. Influence of instantaneous fertility decline to replacement level on population growth: an alternative model. Demography 13 (4), 513–519.

O'Neill, B.C., Scherbov, S., Lutz, W., 1999. The long-term effect of the timing of fertility decline on population size. Population and Development Review 25 (4), 749–756.

Preston, S.H., 1986. The relation between actual and intrinsic growth rates. Population Studies 40 (3), 343–351.

Preston, S.H., Guillot, M., 1997. Population dynamics in an age of declining fertility. Genus 53 (3–4), 15–31.

Raymer, J., Abel, G., Rogers, A., 2012. Does specification matter? Experiments with simple multiregional probabilistic population projections. Environment and Planning A 44, 2664–2686.

Rogers, A., 1975. Introduction to Multiregional Mathematical Demography. John Wiley & Sons, New York.

Rogers, A., 1995. Multiregional Demography: Principles, Methods and Extensions. John Wiley & Sons, New York.

Rogers, A., Willekens, F., 1978. The spatial reproductive value and the spatial momentum of zero population growth. Environment and Planning A 10 (5), 503–518.

Schoen, R., 2002. On the impact of spatial momentum. Demographic Research 6 (3), 49–66.

Schoen, R., 2005. Intrinsically dynamic population models. Demographic Research 12 (3), 51–76.

Schoen, R., 2006. Dynamic Population Models. Springer, Dordrecht, The Netherlands.

Schoen, R., Jonsson, S.H., 2003. Modeling momentum in gradual demographic transitions. Demography 40 (4), 621–635.

Schoen, R., Kim, Y.J., 1991. Movement toward stability as a fundamental principle of population dynamics. Demography 28 (3), 455–466.

Schoen, R., Kim, Y.J., 1998. Momentum under a gradual approach to zero growth. Population Studies 52 (3), 295–299.

United Nations, 2011. World Population Prospects: The 2010 Revision. Department of Economics and Social Affairs, Population Division. Retrieved from: http://esa.un.org/wpp/index.htm (accessed on 22.05.13.).

Vincent, P., 1945. Potentiel d'accroissement d'une population [Growth potential of a population]. Journal de la Société de Statistique de Paris, 1re Série, Janvier-Février 86 (1–2), 16–39.