Population Dynamics: Two-Sex Demographic Models

Robert Schoen, Pennsylvania State University, University Park, PA, USA

This article is a replacement of the previous edition article by R.A. Pollak, volume 17, pp. 11773–11777, © 2001, Elsevier Ltd.

Abstract

Demographic models of marriage and fertility that incorporate the behavior of both males and females encounter the so-called 'two-sex problem.' That problem arises because observed one-sex male and female rates of marriage and birth are influenced by the total age–sex composition. The leading two-sex solution that has been advanced is the harmonic mean approach, which assumes that the sum of male and female marriage or birth rates is independent of such compositional effects. Although other serious alternatives have been proposed, the harmonic mean approach is based on the most plausible demographic foundation.

The marriage squeeze is the best known manifestation of two-sex population dynamics. Using the harmonic mean approach, analyses indicate that over the past 50 years marriage squeezes have had significant impacts on the timing of marriage. Nonetheless, marriage squeezes have had less influence on the probability of ever marrying, and are unlikely to have played a major role in large-scale social change.

Human populations have two sexes, males and females, but demographic models generally depict only one sex. The major reason is the so-called 'two-sex problem,' the fact that conventional marriage and birth rates are shaped by the composition of the entire population.

Here we examine the nature of the two-sex problem, alternative approaches that have been proposed to deal with it, and touch on some demographic models that incorporate the experience of both sexes. Then we consider the 'marriage squeeze' and the 'birth squeeze,' real world manifestations of two-sex demography, and examine their impact on population dynamics.

The Two-Sex Problem

To begin, let us consider a hypothetical population of 100 males and 100 females, which experiences 10 marriages during the course of a year. Conventional one-sex incidence/exposure rates of marriage are given by the number of marriages divided by the population at risk. Here, the male marriage rate, W_{m}, is $10/100 = 0.1$. That is identical to the female marriage rate, W_{f}. If the underlying propensity to marry is unchanged and the number of males and females continues to be equal, rates of 0.1 will continue to yield the annual number of marriages in the population.

Now assume that the propensity to marry and the size of the female population remain unchanged, but the size of the male population increases to 200. Applying the male rate to the male population gives 20 marriages, while applying the female rate to the female population yields 10 marriages. Given the traditional definition of marriage as involving one male and one female, it is clear that both the male and female rates cannot persist when relative population size changes. In a real population, we would expect the number of observed marriages to increase, but not to double, because the number of females did not double. Hence the male marriage rate would fall below 0.1 while the female marriage rate would rise above 0.1. The two-sex problem is to determine exactly how male and female marriage rates would change. While here we only consider traditional marriage, a 'two-sex' problem would still exist if partnerships were formed independent of gender, as long as propensities to partner varied by age or some other demographic characteristic.

The Harmonic Mean Solution

Schoen (1981) argued that if there were no change in the propensity to marry, the rise in the marriage rate for one sex would exactly offset the fall in the marriage rate for the other sex, leaving the sum of the male and female rates constant. That sum of male and female rates, termed the 'magnitude of marriage attraction,' would reflect the underlying propensity to marry between males and females, and would be independent of population composition. Magnitudes of marriage attraction, like rates, could be made specific to age and other characteristics of the men and women marrying. If two populations have the same magnitudes of marriage attraction, then, in every case, the sum of the characteristic-specific male and female marriage rates would be equal, even though the corresponding male and female rates might differ.

The above approach is a harmonic mean solution because the magnitude of marriage attraction, as the sum of male and female rates, equals the number of marriages divided by half the harmonic mean of the number of males and females. The magnitude of marriage attraction is zero if either the number of males or females is zero, and if one sex is large relative to the other, is proportional to the number in the less numerous sex. If the number in both sexes increases by factor f, then the number of marriages increases by factor f. The denominator of the magnitude of marriage attraction can also be interpreted as the total number of male–female pairs divided by the sum of the male and female populations. Hence the 'two-sex population' denominator is the average number of possible pairs per person.

An important property of a two-sex solution is sensitivity to the age–sex composition of the entire 'marriage market,' i.e., all persons eligible to marry. For example, the chance that a 25 year old man will marry a 24 year old woman should be influenced by competition from the number of men aged 24 and women aged 23, and indeed by the number of eligible men and women at all ages. It is not obvious that the harmonic mean approach, which determines the magnitude of marriage attraction from just the number of men aged 25 and women aged 24, does so. Yet, the harmonic mean approach is sensitive to the marriage market because of the intimate relationship between population stocks (numbers of persons) and flows (here, marriages).

Consider two points. First, the number of persons in the (midyear) population eligible to marry and the number of observed marriages reflect outcomes from the competitive environment of the marriage market. Second, recall that rates are based on the experience of a given year. Now assume that we are at the beginning of a year of observation and know the array of magnitudes of marriage attraction and the initial number of unmarried persons by age and sex. To calculate the number of marriages by age and sex, we need the midyear, not beginning of the year, populations. Those midyear populations are affected by the marriages that occur during the first half of the year, thus the size and composition of the midyear populations bears the imprint of the competitive context of the marriage market.

Because of its desirable properties, the harmonic mean approach is generally accepted as 'a' solution to the two-sex problem. However, there is no consensus as to 'the' solution, and a number of other serious approaches have been advanced.

Alternative Two-Sex Solutions

Schoen (1988: Chapter 6) reviewed a number of solutions to the two-sex problem that have been advanced. Here, we focus on 4 approaches: (1) other simple means or combinations of rates; (2) iterative proportional fitting; (3) panmictic circles; and (4) the birth matrix-mating rule approach. Since the essence of the two-sex problem is how population composition impacts male and female rates, methods based only on relative numbers of males and females are not explored.

Other Simple Means or Combinations of Rates

The simple arithmetic mean, i.e., half the sum of the male and female rates, is an obvious possibility. However, it has received little attention because it yields marriages even when there are no persons of one sex.

The geometric mean, i.e., the square root of the product of male and female rates, is a simple average that avoids the problem of marriages when there are no persons of one sex. Goodman (1967) discussed a geometric mean variant where the two sexes are given unequal weight, that is where the male rate would be raised to the power α $(0 \leq \alpha \leq 1)$ and the female rate would be raised to the power $(1 - \alpha)$. Parameter α would indicate the 'dominance' of one sex.

Rather than averaging male and female rates, one could simply take the minimum of the two rates. That 'minimum' approach avoids the problem of marriages in the absence of one sex, and has been incorporated into more sophisticated models.

Iterative Proportional Fitting

McFarland (1975) applied a variation of the statistical technique of iterative proportional fitting (IPF), also known as the Deming-Stephan procedure or biproportional adjustment. Essentially, IPF begins by creating a marriage array, i.e., a cross-tabulation of marriages by age of bride and age of groom. The rows of the array contain elements showing the numbers of marriages for a given age of the bride and each age of the groom, plus an additional element equal to the total unmarried female population that age minus the total number of women marrying at that age. The columns contain elements showing similar numbers of marriages for a given age of the groom, plus an additional element equal to the total unmarried male population at that age minus the number of men marrying at that age. The row and column sums of the array, i.e., the marginals, are thus the total marriage eligible populations at each age. To apply IPF, change the marginals to the values for the population whose marriages are to be found. Then repetitively, or iteratively, multiply each row of the array by a factor that will yield that row's marginal. Next multiply each column of the array by a factor that will yield that column's marginal. Continue until no further row or column adjustments are needed. The process will always converge to a unique solution, and the resulting array will give the desired distribution of marriages by age and sex. If only one marriage age is involved, IPF is identical to the geometric mean approach. IPF is quite flexible, preserves the cross-product ratios of the original array, and is a maximum entropy (or minimum information) solution.

Marriage within Panmictic Circles

Henry (1972) viewed the marriage process as one involving men and women meeting and marrying within overlapping 'circles,' or large social groupings, within which marriage is 'panmictic' in the sense that age does not influence partner choice. Henry set forth a process for dividing male and female population distributions into a series of such circles, arguing that a maximum of six circles would be sufficient to characterize national level data. Within circles, male and female age-circle-specific marriage rates can be calculated, and those rates applied to other populations that have been decomposed into panmictic circles. The male rate yields one number of marriages and the female rate yields another, with the minimum number used. The procedure is not readily visualized, but seems to respond to competition in the marriage market.

The Birth Matrix-Mating Rule Model

Pollak (1990), building on the work of Feeney (1972), advanced the birth matrix-mating rule (BMMR) model consisting of three fundamental components. The first component is a birth matrix, indicating the number of children born to each possible union type (e.g., males age y and females age x). The second is a mating rule, which gives the number of unions of each kind from the number of males and females in each category. The third has the applicable male and female age-specific death rates.

Under a number of possible mating rules, the BMMR model provides the long term male and female age distributions and the age pattern of marriage. The mating rule used must satisfy two fairly weak conditions: it must not yield negative numbers and it cannot yield more male (female) unions in an age category than there are males (females) in that category. The model yields a unique result if an increase in the number of persons in any age category does not decrease the number of births.

The BMMR model offers a framework for two-sex model solutions, rather than a specific solution. The harmonic mean, geometric mean, and IPF approaches, among others, can provide satisfactory rules whose demographic implications can be found using the BMMR approach.

Evaluating Alternative Two-Sex Solutions

Demography is empirically oriented, so it is logical to try and evaluate alternative approaches by applying them to actual data. The problem is that the standard for comparison – the correct result – is unknown. The most promising approach is to compare the same population over time, but that still requires the rather strong assumption that all of the propensities have remained constant. Schoen (1981) made such comparisons for Sweden, using single year of age male and female population and marriage data for the years 1961, 1962, 1963, and 1964. The harmonic mean, iterative adjustment, and panmictic circles approaches were each applied to the data, using 1961 data to predict 1962 marriages, 1962 data to predict 1963 marriages, and 1963 data to predict 1964 marriages. None of the three approaches performed well. All of them tended to be off in the same direction, but in an inconsistent and erratic fashion. A recourse to data thus seems unlikely to provide a useful evaluation of alternative two-sex solutions.

Another approach has been to posit a set axioms or conditions that a satisfactory method must satisfy, e.g., those set forth by Parlett (1972) or those required by the BMMR model. That approach has not worked well either, as no set of axioms has been generally accepted, and the leading two-sex solutions satisfy virtually all of the required properties that have been proposed.

There appears to be no alternative to evaluating two-sex solutions on theoretical or conceptual criteria. To be considered an acceptable solution, a two-sex approach should not only yield reasonable results empirically, but be based on a plausible, demographically meaningful rationale. In that respect, the harmonic mean emerges as the most satisfactory approach. No clear explanation has been offered for why the cross-product ratios of a marriage matrix should be preserved, why marriage should be independent of age within a few large 'circles,' or why the geometric mean captures the interaction between the sexes. In contrast, the harmonic mean method is based on magnitudes of attraction that reflect the marginal change in the number of marriages (or births) given a marginal change in the number of couples, and that imply a two-sex population at risk equal to the average number of pairs per person.

Two-Sex Demographic Models

Traditional one-sex demographic models can be extended to incorporate two-sex marriage or two-sex fertility. Along the three dimensions of single state/multistate, stationary/stable, and one-sex/two-sex, Schoen (1988: p. 120) identified six distinct two-sex nuptiality models and two two-sex fertility models.

The conventional, one-sex life table follows a cohort, or closed group of persons, over time, often recognizing more than one cause of decrement (loss) from the cohort. In particular, a one-sex nuptiality–mortality life table follows single person from birth to decrement from either first marriage or death, based on rates from a source population. The corresponding two-sex nuptiality–mortality life table follows a male cohort and a female cohort from birth to first marriage or death. The source population provides the one-sex male and female rates as well as the two-sex magnitudes of marriage attraction.

The classical one-sex stable population depicts the implications of a set of male or, more usually, female age-specific birth and death rates, showing the long term exponential population growth and constant age composition that those rates imply. Similarly, a two-sex (fertility) stable population shows the implications of the source population's male and female death rates and magnitudes of fertility attraction, i.e., sums of male and female age-specific birth rates.

A number of two-sex life table and stable population models have been constructed and analyzed, though such analyses are still infrequent. Two-sex modeling demands extensive input data, and is hampered by a lack of consensus on the appropriate two-sex methodology.

Manifestations of Two-Sex Behavior

Two-sex population dynamics, or the interaction between the sexes with regard to marriage and birth, is important to study because it influences marriage and fertility behavior. An imbalance between the number of men and women at the prime ages of marriage (or reproduction) alters the marriage (or fertility) rates of both sexes, and the effect can be substantial. A gender imbalance at the marriage ages produces what has been termed a 'marriage squeeze.' The analogous phenomenon, a 'birth squeeze,' can arise with respect to fertility. We examine each in turn, using measures based on the harmonic mean approach.

The Marriage Squeeze

Marriage squeezes typically arise when the number of births is rapidly rising (or falling). In most populations, men marry women several years younger than themselves. Thus when the number of births is increasing, men are looking for partners born in larger cohorts while women are looking for partners born in smaller cohorts. The result is a marriage squeeze disadvantaging females. If the number of births is falling, then the same process yields a marriage squeeze disadvantaging males.

Some observers have theorized that the marriage squeeze played a key role in social change during the twentieth century. Heer and Grossbard-Shechtman (1981) argued that a marriage squeeze against females reduced the proportion of US women who married during the 1960s, changing the 'terms of trade' between men and women and increasing women's labor force participation. Davis and van den Oever (1982) interpreted the feminist movement as a response to lower fertility, less marriage, and the marriage squeeze. Guttentag and Secord (1983) argued that whenever there are *Too Many Women* (the title of their book), as in the United States during the 1960s and 1970s, marriage and family life are devalued and sexual libertarianism prevailed. Such claims by serious researchers deserve careful consideration and analysis.

Because two-sex dynamics change one-sex rates, we want a marriage squeeze measure that reflects how compositional changes have influenced male and female behavior. One such measure is marriage squeeze index S (cf Schoen, 1988). Index S is the male life table number of male marriages, minus the female life table number of female marriages, divided by the total number of marriages in the two-sex nuptiality life table. If $S > 0$, there is a marriage squeeze against females, if $S < 0$, there is a marriage squeeze against males, and if $S = 0$, there is no marriage squeeze at all. The larger the magnitude of S, the greater the squeeze. For example, if there were 98 marriages in a male life table, 92 marriages in a female life table, and 96 marriages in a two-sex table, then $S = 6/96 = 0.0625$.

Schoen (1983) looked at index S in 25 countries during the period 1966–75, using data from the 1976 United Nations Demographic Yearbook. The largest marriage squeezes were found in Jordan 1970 ($S = 0.14$) and Costa Rica 1973 ($S = 0.12$). More developed countries had S values that were either small or negative, the most negative being Sweden 1975 ($S = -0.04$). Schoen and Baj (1985) examined trends in index S in five Western countries (Belgium, England and Wales, Sweden, Switzerland, and the United States) during the period 1910–75. In the post–World War II period, S values were small, though Switzerland's was about 0.03 and Sweden's became negative. However, in the 1910s and 1920s, England and Wales, Sweden, and Switzerland had S values of 0.125 or above.

Schoen (1983) also looked at the marriage squeeze and its implications for the United States during the period 1950–90, assuming that 1970 magnitudes of marriage attraction applied over that period. Index S was always quite small, varying between 0.0013 and 0.0073, showing a slight marriage squeeze against females. Closer examination, however, revealed significant changes in the distribution of male and female marriages. The male mean age at marriage varied by 0.52 years, the female mean age at marriage by 0.62 years, and the mean difference between partner marriage ages fluctuated between 1.84 and 2.99 years. The variance of the distribution of both male and female ages at marriage also changed substantially.

In sum, studies suggest that marriage squeezes of some magnitude do characterize the contemporary world. While marriage squeeze effects on the proportion ever marrying appear modest, at least in Western countries, they have exerted substantial influence on the timing and distribution of marriages, including the age difference between spouses. Still, the evidence falls short of supporting the marriage squeeze as a cause of large-scale social change. In the 1910s and 1920s, sizable marriage squeezes in the West passed with little comment, and most social change since the 1960s, when the marriage squeeze has been small, seems attributable to more fundamental social and economic transformations.

The Birth Squeeze

The same demographic dynamic that produces marriage squeezes produces birth squeezes, even when marriage is not a necessary societal prelude to fertility.

The standard measure of fertility is the total fertility rate (TFR_f), the sum of age specific female birth rates over the female reproductive age span. The TFR_f can be interpreted as the average number of children a woman would bear in her lifetime in the absence of mortality. Fertility analysis has traditionally emphasized female birth rates because of the narrower female reproductive age span and the fact that data on fathers are often incomplete. Though less frequently calculated, TFR_m, the male total fertility rate, is the sum of male fertility rates over the male life course.

The approach expressed here, i.e., that two-sex dynamics affect one-sex rates, but that the sum of male and female rates is composition independent, applies to fertility as well as marriage. Following Schoen (1985), define the two-sex total fertility rate, TFR2, as one-half the sum of TFR_m and TFR_f. Paralleling index S, birth squeeze index U can be defined as $(TFR_m - TFR_f)/TFR2$. When $U > 0$, there is a birth squeeze against females, when $U < 0$, there is a birth squeeze against males, and when $U = 0$, there is no birth squeeze.

Schoen (1985) examined index U for 22 countries during the 1963–74 period, using data from the United Nations Demographic Yearbooks. Developed countries in the sample typically had small or negative values of U. For the United States 1970, U was 0.12. Four countries, Phillipines 1973, Jordan 1974, Tunisia 1971, and Puerto Rico 1970, had U values over 0.25. The Phillipines 1973, where U was 0.30, was the highest, having a TFR_m of 4.6 and a TFR_f of 3.4.

Since population growth is associated with birth squeezes against females, it is not surprising that growing populations have sizable values of U. Less obviously, when $TFR2 > TFR_f$, an argument can be made that the conventional TFR is understating the population's actual propensity for fertility.

Conclusion

Conventional one-sex demographic rates are unable to capture the dynamics underlying two-sex marriage and fertility behavior, or to model such demographic phenomena as the marriage squeeze. The harmonic mean approach, the leading two-sex solution to date, affords an opportunity for such analyses. The results have provided strong evidence that marriage squeezes have had considerable influence on the timing and pattern of marriage, but little support for a belief that marriage squeezes have caused major societal change.

See also: Demographic Measurement: General Issues and Measures of Fertility; Demographic Measurement: Nuptiality, Mortality, Migration, and Growth; Demographic Techniques: Data Adjustment and Correction; Population Dynamics: Theory of Nonstable Populations.

Bibliography

Das Gupta, Prithwis, 1973. Growth of the U.S. population, 1940–1971, in the light of an interactive two-sex model. Demography 10, 543–565.

Davis, Kingsley, van den Oever, P., 1982. Demographic foundations of new sex roles. Population and Development Review 8, 376–395.

Feeney, Griffith, M., 1972. Marriage Rates and Population Growth: The Two-Sex Problem in Demography (Ph.D. thesis). University of California, Berkeley, CA.

Goodman, Leo A., 1967. On the age-sex composition of the population that would result from given fertility and mortality conditions. Demography 4, 423–441.

Guttentag, Marcia, Secord, Paul F., 1983. Too Many Women? The Sex Ratio Question. Sage, Beverly Hills.

Heer, David M., Grossbard-Shechtman, Amyra, 1981. The impact of the female marriage squeeze and the contraceptive revolution on sex roles and the women's liberation movement in the United States, 1960 to 1975. Journal of Marriage and the Family 43, 49–65.

Henry, Louis, 1972. Nuptiality. Theoretical Population Biology 3, 135–152.

Iannelli, Mimmo, Martcheva, Maia, Milner, Fabio A., 2005. Gender Structured Population Modeling: Mathematical Methods, Numerics, and Simulations. Society for Industrial and Applied Mathematics, Philadelphia.

McFarland, David D., 1975. Models of marriage formation and fertility. Social Forces 54, 66–83.

Parlett, Beresford, 1972. Can there be a marriage function? In: Greville, T.N.E. (Ed.), Population Dynamics. Academic, New York.

Pollak, Robert A., 1990. Two-sex demographic models. Journal of Political Economy 98, 399–420.

Schoen, Robert, 1981. The harmonic mean as the basis of a realistic two-sex marriage model. Demography 18, 201–216.

Schoen, Robert, 1983. Relationships in a simple harmonic mean two-sex fertility model. Journal of Mathematical Biology 18, 201–211.

Schoen, Robert, 1985. Population growth and the birth squeeze. Social Science Research 14, 251–265.

Schoen, Robert, 1988. Modeling Multigroup Populations. Plenum, New York.

Schoen, Robert, Baj, John, 1985. The impact of the marriage squeeze in five western countries. Sociology and Social Research 70, 8–19.