Overview of demographic methods

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Outline

- Exponential growth
  - Wachter 2014, Chapter 1, pp. 5–29

- Periods and cohorts
  - Wachter 2014, Chapter 2, pp. 30–47
  - Fleurence, Hollenbeak 2007
  - Weeks 2015, Chapter 7, pp. 251–297

Exponential growth

(Wachter 2014, Chapter 1, pp. 5–29)

- Balancing equation
- Population growth rate (R)
- Exponential function
- Doubling times

Balancing equation

- Balancing equation for the world, 2010–2011

$$K(2011) = K(2010) + B(2010) - D(2010)$$

  - $K(2010)$: world population at start of 2010
  - $B(2010)$: births during 2010
  - $D(2010)$: deaths during 2010
  - $K(2011)$: population at start of 2011

World population 2010 to 2011

Population 1 January 2010	**6,851 million**
+ Births 2010	+140 million
+ Deaths 2010	–57 million
= Population 1 January 2011	6,934 million

Source: 2010 Population Data Sheet of the Population Reference Bureau (PRB). Wachter 2014, p. 6.

General form of balancing equation

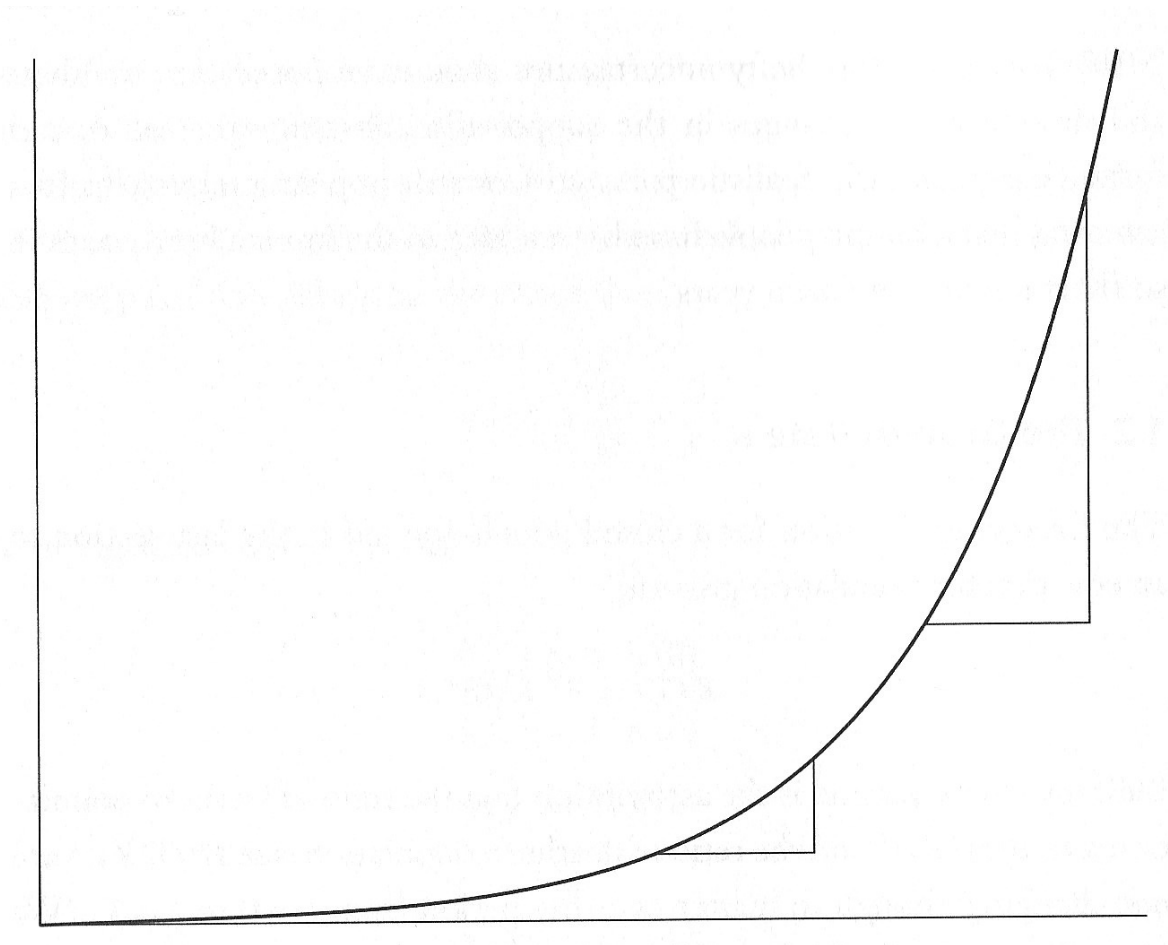
- For closed population

$$K(t + n)= K(t) + B(t) – D(t)$$

  – n: length of a period, e.g. 1 year or 10 years
  – $B(t)$, $D(t)$: births, deaths during period from t to $t+n$

- Equation for national or regional populations are more complicated due to migration
  – Closed population examples are used to understand concepts

K(t) with ever-changing slope



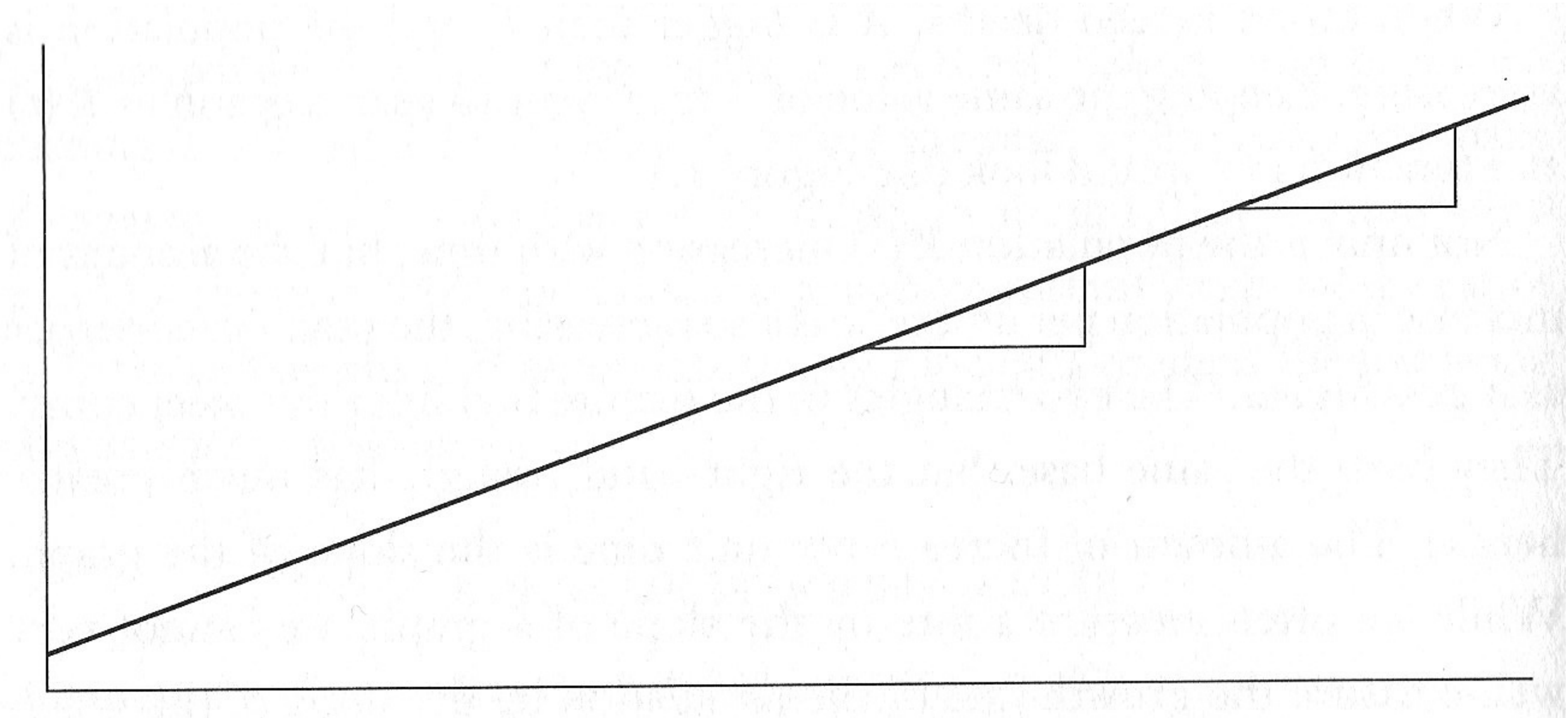
Source: Wachter 2014, p. 10.

Constant slope

- Previous graph, we cannot measure growth rate by graph slope, because it varies
  - Slope changes even when B/K and D/K are fixed

- We need a measure of growth that stays fixed when B/K and D/K are fixed
  - Take logarithms of $K(t)$
  - Usual way of converting multiplication into addition
  - log $K(t)$ versus t has constant slope...

Log *K(t)* with constant slope



Source: Wachter 2014, p. 10.

Population growth rate (R)

$$R = \frac{1}{n} log\left(\frac{K(t+n)}{K(t)}\right)$$

- *n*: length of a period, e.g. 1 year or 10 years
- *K*(*t*): population at the beginning of the interval
- *K*(*t*+*n*): population at the end of the interval

Example of slope R

Population 1 January 2010	**6,851 million**
+ Births 2010	+140 million
+ Deaths 2010	–57 million
= Population 1 January 2011	6,934 million

- R = $\log[K(t+n)/K(t)]$ / n = log(6,934/6,851) / 1 = 0.012042
- World population has been growing at a rate of about 12 per thousand per year between 2010 and 2011

Exponential function

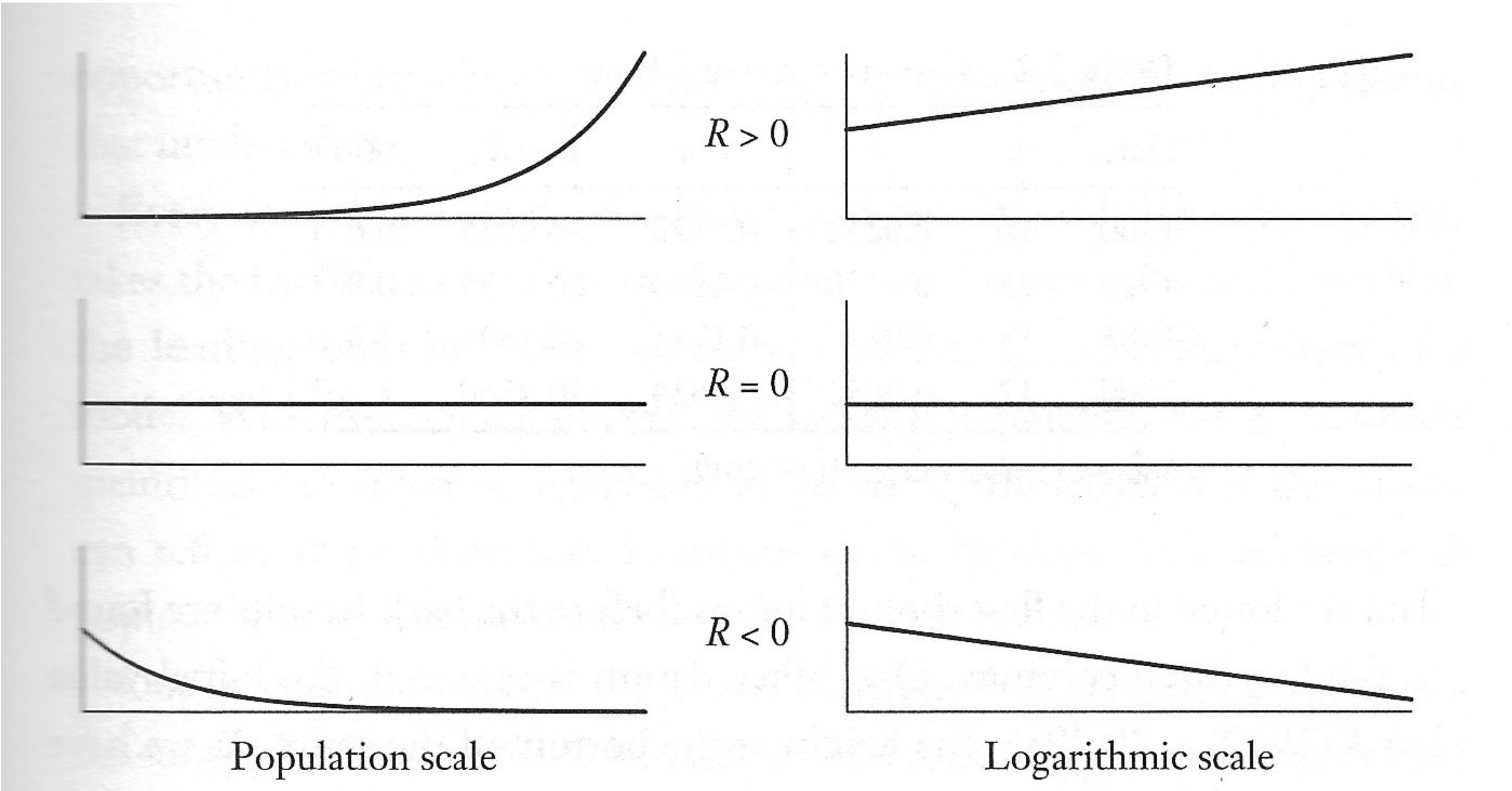
- Population over time when ratios of births and deaths to population remain constant

$$\log K(t+n) = \log K(t) + \mathrm{R}n$$

$$K(t+n) = K(t) \exp(\mathrm{R}n)$$

$$K(t+n) = K(t)\, e^{\mathrm{R}n}$$

Trajectories of exponential growth



Source: Wachter 2014, p. 15.

Doubling times

$$K(t+n) = K(t)\ \exp(\mathrm{R}n)$$

$$K(t_{\mathrm{double}}) = 2K(t) = K(t)\ \exp(\mathrm{R}t_{\mathrm{double}})$$

$$2 = \exp(\mathrm{R}t_{\mathrm{double}})$$

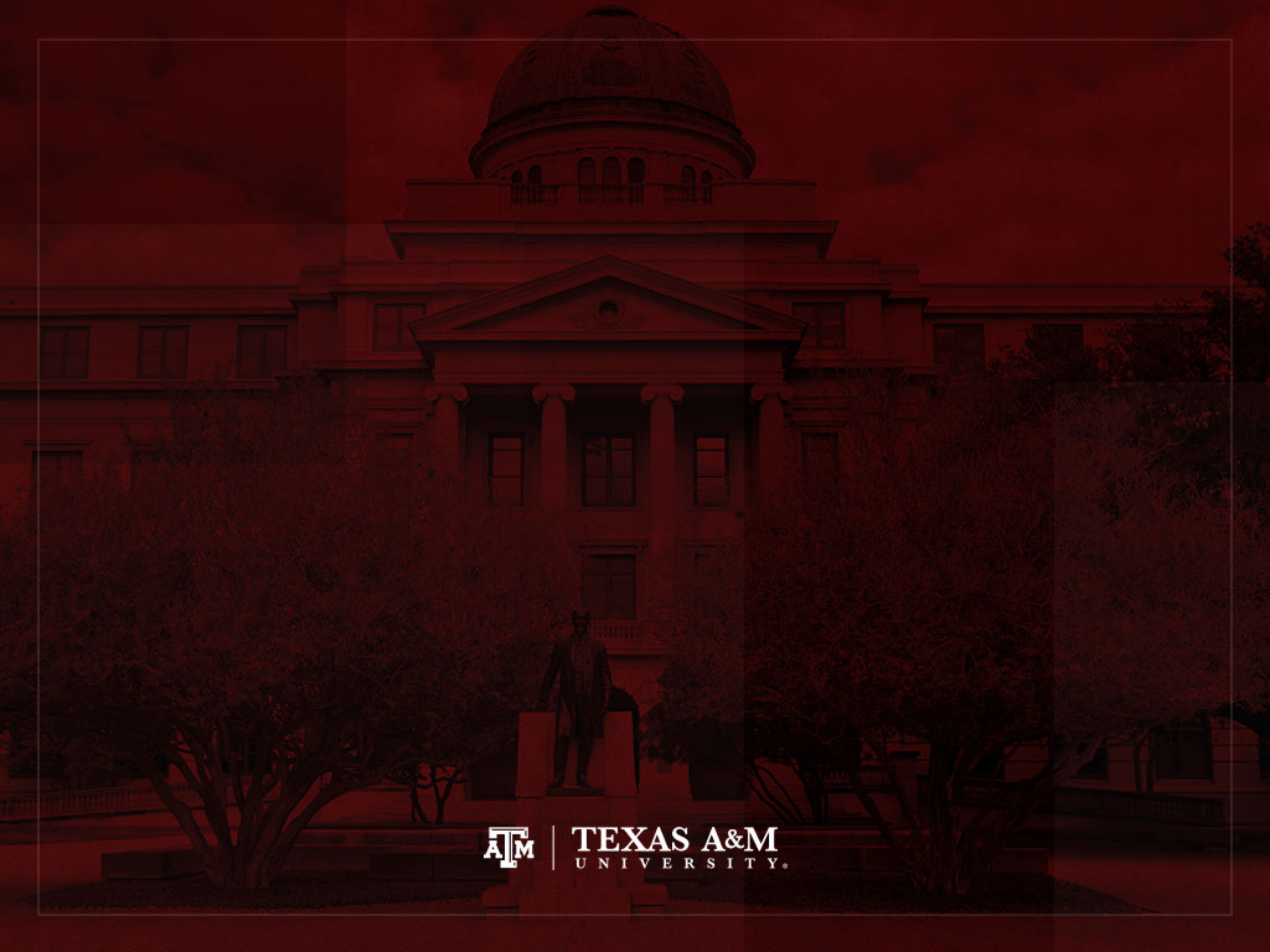
$$\log(2) = \mathrm{R}t_{\mathrm{double}}$$

$$t_{\mathrm{double}} = \log(2)\ /\ \mathrm{R} \approx 0.6931\ /\ \mathrm{R}$$

World population and doubling times

Date	Population	Growth rate (R)	Doubling time ~ (0.6931 / R)
8000 B.C.	5 million	0.000489	1417 years
1 A.D.	250 million	–0.000373	–1858 years
600	200 million	0.000558	1272 years
1000	250 million	0.001465	473 years
1750	750 million	0.004426	157 years
1815	1,000 million	0.006957	100 years
1950	2,558 million	0.018753	37 years
1975	4,088 million	0.015937	43 years
2000	6,089 million		

Source: Estimates drawn from Cohen (1995) and IDB (2012). Wachter 2014, p. 25.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Periods and cohorts

(Wachter 2014, Chapter 2, pp. 30–47)
(Fleurence, Hollenbeak 2007)

- Lexis diagram
- Ratios
- Rates, person-years
- Probabilities
- Stable and stationary populations

Exponential population growth model

- The exponential model treats all people as if they were alike
  - No mention to *age*
  - However, people are aging in the population
- Time enters demography in two ways
  - Chronological time: calendar dates, same for everyone
  - Personal time: age for each set of people who share same birthdate

Lexis diagram

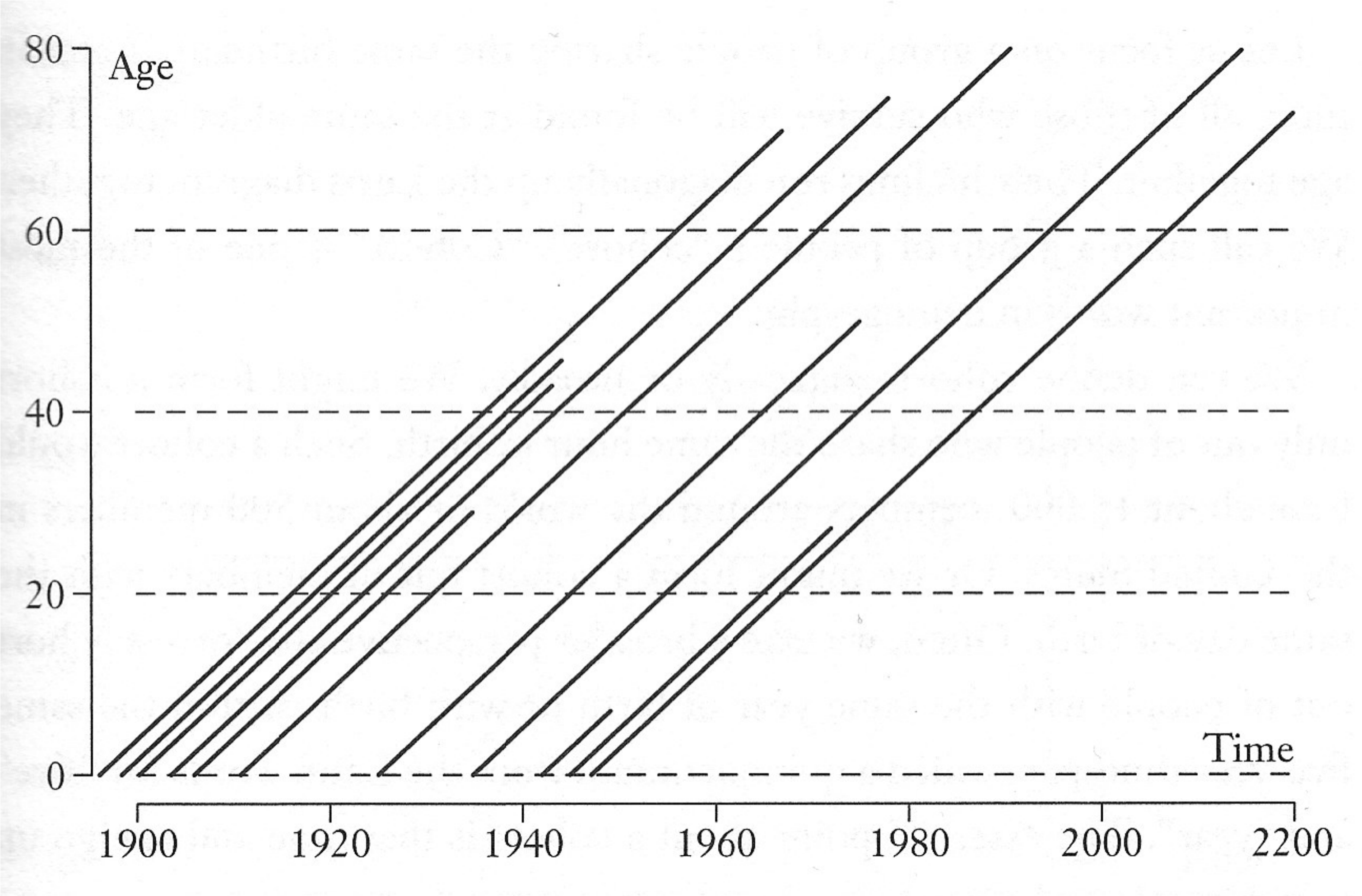
- Lexis diagram provides relationships between chronological time *t* (horizontal) and age *x* (vertical)

- Each person has a lifeline on a Lexis diagram
  - Starting at (t_b, 0), where t_b is the person's birthdate and 0 is the person's age at birth

- Line goes up to the right with a slope equal to 1
  - People age one year in one calendar year

- Lifeline goes up until time and age of the person's death

Lexis diagram



Source: Wachter 2014, p. 31.

Exploring Lexis diagram

- To find population size
  - Draw vertical line upward from the time point
  - Count how many lifelines cross vertical line

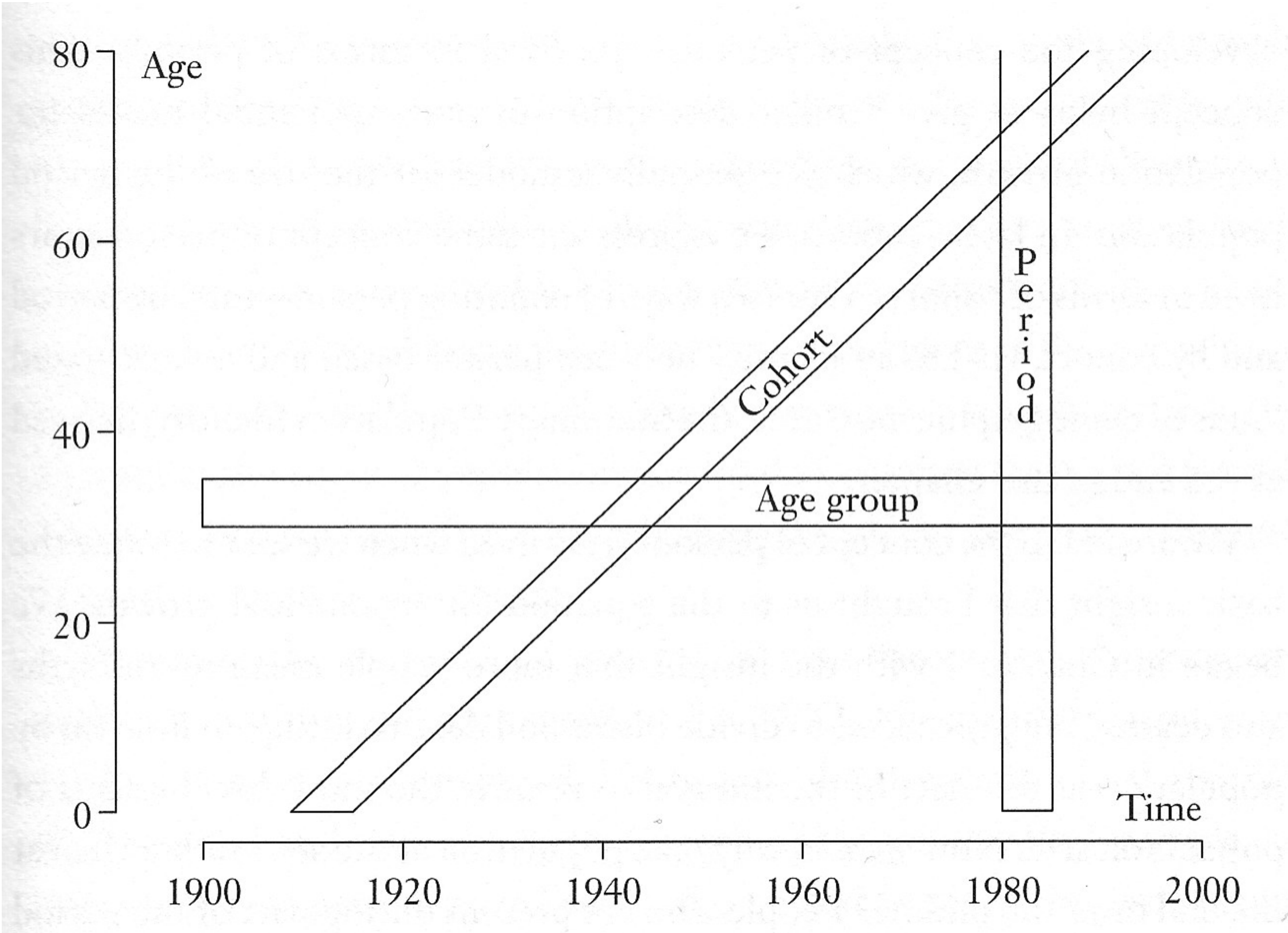
- To find how many people survive to some age
  - Draw horizontal line across at the height corresponding to that age
  - Count how many lifelines cross that horizontal line

- Immigrants start at age and time of immigration

Cohort

- Group of people sharing the same birthdate
- Group of individuals followed simultaneously through time and age
- Their lifelines run diagonally up the Lexis diagram together
- In a cohort, time and age go up together
- A cohort shares experiences

Age, period, cohort



Source: Wachter 2014, p. 33.

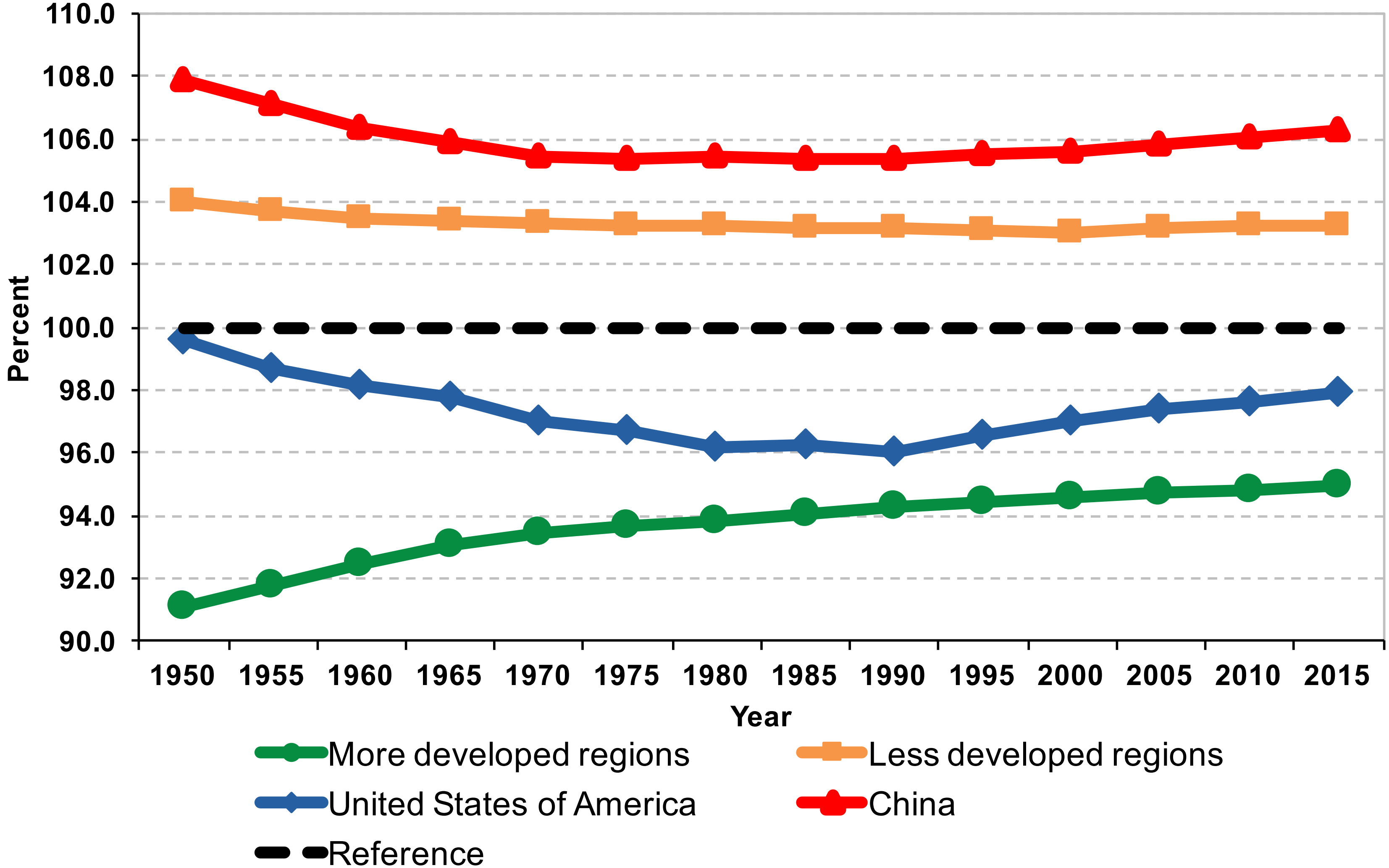
Ratios

- Describe a relationship between two numbers
  - Compare the size of one number to the size of another number
  - Compare the relative sizes of categories
  - Indicate how many times the first number contains the second
  - Denominator is not at “risk” of moving to numerator
  - Optional: multiply by 100 to get percentage

$$Sex\ ratio = \frac{Population\ of\ males}{Population\ of\ females}$$

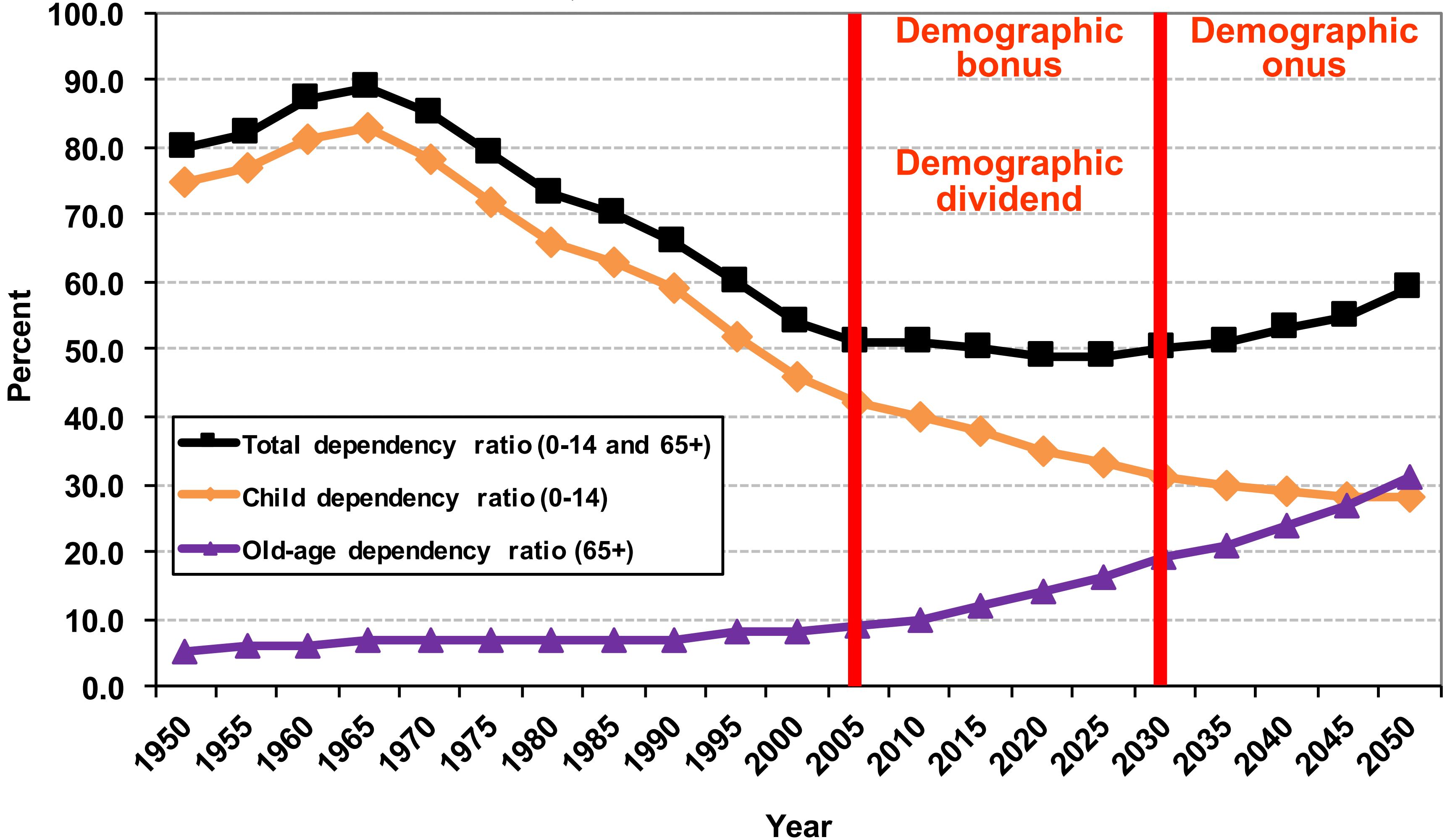
$$Total\ dependency\ ratio = \frac{Pop.\ children\ (0\ to\ 14)\ +\ Elderly\ pop.\ (65+)}{Working\ age\ population\ (15\ to\ 64)}$$

Sex ratios, 1950–2015



Source: United Nations, World Population Prospects 2017
https://esa.un.org/unpd/wpp/Download/Standard/Population/

Dependency ratios, Brazil, 1950–2050



Source: United Nations - http://esa.un.org/unpp (medium variant).

Rates

(Fleurence, Hollenbeak 2007)

- Rates are an instantaneous measure that range from zero to infinity
  - Rates describe the number of occurrences of an event for a given number of individuals per unit of time
  - Time is included directly in the denominator
  - Rates take into account the time spent at risk
- Incidence rate describes the number of new cases of an event during a given time period over the total **person-years** of observation
  - **Numerator**: number of events (e.g. births, deaths, migrations)
  - **Denominator**: number of "**person-years** of exposure to risk" experienced by a population during a certain time period

Person-years

- **Person-years** is the sum of each individual's time at risk of experiencing an event (e.g. birth, death, migration)
  - For those who do not experience event, person-years is the sum of time until end of period
  - For those who experience event, it is the time until the event

- **Period person-years lived** take into account that people are present during part of the period (fraction of years)
  - Each full year that a person is present in a period, he/she contributes one "person-year" to the total of PPYL
  - Each month a person is present in the population, he/she contributes 1 person-month or 1/12 person-year, to PPYL

Example of person-years

Hypothetical population increasing at the rate of 0.001 per month				
			Approximation for person-years	
Month	**Population**	**Person-years (population / 12)**	**Mid-period**	**Average of start and end**
January	200.00	16.67		200.00
February	200.20	16.68		
March	200.40	16.70		
April	200.60	16.72		
May	200.80	16.73		
June	201.00	16.75		
July	201.20	16.77	201.20	
August	201.40	16.78		
September	201.61	16.80		
October	201.81	16.82		
November	202.01	16.83		
December	202.21	16.85		202.21
Period person-years lived (PPYL)		**201.10**	**201.20**	**201.11**

Calculating person-years

- Whenever we know the population sizes on each month over the period of a year
  - We can add up the person-years month by month
  - Take the number of people present on each month and divide by 12
  - Add up all monthly contributions

- When our subintervals are small enough
  - Our sum is virtually equal to the area under the curve of population as a function of time during the period

Approximation for PPYL

- When sequence of population sizes throughout a period are unknown
- Take the population in the middle of the period and multiply by the length of the period
- Or take the average of the starting and ending populations and multiply by the length of the period

Examples of rates

- Express the number of actual occurrences of an event (e.g. births, deaths, homicides) vs. number of possible occurrences per some unit of time
- Examples

$$Crude\ birth\ rate = \frac{Number\ of\ births}{Total\ population} \times 1{,}000$$

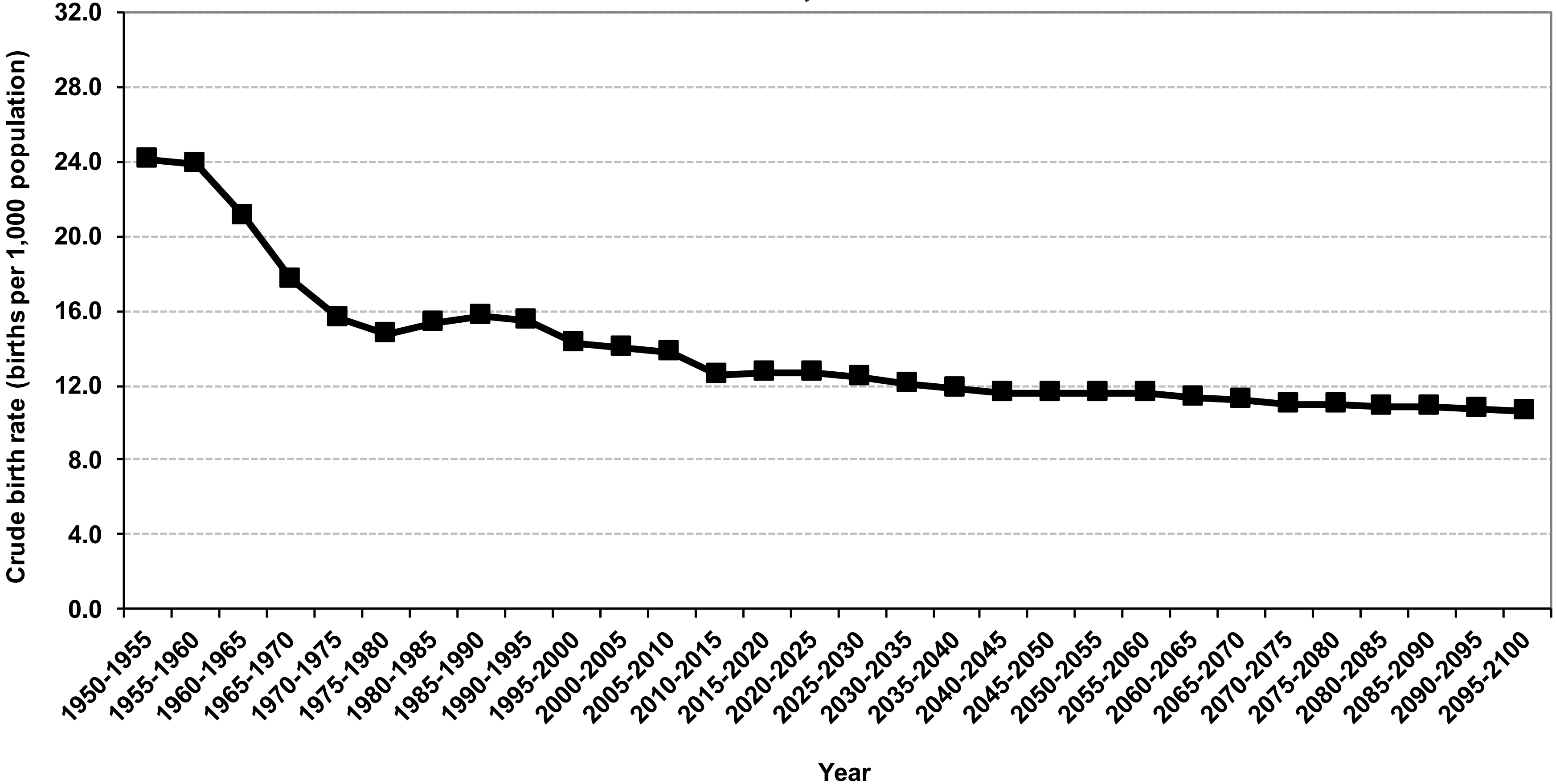
$$Crude\ death\ rate = \frac{Number\ of\ deaths}{Total\ population} \times 1{,}000$$

CBR and CDR

- Crude Birth Rate (CBR or *b*)
  - Number of births to members of the population in the period divided by the total period person-years lived

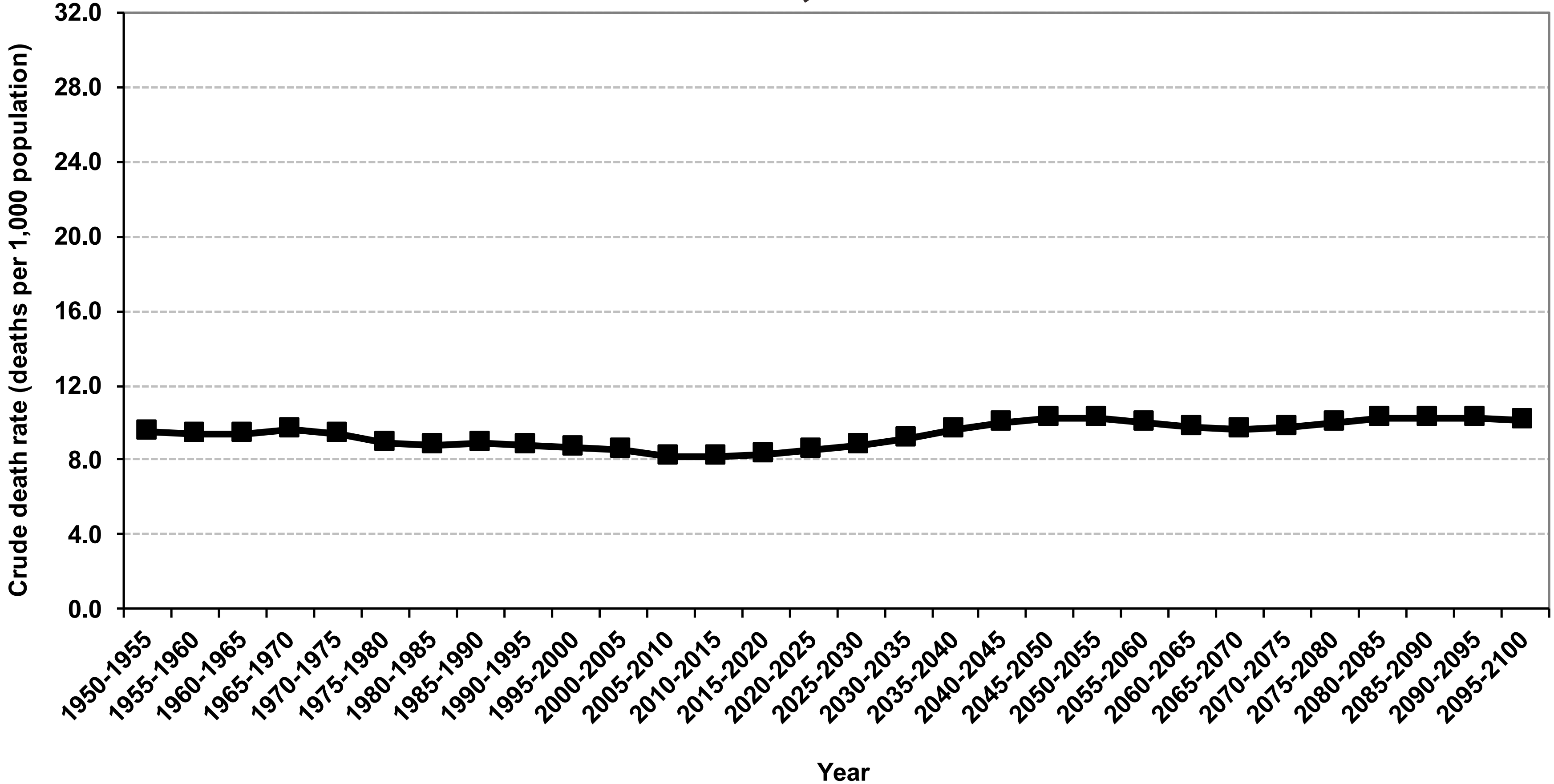
- Crude Death Rate (CDR or *d*)
  - Number of deaths to members of the population in the period divided by the total period person-years lived

Crude birth rates, United States, 1950–2100



Source: United Nations, World Population Prospects 2017
https://esa.un.org/unpd/wpp/Download/Standard/Population/
(medium variant).

Crude death rates, United States, 1950–2100



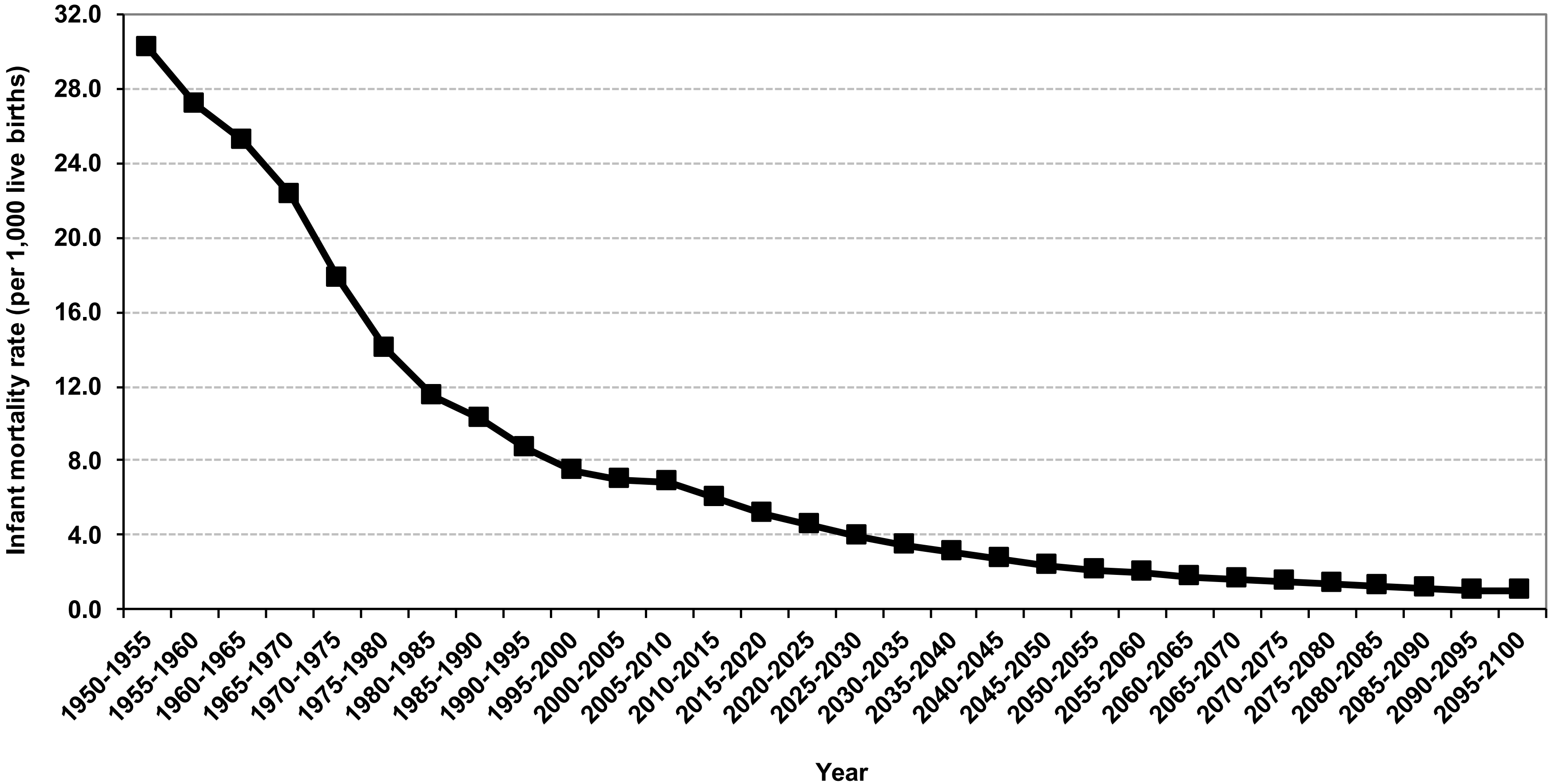
Source: United Nations, World Population Prospects 2017
https://esa.un.org/unpd/wpp/Download/Standard/Population/
(medium variant).

Infant mortality rate (IMR)

$$IMR = \frac{the\ number\ of\ deaths\ under\ age\ 1\ in\ the\ period}{the\ number\ of\ live\ births\ in\ the\ period}$$

- IMR is a period measure
- It uses current information from vital registration
- It can be computed for countries without reliable census or other source for a count of the population at risk by age
- Infants born by teenagers and by older mothers are at higher risk

Infant mortality rates, United States, 1950–2100



Source: United Nations, World Population Prospects 2017
https://esa.un.org/unpd/wpp/Download/Standard/Population/
(medium variant).

Migration indices

- Crude or gross rate of out-migration

$$OMigR = OM / p * 1{,}000$$

- Crude or gross rate of in-migration

$$IMigR = IM / p * 1{,}000$$

- Crude net migration rate

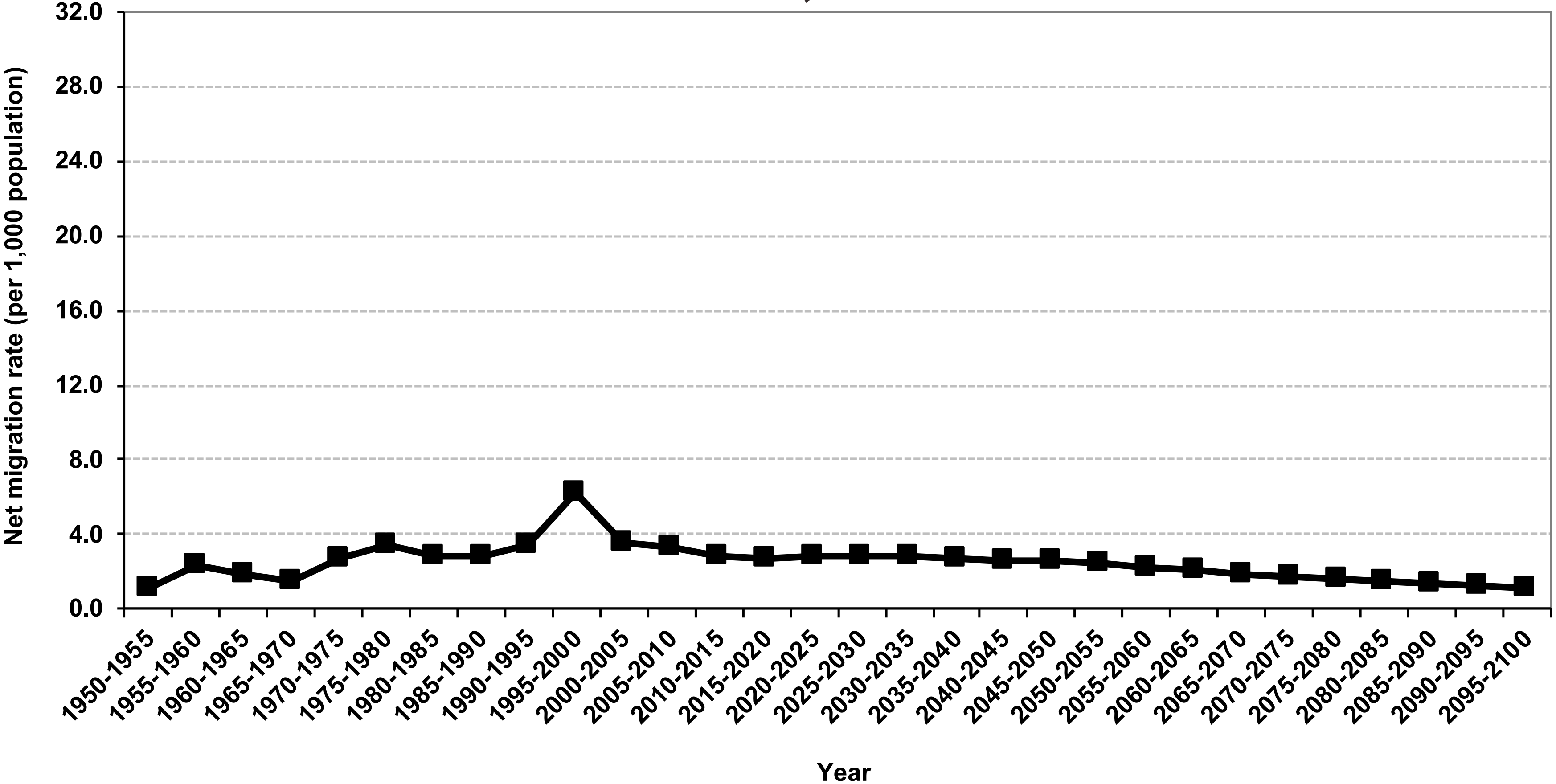
$$CNMigR = IMigR – OMigR$$

- Net migration rate

$$NMigR = IM – OM / \text{person-years lived} * 1{,}000$$

Source: Weeks 2015, Chapter 7, pp. 251–297.

Net migration rates, United States, 1950–2100



Source: United Nations, World Population Prospects 2017
https://esa.un.org/unpd/wpp/Download/Standard/Population/
(medium variant).

Other migration indices

- Total or gross migration rate

$$TMigR = IMigR + OMigR$$

- Migration effectiveness

$$E = (IMigR – OMigR) / (IMigR + OMigR) * 100$$

$$E = CNMigR / TMigR * 100$$

- Migration ratio

$$MigRatio = (IM – OM) / (b – d)$$

- Percent of total growth due to migration

$$MigPct = \frac{IM - OM}{(IM - OM) + (b - d)} * 100$$

Source: Weeks 2015, Chapter 7, pp. 251–297.

Probabilities

(Fleurence, Hollenbeak 2007)

- Probabilities describe the likelihood that an event will occur for a single individual in a given time period and range from 0 to 1
  - Does not include time in the denominator
  - Divides the number of events by the total number of people at risk in the relevant time frame
- Conversion between rates and probabilities:

$$\text{probability: } p = 1 - e^{-rt}$$

$$\text{rate: } r = -1/t * \ln(1-p)$$

- An approximation for the denominator is the population at the beginning of the period

Stable and stationary populations

- Stable population
  - Birth and death rates are constant
  - Population size might be growing, constant or declining

- Stationary population
  - Birth and death rates are constant
  - Number of births equals number of deaths
  - Total population is the same from year to year

References

Fleurence RL, Hollenbeak CS. 2007. “Rates and probabilities in economic modelling: Transformation, translation and appropriate application.” Pharmacoeconomics, 25(1): 3–6.

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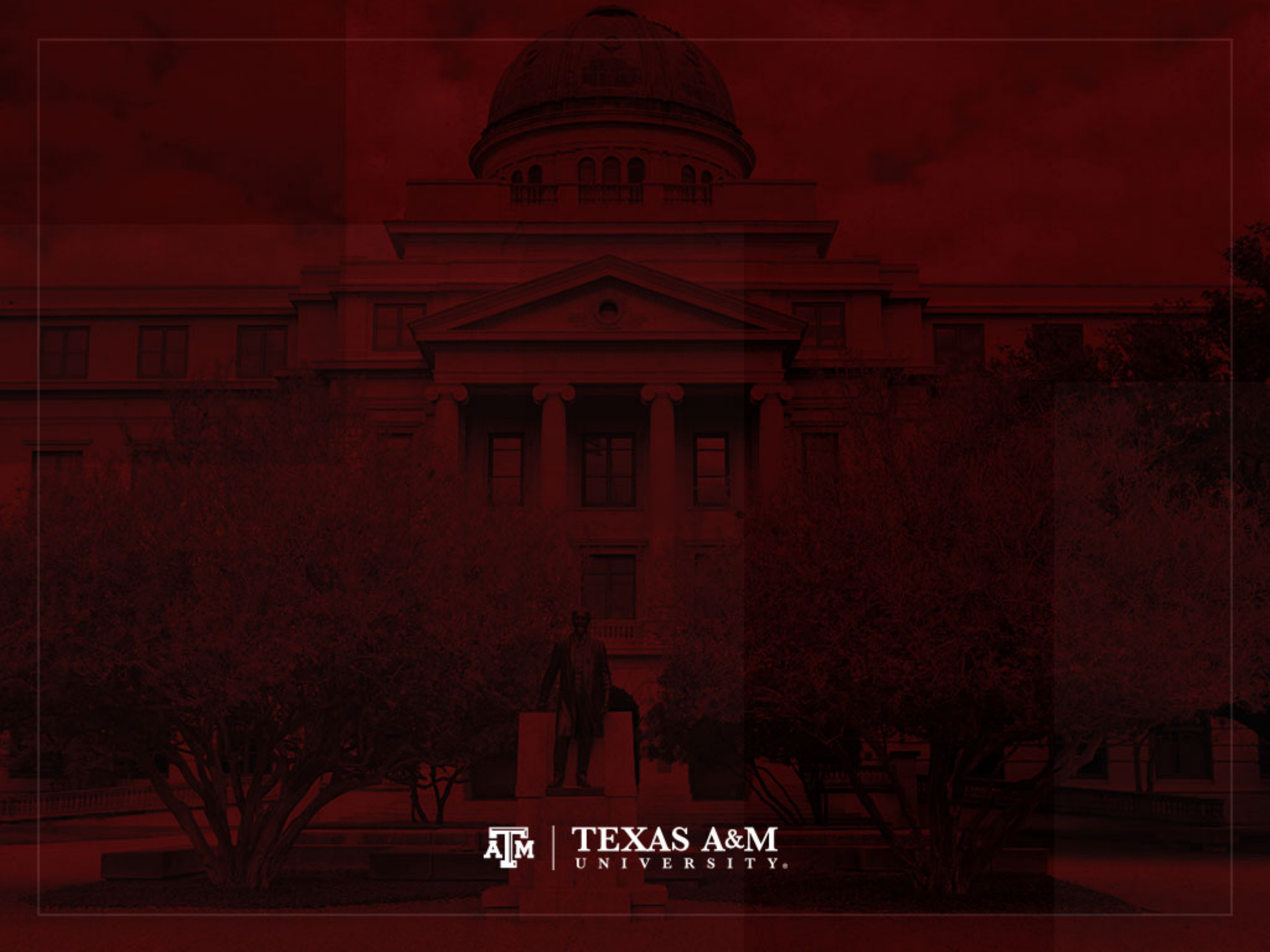
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