

# An introduction to demography

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# Introduction

- Definition of demography
- Demographic equation
- Variables and observations
- Demographic models
- Cohorts and generations
- Lexis diagram
- Person-years
- Rates, probabilities, ratios



# Definition of demography

- The scientific study of human population
- The term was coined by the Belgian statistician Achille Guillard in his 1855 book
  - *Éléments de Statistique Humaine ou Démographie Comparée*



# Demography is destiny

- This phrase is attributed to the French mathematician and philosopher, Auguste Comte (1798–1857)
  - He is known as the “father of sociology”
  - Demography shapes the world, even if it does not determine it
  - Population change is an underlying component of almost everything happening in the world today, and therefore in the future as well



# John Graunt (1620–1674)

- English statistician
  - Considered to be the founder of demography
  - Analyzed vital statistics of the London population
  - Studied the bills of mortality (weekly statistics of deaths) in early modern London
  - More specifically, studied death records that had been kept by London parishes since 1532
- Noticed certain regularities in death phenomena
  - Published in the book “Natural and Political Observations Made upon the Bills of Mortality” (1662)



# Graunt's substantive contributions

- Recognized the phenomenon of rural-urban migration
  - Urban death rate exceeded rural death rate
- Population was divided almost evenly by sex
  - Male birth rate was higher than female birth rate
    - Less females are born than males
  - Male death rate was higher than female death rate
    - Females live longer than males
- Presented mortality in terms of survivorship
  - He was the first to attempt to construct a life table...



# Graunt's life table

Age	Number surviving	Age	Number surviving
0	100	46	10
6	64	56	6
16	40	66	3
26	25	76	1
36	16	86	0

# Graunt's methodological contributions

- Paid attention to quality of data
- Exhibited a healthy skepticism
- Questioned the validity and reliability of data



# Poston's definition

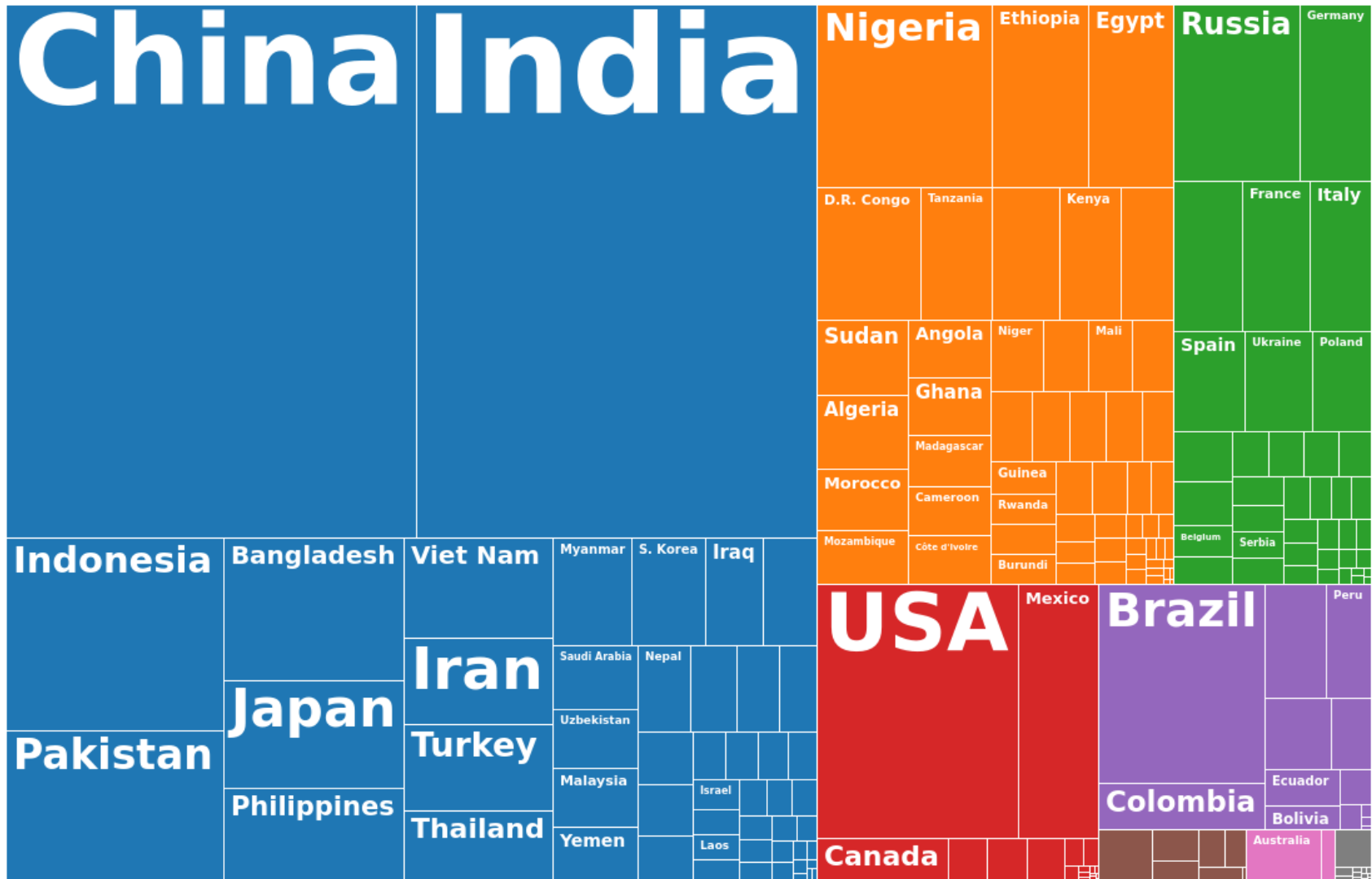
- Demography is the scientific study of the size, composition, and spatial distribution of human populations
- It investigates changes in population size, composition, and distribution, resulting from fertility, mortality, and migration
- Demography helps understand what the past says about the future, given expected population changes



# List of countries ordered by their population size

Total: 7,794,798,729

Year: 2020



# Concerns of demography

- Population size
- Population growth or decline
- Population processes/components
- Population distribution
- Population structure
- Population characteristics



# Primary demographic questions

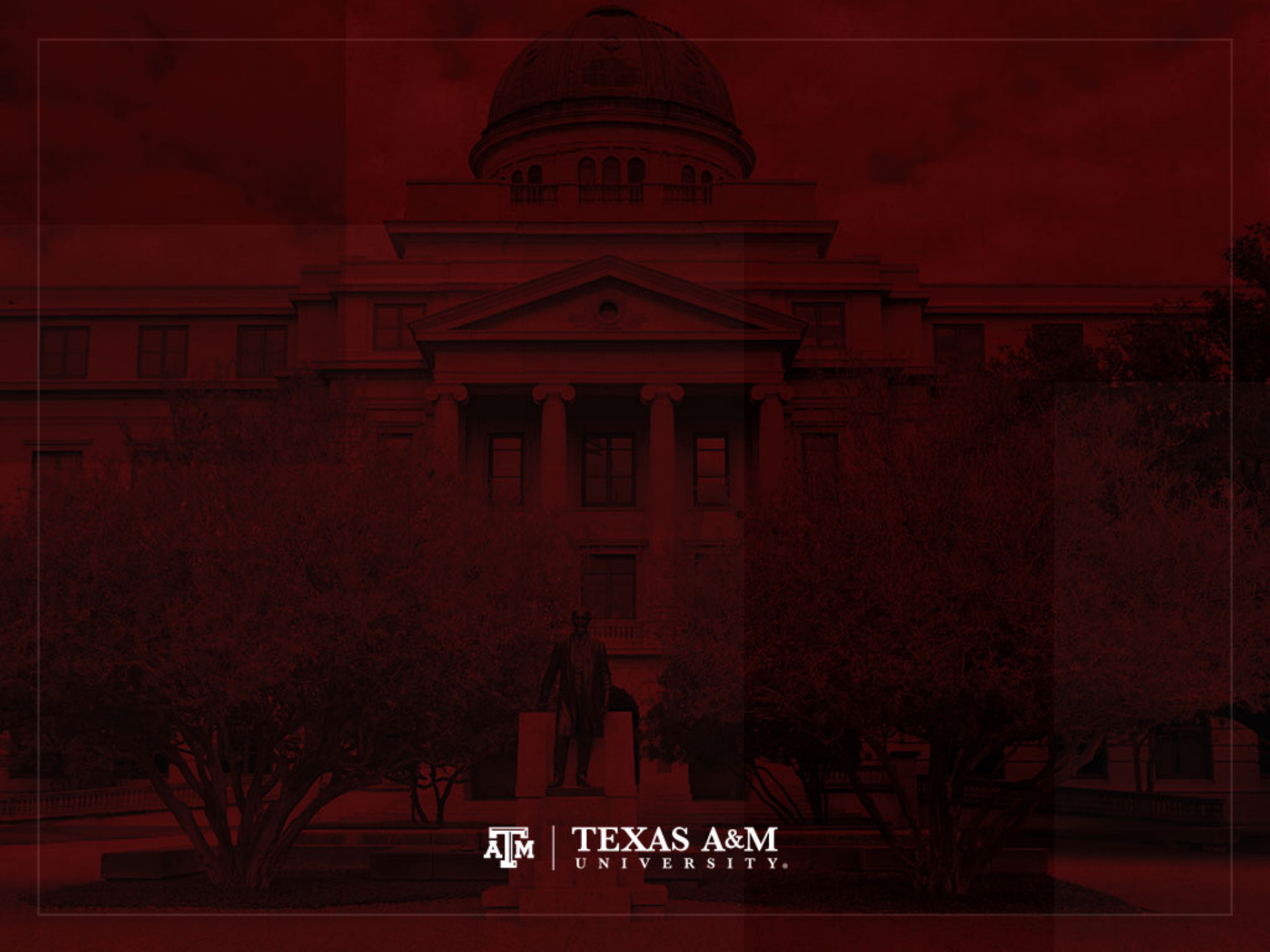
- How large (or small) is the population?
- How is the population composed, in terms of age, sex, race, marital status, and so forth?
  - What are the characteristics of the population?
- How is the population distributed spatially?
  - Populations are not randomly distributed in space
- How population changes happen over time?



# Demographic components

- These demographic questions are answered in terms of the three demographic processes (components of demographic change)
  - Fertility
  - Mortality
  - Migration





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# Demographic equation

- Population size can change only through the processes of fertility, mortality, and migration
- Two ways of entering a population
  - Being born or moving into it
- Two ways of leaving a population
  - Dying or moving out of it
- Population can only change by way of a limited, countable number of events





# Basic demographic equation

$$P_{t+1} = P_t + B_{t \text{ to } t+1} - D_{t \text{ to } t+1} + I_{t \text{ to } t+1} - E_{t \text{ to } t+1}$$

- $P_{t+1}$ : population at time  $t+1$
- $P_t$ : population at time  $t$
- $B_{t \text{ to } t+1}$ : births between times  $t$  and  $t+1$
- $D_{t \text{ to } t+1}$ : deaths between times  $t$  and  $t+1$
- $I_{t \text{ to } t+1}$ : immigrants (or in-migrants) to the population between times  $t$  and  $t+1$
- $E_{t \text{ to } t+1}$ : emigrants (or out-migrants) from the population between times  $t$  and  $t+1$





# Components of equation

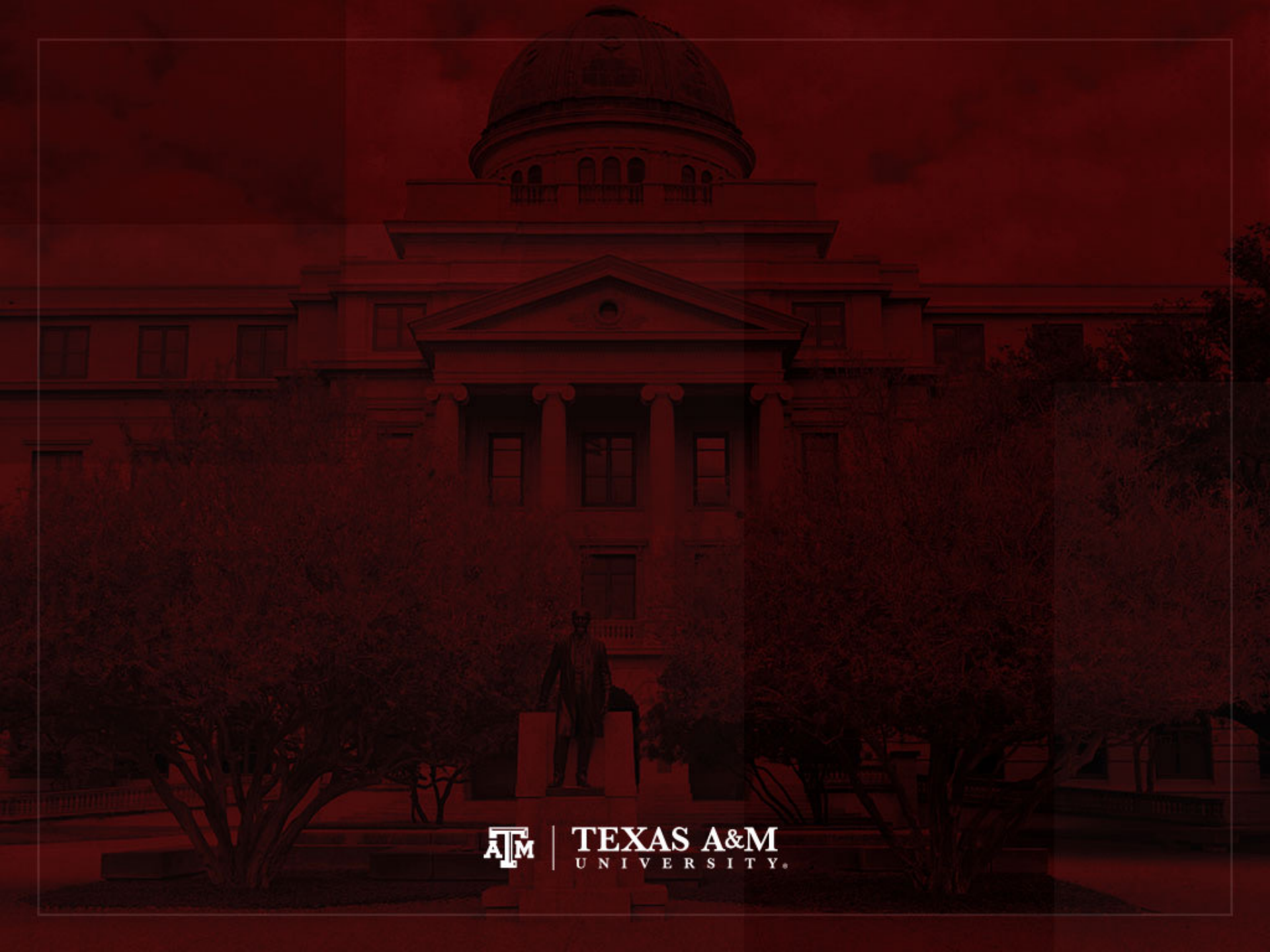
- $P_{t+1} = P_t + B_{t \text{ to } t+1} - D_{t \text{ to } t+1} + I_{t \text{ to } t+1} - E_{t \text{ to } t+1}$
- Natural increase:  $B_{t \text{ to } t+1} > D_{t \text{ to } t+1}$
- Natural decrease:  $B_{t \text{ to } t+1} < D_{t \text{ to } t+1}$ 
  - Negative natural increase



# Migration components of equation

- $I_{t \text{ to } t+1} - E_{t \text{ to } t+1}$ 
  - Net international migration
    - Immigration minus emigration
  - Net internal migration
    - In-migration minus out-migration
- $I_{t \text{ to } t+1} < E_{t \text{ to } t+1}$ 
  - Negative net international migration (sending countries)
  - Negative net internal migration (net out-migration)
- $I_{t \text{ to } t+1} > E_{t \text{ to } t+1}$ 
  - Positive net international migration (receiving countries)
  - Positive net internal migration (net in-migration)





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# Variables and observations

- **Variables**

- Characteristics that can change values from case to case
- E.g. gender, age, race/ethnicity, number of children, place of residence, income...

- **Observations (cases)**

- Refer to the entity from which data are collected
- Also known as "unit of analysis"
- E.g. individuals, households, states, countries...



# Variables

- **Variable:** a characteristic/phenomenon whose value varies (changes) from case to case, and is empirically quantifiable
- **Dependent variable:** a variable whose variation depends on another variable
- **Independent variable:** a variable whose variation produces (“causes”) variation in another variable



# Causation

- Theories and hypotheses are often stated in terms of the relationships between variables
  - Causes: independent variables
  - Effects or results: dependent variables

y	x	Use
Dependent variable	Independent variable	Econometrics
Explained variable	Explanatory variable	
Response variable	Control variable	Experimental science
Predicted variable	Predictor variable	
Outcome variable	Covariate	
Regressand	Regressor	



# Association vs. causation

- Association and causation are different
  - Strong associations may be used as evidence of causal relationships (causation)
  - Associations do not prove variables are causally related
- We might have problems of reverse causality
  - e.g., immigration increases competition in the labor market and affects earnings
  - Availability of jobs and income levels influence migration

**Migration**  **Earnings**





# Observations

- Observations (cases) are collected information used to test hypotheses
- Decide how variables will be measured and how cases will be selected and tested
- Measure social reality: collect numerical data
- Information can be organized in databases
  - Variables as columns
  - Observations as rows





# Example of a database

Observation	Salary per hour	Years of schooling	Years of experience in the labor market	Female	Marital status (married)
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
...	...	...	...	...	...
525	11.56	16	5	0	1
526	3.50	14	5	1	0



# Coronavirus pandemic, August 24, 2020

#	Country, Other	Total Cases	New Cases	Total Deaths	New Deaths	Total Recovered	Active Cases	Serious, Critical	Tot Cases/ 1M pop	Deaths/ 1M pop	Total Tests	Tests/ 1M pop	Population
	World	23,809,061	+6,189	817,005	+431	16,358,235	6,633,821	61,715	3,054	104.8			
1	<a href="#">USA</a>	5,915,630		181,114		3,217,981	2,516,535	16,483	17,856	547	76,883,479	232,071	331,293,410
2	<a href="#">Brazil</a>	3,627,217		115,451		2,778,709	733,057	8,318	17,046	543	14,144,344	66,473	212,784,888
3	<a href="#">Mexico</a>	563,705	+3,541	60,800	+320	389,124	113,781	3,346	4,365	471	1,263,835	9,787	129,132,739
4	<a href="#">India</a>	3,164,881		58,546		2,403,101	703,234	8,944	2,290	42	35,902,137	25,978	1,382,011,722
5	<a href="#">UK</a>	326,614		41,433		N/A	N/A	72	4,807	610	15,177,265	223,394	67,939,531
6	<a href="#">Italy</a>	260,298		35,441		205,662	19,195	65	4,306	586	8,053,551	133,231	60,448,212
7	<a href="#">France</a>	244,854		30,528		85,199	129,127	399	3,750	468	6,000,000	91,890	65,295,389
8	<a href="#">Spain</a>	420,809		28,872		N/A	N/A	658	9,000	617	8,517,446	182,162	46,757,536
9	<a href="#">Peru</a>	600,438		27,813		407,301	165,324	1,525	18,174	842	3,006,993	91,014	33,038,913
10	<a href="#">Iran</a>	361,150		20,776		311,365	29,009	3,848	4,292	247	3,062,422	36,392	84,150,494
11	<a href="#">Colombia</a>	551,696		17,612		384,171	149,913	1,493	10,825	346	2,508,972	49,231	50,962,919
12	<a href="#">Russia</a>	961,493		16,448		773,095	171,950	2,300	6,588	113	34,600,000	237,077	145,943,991
13	<a href="#">South Africa</a>	611,450		13,159		516,494	81,797	539	10,291	221	3,564,065	59,983	59,418,339
14	<a href="#">Chile</a>	399,568		10,916		372,464	16,188	1,014	20,875	570	2,231,463	116,583	19,140,575
15	<a href="#">Belgium</a>	82,092	+156	9,996	+4	18,242	53,854	89	7,079	862	2,144,563	184,921	11,597,214
16	<a href="#">Germany</a>	236,117		9,336		209,600	17,181	245	2,817	111	10,197,366	121,652	83,824,401
17	<a href="#">Canada</a>	125,647		9,083		111,694	4,870	62	3,325	240	5,169,166	136,782	37,791,278
18	<a href="#">Argentina</a>	350,867		7,366		256,789	86,712	1,960	7,753	163	1,105,878	24,435	45,257,261
19	<a href="#">Indonesia</a>	155,412		6,759		111,060	37,593		567	25	2,056,166	7,506	273,950,524
20	<a href="#">Iraq</a>	207,985		6,519		150,389	51,077	661	5,154	162	1,457,665	36,125	40,350,522

# Coronavirus pandemic, August 31, 2021

#	Country, Other	Total Cases	New Cases	Total Deaths	New Deaths	Total Recovered	New Recovered	Active Cases	Serious, Critical	Tot Cases/ 1M pop	Deaths/ 1M pop	Total Tests	Tests/ 1M pop	Population
	World	218,171,757	+278,500	4,527,970	+4,700	195,040,717	+304,214	18,603,070	113,811	27,989	580.9			
1	<a href="#">USA</a>	39,953,651	+6,943	656,482	+89	30,945,115	+650	8,352,054	25,541	119,888	1,970	582,550,800	1,748,051	333,257,237
2	<a href="#">Brazil</a>	20,752,281		579,643		19,692,898		479,740	8,318	96,831	2,705	56,897,224	265,485	214,314,149
3	<a href="#">India</a>	32,808,018	+40,198	438,962	+370	31,982,180	+29,967	386,876	8,944	23,506	314	521,541,098	373,663	1,395,753,675
4	<a href="#">Mexico</a>	3,341,264	+5,564	258,491	+326	2,686,568	+16,627	396,205	4,798	25,603	1,981	9,723,416	74,506	130,505,007
5	<a href="#">Peru</a>	2,149,591		198,263		N/A	N/A	N/A	1,333	64,158	5,917	16,733,426	499,437	33,504,611
6	<a href="#">Russia</a>	6,918,965	+17,813	183,224	+795	6,181,054	+18,624	554,687	2,300	47,388	1,255	178,700,000	1,223,912	146,007,206
7	<a href="#">Indonesia</a>	4,089,801	+10,534	133,023	+532	3,760,497	+16,781	196,281		14,771	480	32,216,075	116,354	276,880,593
8	<a href="#">UK</a>	6,757,650		132,485		5,427,062		1,198,103	982	98,940	1,940	266,714,771	3,905,032	68,300,272
9	<a href="#">Italy</a>	4,534,499		129,146		4,263,960		141,393	548	75,126	2,140	83,728,076	1,387,181	60,358,447
10	<a href="#">Colombia</a>	4,907,264		124,883		4,737,467		44,914	8,155	95,264	2,424	24,121,717	468,271	51,512,348
11	<a href="#">France</a>	6,746,283		114,308		6,225,201		406,774	2,270	103,089	1,747	124,769,146	1,906,579	65,441,374
12	<a href="#">Argentina</a>	5,178,889		111,607		4,869,104		198,178	2,713	113,380	2,443	22,017,526	482,024	45,677,243
13	<a href="#">Iran</a>	4,992,063	+31,319	107,794	+643	4,205,927	+30,522	678,342	7,879	58,565	1,265	28,213,229	330,985	85,240,218
14	<a href="#">Germany</a>	3,950,247	+3,231	92,682	+11	3,738,000	+6,100	119,565	1,096	46,973	1,102	68,329,706	812,527	84,095,254
15	<a href="#">Spain</a>	4,847,298		84,146		4,338,145		425,007	1,685	103,628	1,799	60,618,810	1,295,943	46,775,830
16	<a href="#">South Africa</a>	2,770,575		81,830		2,533,956		154,789	546	46,041	1,360	16,426,011	272,965	60,176,262
17	<a href="#">Poland</a>	2,888,670	+285	75,345	+5	2,657,084	+30	156,241	60	76,423	1,993	19,778,356	523,259	37,798,415
18	<a href="#">Turkey</a>	6,366,438		56,458		5,823,111		486,869	633	74,555	661	76,140,298	891,652	85,392,352
19	<a href="#">Ukraine</a>	2,286,296	+1,356	53,789	+51	2,207,940	+1,257	24,567	177	52,646	1,239	11,980,323	275,866	43,428,075
20	<a href="#">Chile</a>	1,638,675	+345	36,937	+14	1,595,747	+577	5,991	687	84,876	1,913	20,276,691	1,050,240	19,306,720

Source: <https://www.worldometers.info/coronavirus/>.

# Coronavirus pandemic, January 17, 2022

#	Country, Other	Total Cases	New Cases	Total Deaths	New Deaths	Total Recovered	New Recovered	Active Cases	Serious, Critical	Tot Cases/ 1M pop	Deaths/ 1M pop	Total Tests	Tests/ 1M pop	Population
	World	331,459,057	+138,304	5,563,652	+219	269,090,164	+64,428	56,805,241	97,247	42,523	713.8			
1	<a href="#">USA</a>	67,631,191		874,321		43,165,667		23,591,203	25,869	202,490	2,618	862,458,737	2,582,225	333,998,303
2	<a href="#">Brazil</a>	23,083,297		621,261		21,710,831		751,205	8,318	107,419	2,891	63,776,166	296,783	214,891,229
3	<a href="#">India</a>	37,618,271		486,784		35,394,882		1,736,605	8,944	26,852	347	705,411,425	503,527	1,400,939,318
4	<a href="#">Russia</a>	10,834,260		321,990		9,878,371		633,899	2,300	74,191	2,205	246,800,000	1,690,051	146,031,061
5	<a href="#">Mexico</a>	4,385,415	+17,101	301,469	+59	3,478,130	+34,246	605,816	4,798	33,471	2,301	13,163,932	100,471	131,022,844
6	<a href="#">Peru</a>	2,606,126		203,464		N/A	N/A	N/A	1,038	77,378	6,041	23,289,858	691,497	33,680,346
7	<a href="#">UK</a>	15,305,410		152,075		11,497,602		3,655,733	746	223,644	2,222	434,073,111	6,342,723	68,436,401
8	<a href="#">Indonesia</a>	4,272,421		144,174		4,119,472		8,775		15,369	519	67,715,434	243,593	277,986,279
9	<a href="#">Italy</a>	8,790,302		141,391		6,093,633		2,555,278	1,717	145,717	2,344	156,338,495	2,591,622	60,324,574
10	<a href="#">Iran</a>	6,224,196		132,095		6,066,819		25,282	1,313	72,669	1,542	42,908,102	500,962	85,651,435
11	<a href="#">Colombia</a>	5,568,068		131,130		5,258,204		178,734	342	107,659	2,535	31,171,683	602,704	51,719,680
12	<a href="#">France</a>	14,274,528		127,263		9,198,995		4,948,270	3,895	217,943	1,943	211,520,605	3,229,497	65,496,464
13	<a href="#">Argentina</a>	7,197,323		118,231		6,193,473		885,619	2,099	157,024	2,579	30,753,911	670,959	45,835,727
14	<a href="#">Germany</a>	8,045,348		116,411		7,000,000		928,937	3,212	95,553	1,383	89,622,218	1,064,429	84,197,463
15	<a href="#">Poland</a>	4,323,482		102,309		3,800,051		421,122	1,519	114,430	2,708	28,591,765	756,744	37,782,620
16	<a href="#">Ukraine</a>	3,759,530		98,361		3,556,162		105,007	177	86,769	2,270	17,182,817	396,574	43,328,102
17	<a href="#">South Africa</a>	3,560,921		93,451		3,375,859		91,611	546	58,895	1,546	21,815,463	360,811	60,462,270
18	<a href="#">Spain</a>	8,424,503		90,993		5,331,175		3,002,335	2,251	180,077	1,945	66,213,858	1,415,348	46,782,734
19	<a href="#">Turkey</a>	10,522,099		84,920		9,737,610		699,569	1,128	122,722	990	125,433,490	1,462,964	85,739,301
20	<a href="#">Romania</a>	1,911,546		59,257		1,776,122		76,167	485	100,399	3,112	17,974,573	944,065	19,039,551





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# Demographic models

- Formal demography
- Population studies I
- Population studies II



# Formal demography

## Independent variable

Demographic

## Dependent variable

→ Demographic

## Examples

1. Age composition

→ Birth rate

2. Birth rate

→ Age composition

3. Sex composition of  
in-migrants to a city

→ Sex ratio of the  
total population of the city



# Population studies I (social demography)

## Independent variable

Non-demographic

## Dependent variable

→ Demographic

## Examples

1. Social class  
(sociological) → Death rate
2. Attitude about motherhood  
(social psychology) → Number of children
3. Annual rainfall  
(geographical) → Population density
4. Economic opportunity  
(economic) → Migration





# Population studies II (social demography)

## Independent variable

Demographic

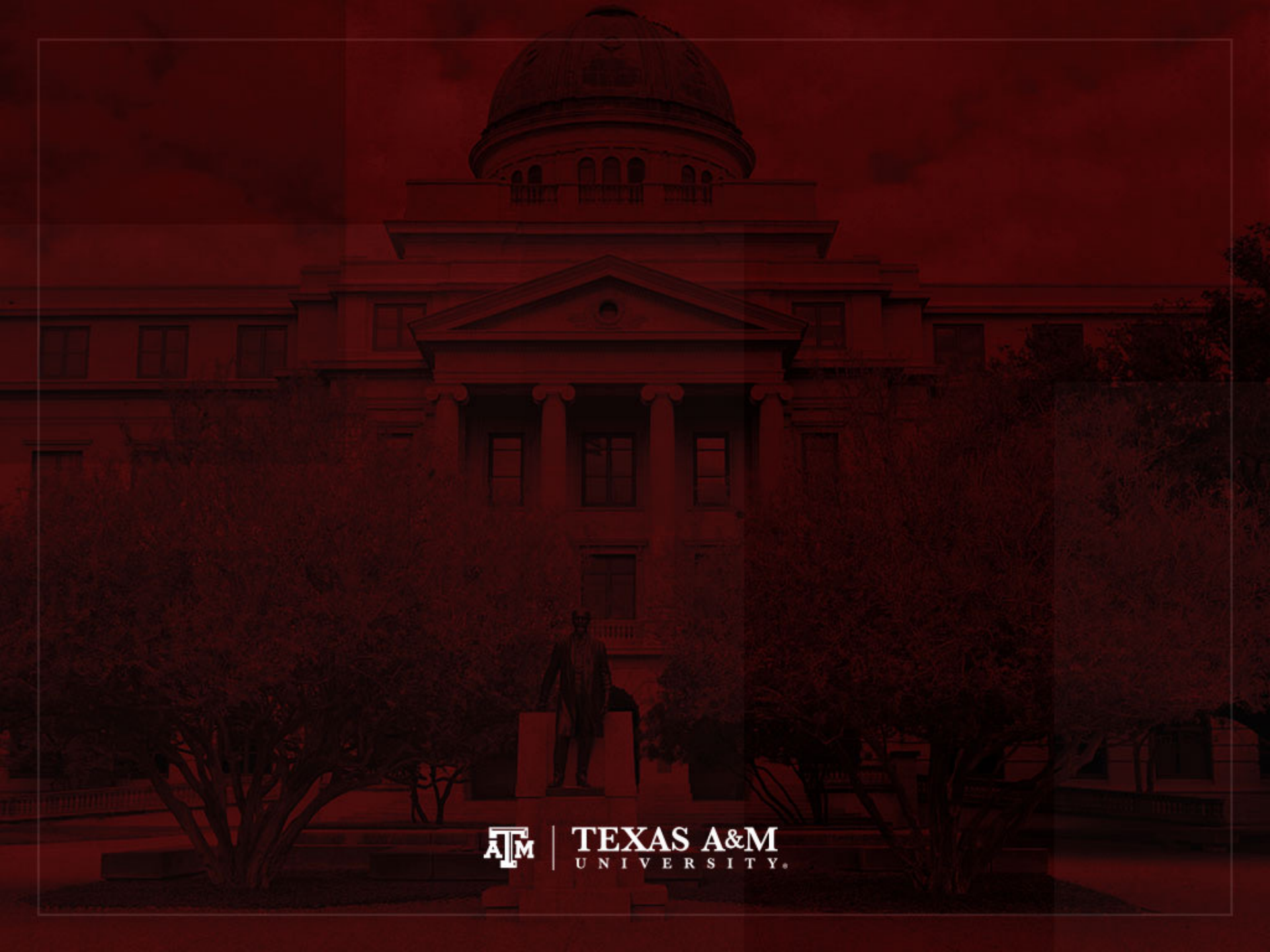
## Dependent variable

→ Non-demographic

## Examples

1. Age composition → Voting behavior  
(political)
2. Migration → Social change  
(sociology)
3. Birth rate → Need for infant & child goods/services  
(public health)





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# Cohorts and generations

- Cohort
  - Group of persons who have experienced a common event during a given time interval
  - Birth cohorts are sometimes referred to as generations
- Why study birth cohorts?
  - If you understand what distinctive opportunities and problems you have faced, you can find common ground with others in your generation and in other generations (Elwood Carlson)



# Examples of cohorts

- People born during the same period who experience similar social circumstances throughout their lives
  - Good Warriors (Greatest Generation): born in the 1900s through the 1920s
  - Lucky Few: from around 1929 to 1945
  - Baby Boomers: between around 1946 and 1964
  - Generation X (Baby Bust Cohort): from mid-1960s to early 1980s
  - Millennials (New Boomers or Generation Y): from early 1980s to early 2000s
  - Generation Z: start in early 2000s



# Lucky Few cohort

- **Lucky Few cohort**, born between 1929–1945
  - They were fewer compared to the much larger number of persons in the following cohort
  - Baby Boomer cohort, born between 1946–1964
- The smaller size of the Lucky Few has enabled them to experience
  - Higher employment rates
  - Greater variety of social opportunities than members in the preceding or following cohorts

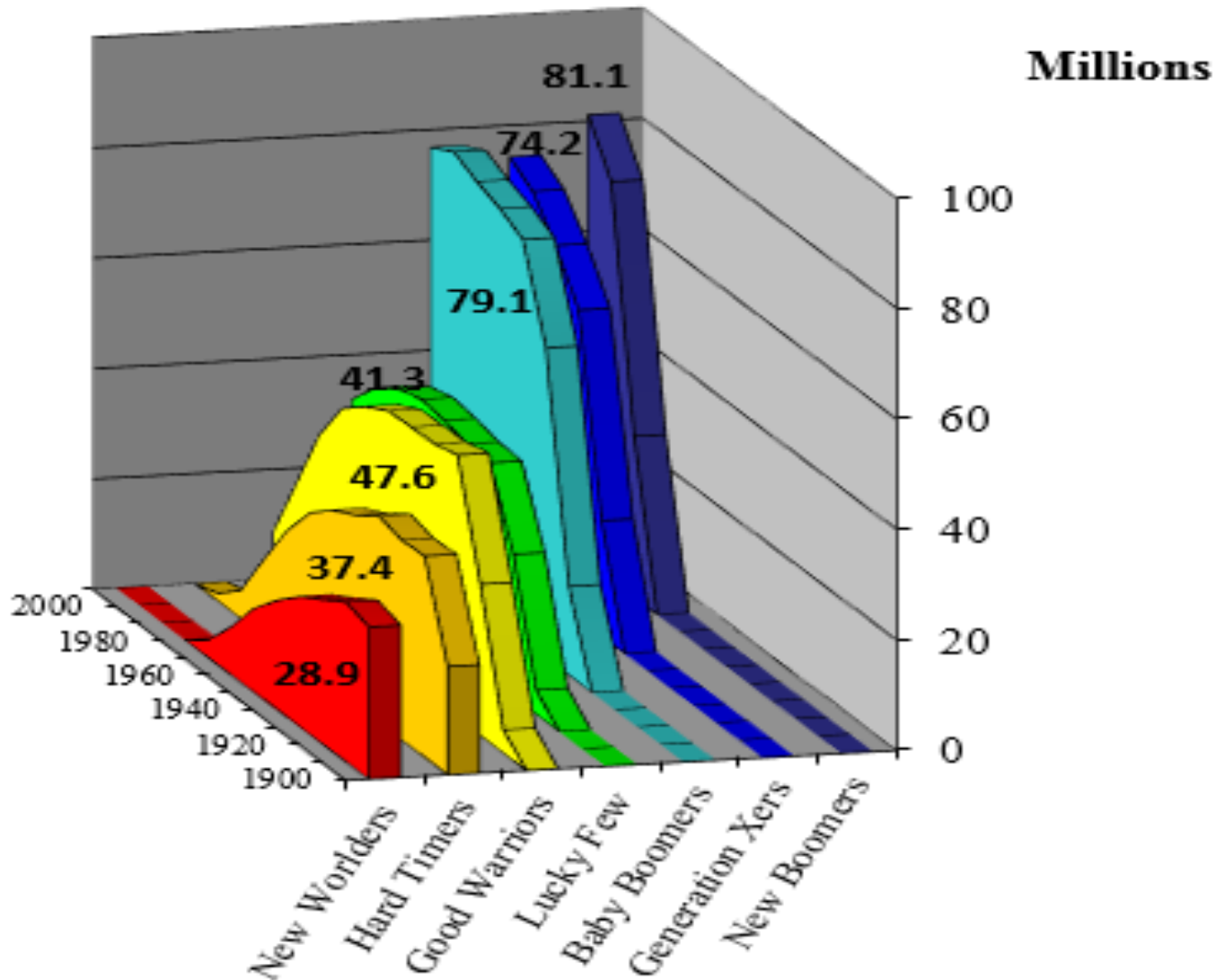


# Eight US birth cohorts

Birth cohort	Years of birth	Age range in 2020	Number born in the U.S., total	Alive in 2019 (include immigrants)	Number born in the U.S., per year
New Worlders	1871–1889	None living	~ 30 million	None	1.6 million
Hard Timers	1890–1908	None living	~ 25 million	None	1.3 million
Good Warriors	1909–1928	92–111	57.6 million	1.7 million	2.8 million
Lucky Few	1929–1945	75–91	44.1 million	20.9 million	2.5 million
Baby Boomers	1946–1964	56–74	75.8 million	69.9 million	4 million
Generation X	1965–1982	38–55	62.2 million	73.9 million	3.4 million
Millennials	1983–2001	19–37	74.5 million	84.9 million	3.9 million
Generation Z	2002–present	0–18	72.4 million	77.3 million	4 million

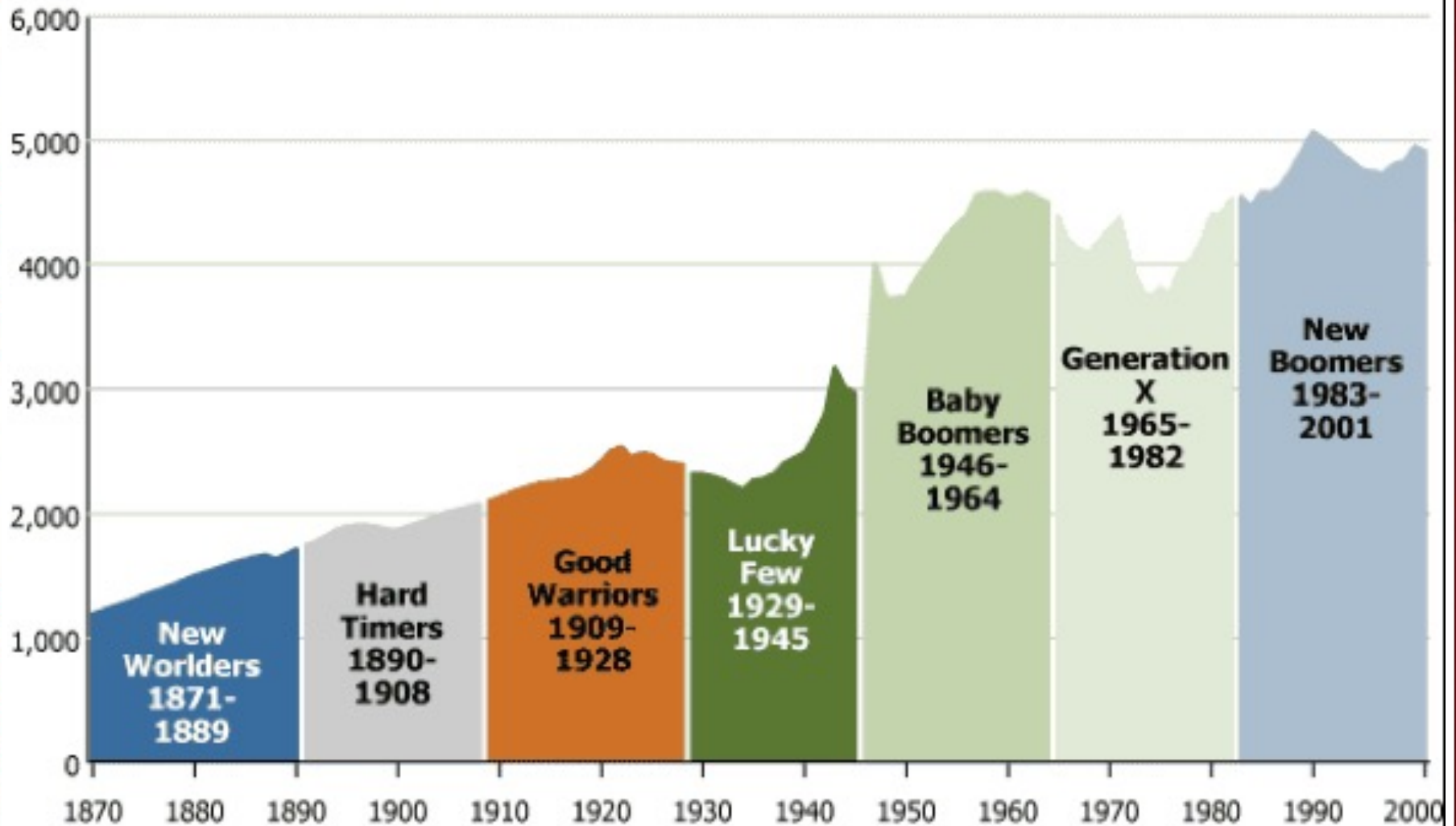


# Seven US birth cohorts by size, 1900–2010



# US birth cohorts

Thousands of people, by year of birth







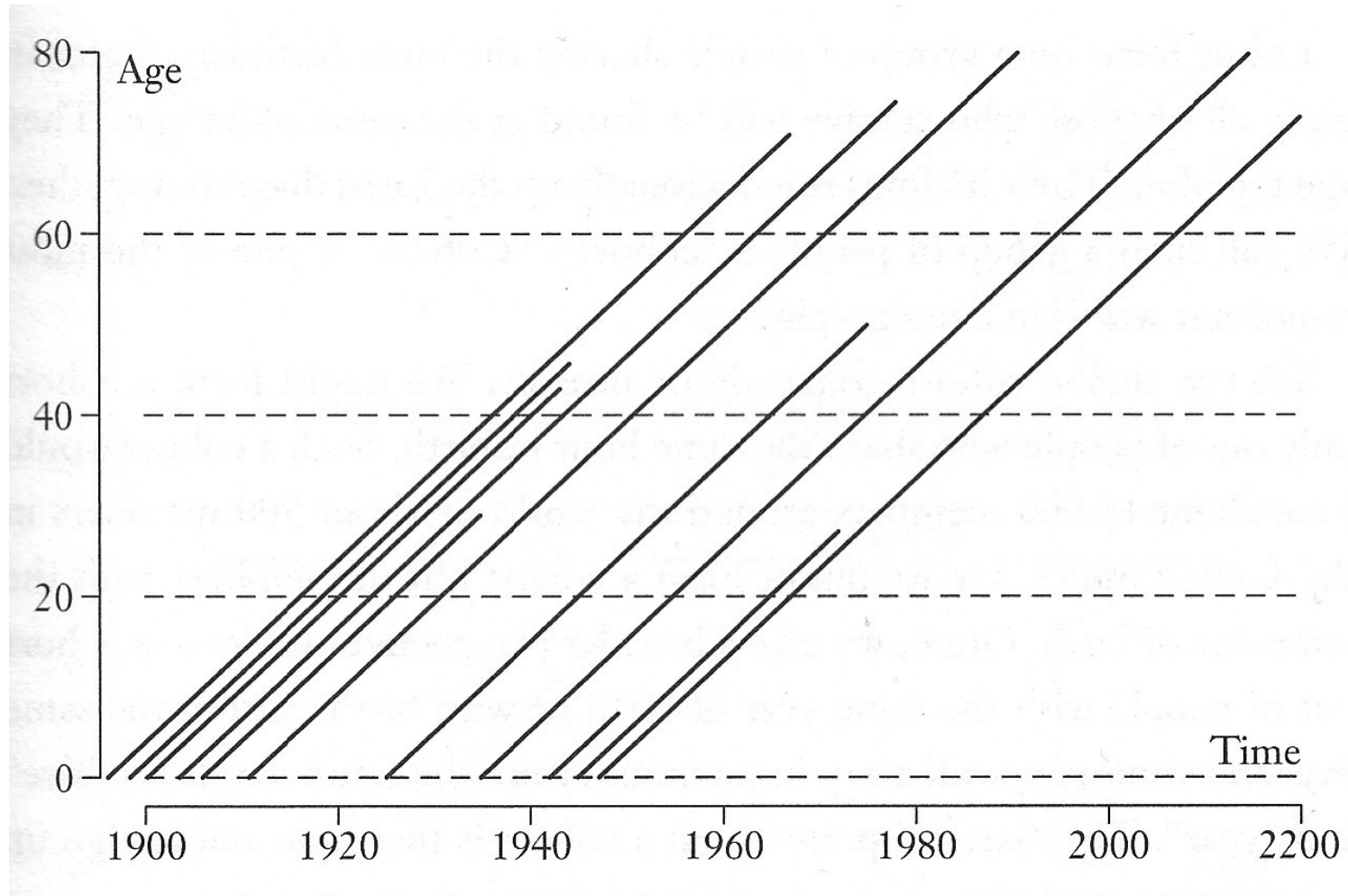
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# Lexis diagram

- Lexis diagram provides relationships between chronological time  $t$  (horizontal) and age  $x$  (vertical)
- Each person has a lifeline on a Lexis diagram
  - Starting at  $(t_b, 0)$ , where  $t_b$  is the person's birthdate and 0 is the person's age at birth
- Line goes up to the right with a slope equal to 1
  - People age one year in one calendar year
- Lifeline goes up until time and age of the person's death



# Lexis diagram



Source: Wachter 2014, p. 31.



# Exploring Lexis diagram

- To find population size
  - Draw vertical line upward from the time point
  - Count how many lifelines cross vertical line
- To find how many people survive to some age
  - Draw horizontal line across at the height corresponding to that age
  - Count how many lifelines cross that horizontal line
- Immigrants start at age and time of immigration



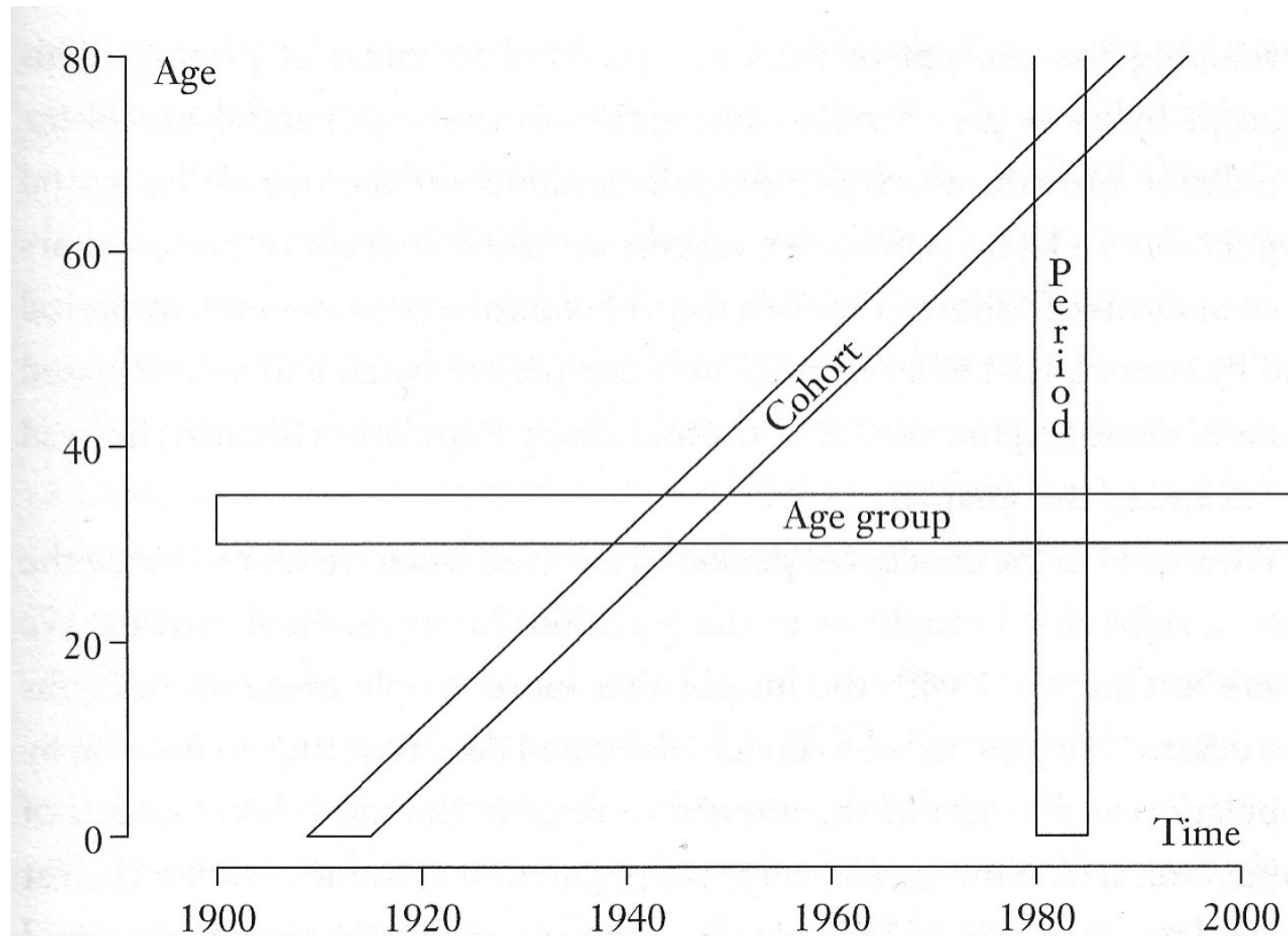
# Cohort in the Lexis diagram

- Group of people sharing the same birthdate
- Group of individuals followed simultaneously through time and age
- Their lifelines run diagonally up the Lexis diagram together
- In a cohort, time and age go up together
- A cohort shares experiences





# Lexis diagram: Age, period, cohort



# Game of pretend

- When we calculate a period measure, we pretend that age-specific rates we see today for different age groups continue unchanged into the future
- We are creating an imaginary cohort whose life experience is pieced together from the experiences of different people found at different ages in one period of time

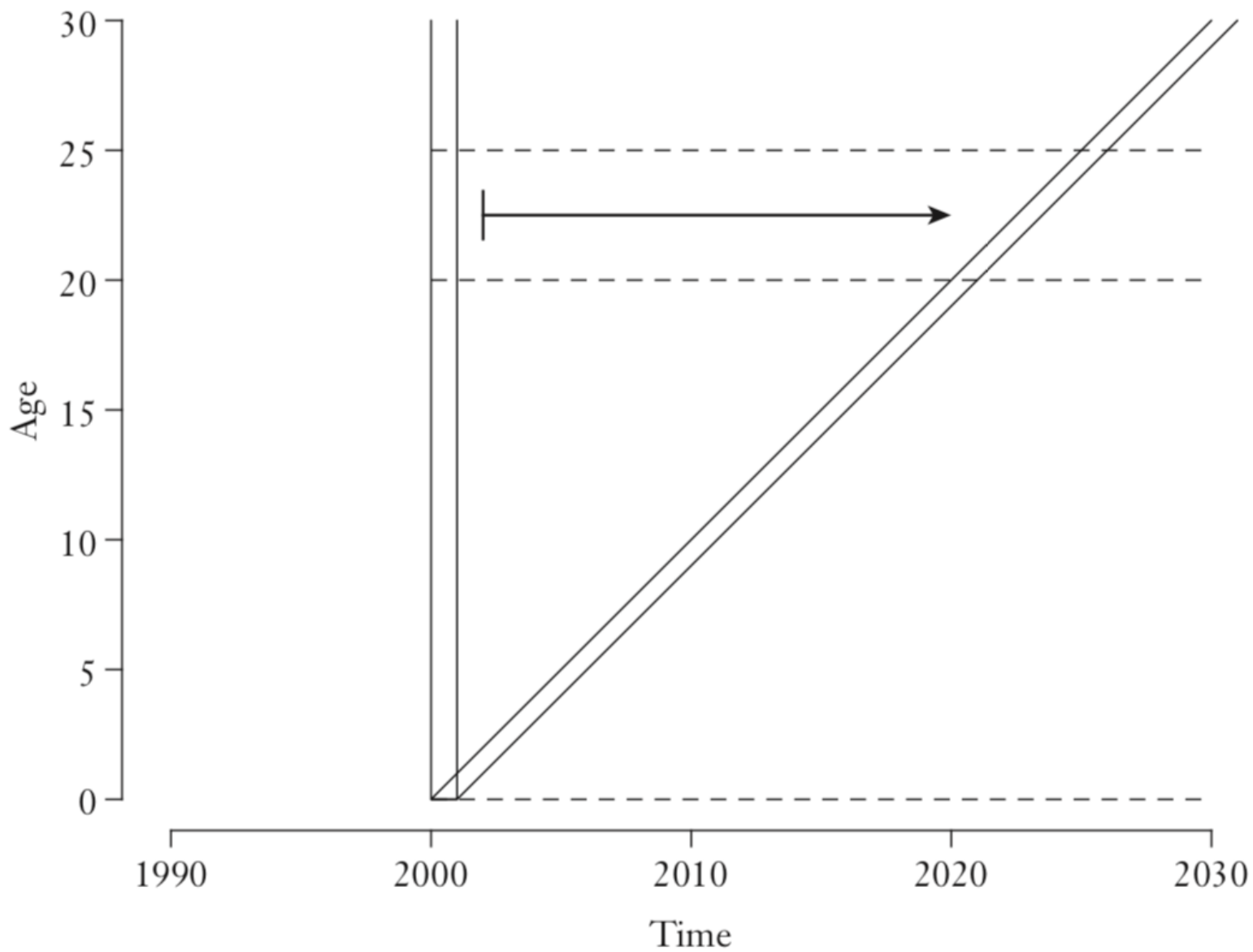


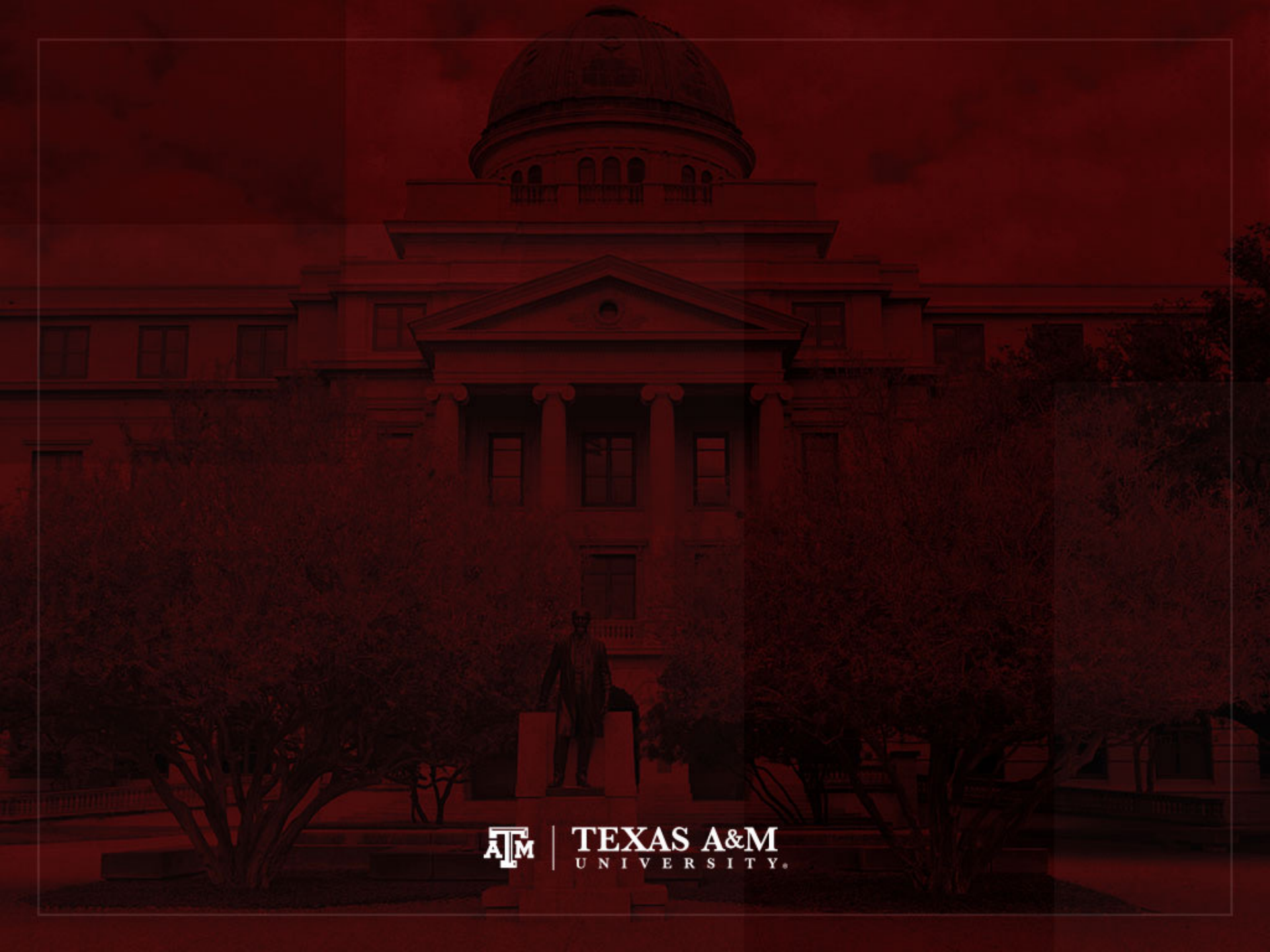
Figure 6.1 From period to cohort on a Lexis diagram





# Synthetic cohort

- We call this imaginary cohort the synthetic cohort
  - *syn*: “together”
  - *thetic*: “pieced”
  - *synthetic*: “pieced together”
- Age-specific cohort rates of the synthetic cohort are the age-specific period rates of the period population
- The concept of a synthetic cohort is central to demography



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# Person-years

- **Person-years** is the sum of each individual's time at risk of experiencing an event (e.g. birth, death, migration)
  - For those who do not experience event, person-years is the sum of time until end of period
  - For those who experience event, it is the time until the event
- **Period person-years lived** (PPYL) take into account that people are present during part of the period (fraction of years)
  - Each full year that a person is present in a period, he/she contributes one “person-year” to the total of PPYL
  - Each month a person is present in the population, he/she contributes 1 person-month, or  $1/12$  person-year, to PPYL



# Example of person-years

Hypothetical population increasing at the rate of 0.001 per month

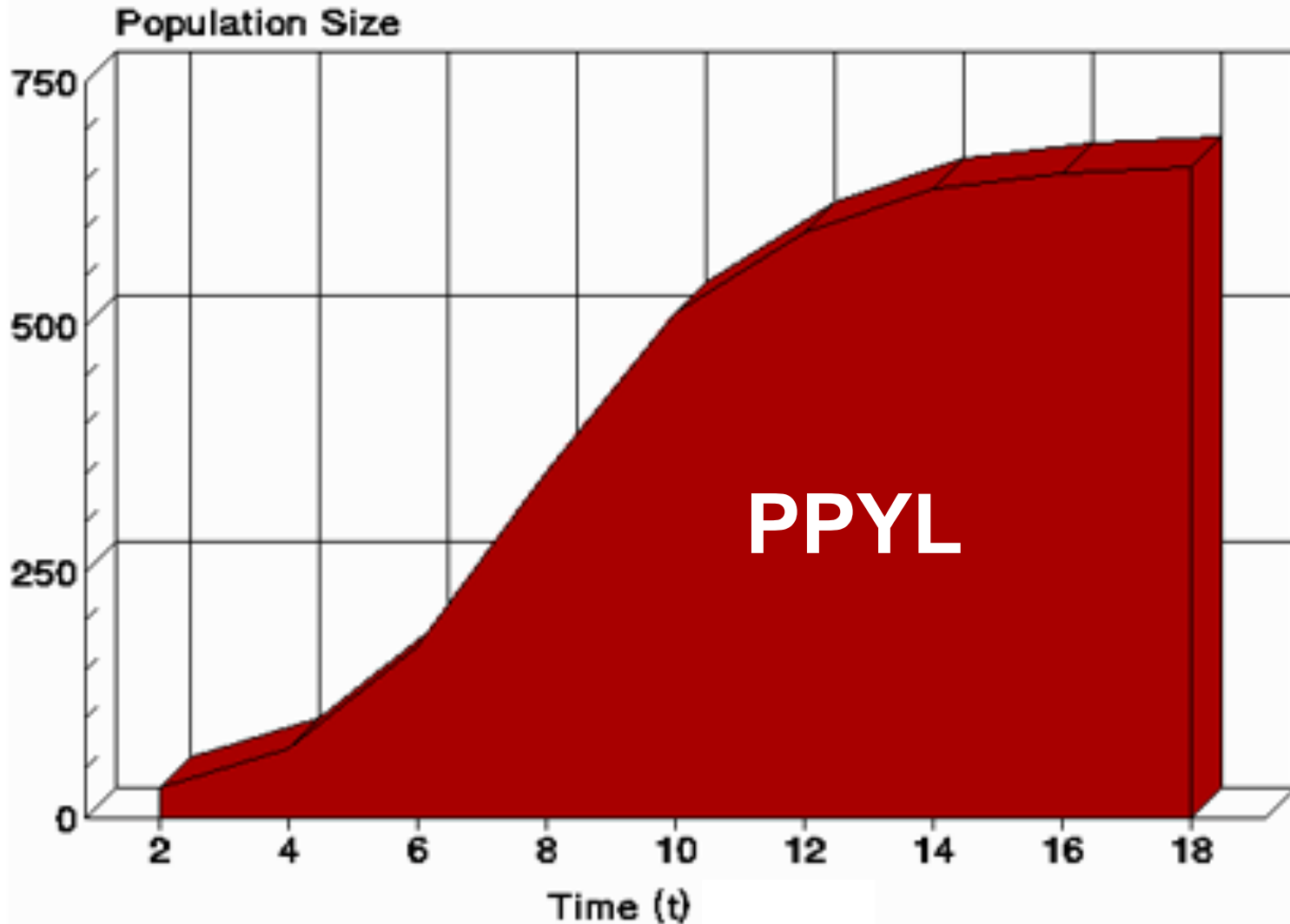
Month	Population	Person-years (population / 12)	Approximation for person-years	
			Mid-period	Average of start and end
January	200.00	16.67		200.00
February	200.20	16.68		
March	200.40	16.70		
April	200.60	16.72		
May	200.80	16.73		
June	201.00	16.75		
July	201.20	16.77	201.20	
August	201.40	16.78		
September	201.61	16.80		
October	201.81	16.82		
November	202.01	16.83		
December	202.21	16.85		202.21
<b>Period person-years lived (PPYL)</b>		<b>201.10</b>	<b>201.20</b>	<b>201.11</b>

# Calculating person-years

- Whenever we know the population sizes on each month over the period of a year
  - We can add up the person-years month by month
  - Take the number of people present on each month and divide by 12
  - Add up all monthly contributions
- When our subintervals are small enough
  - Our sum is virtually equal to the area under the curve of population as a function of time during the period...



# Person-years and areas



Source: <https://www2.palomar.edu/users/warmstrong/lmexer9.htm>.





# Approximation for PPYL

- When sequences of population sizes throughout a period are unknown
  - Take the population in the middle of the period and multiply by the length of the period
    - E.g., for 2005–2015, we take the mid-period count of 308,745,000 people in the U.S. from the 2010 Census and multiply by 10 years to obtain 3,087,450,000 person-years in the period
  - Or take the average of the starting and ending populations and multiply by the length of the period







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# Rates, probabilities, ratios

- Rates
  - Describe the number of occurrences of an event for a given number of individuals who had the chance to experience that event per unit of time
- Probabilities
  - Divide the number of events by the total number of people at risk in the relevant time frame
- Ratios
  - Compare the size of one group to the size of another group



# Rates

(Fleurence, Hollenbeak 2007)

- Rates are an instantaneous measure that range from zero to infinity
  - Rates describe the number of occurrences of an event for a given number of individuals per unit of time
  - Rates consider the time spent at risk
- Numerator
  - Number of events (e.g. births, deaths, migrations) in a given time
- Denominator includes time
  - Sum of each individual's time at risk of experiencing an event for a specific population during a certain time period (person-years)
  - We can use approximations for the denominator
    - Population in the middle of the period or
    - Average of starting and ending populations for that period



# Ideal way to estimate rates

- Crude Birth Rate (CBR or  $b$ )
  - Number of births to members of the population in the period divided by the total period person-years lived
- Crude Death Rate (CDR or  $d$ )
  - Number of deaths to members of the population in the period divided by the total period person-years lived

# Usual way to estimate rates

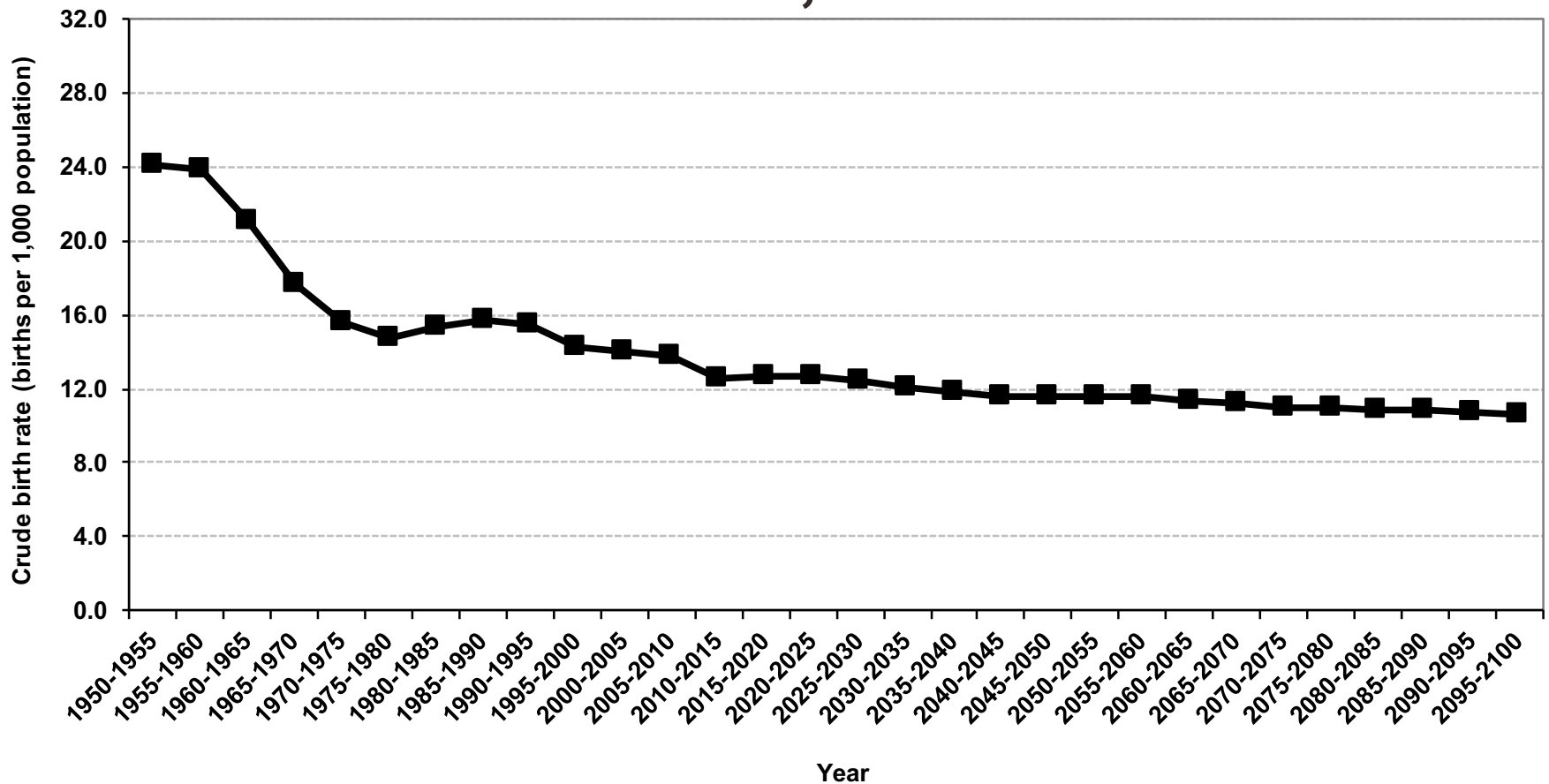
- Express the number of actual occurrences of an event (e.g. births, deaths, homicides) vs. number of possible occurrences per some unit of time
  - Population in the middle of the period as denominator
- Examples

$$\text{Crude birth rate} = \frac{\text{Number of births}}{\text{Total population}} \times 1,000$$

$$\text{Crude death rate} = \frac{\text{Number of deaths}}{\text{Total population}} \times 1,000$$



# Crude birth rates, United States, 1950–2100

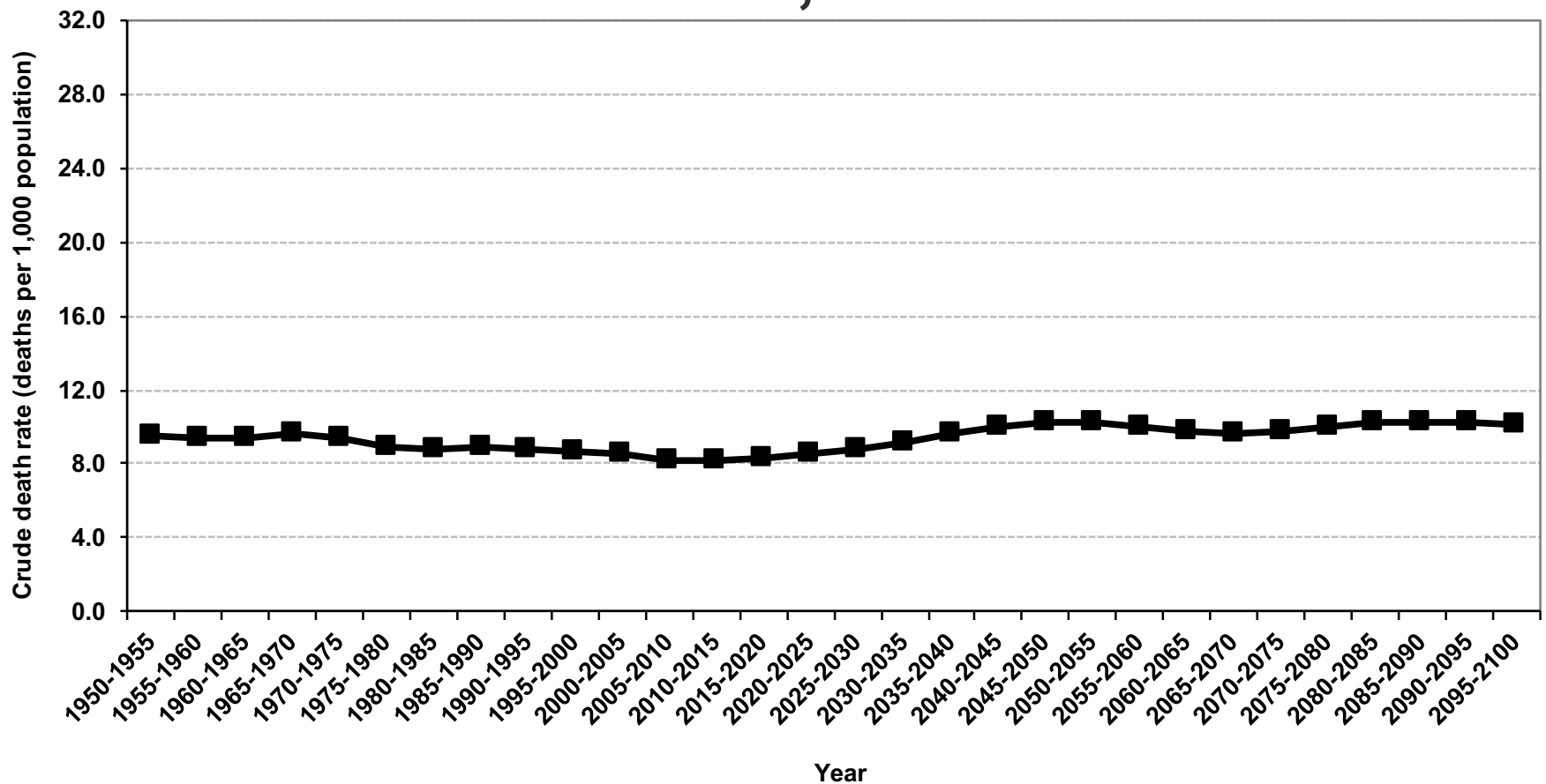


Source: United Nations, World Population Prospects 2017  
<https://esa.un.org/unpd/wpp/Download/Standard/Population/> (medium variant).





# Crude death rates, United States, 1950–2100



Source: United Nations, World Population Prospects 2017  
<https://esa.un.org/unpd/wpp/Download/Standard/Population/> (medium variant).



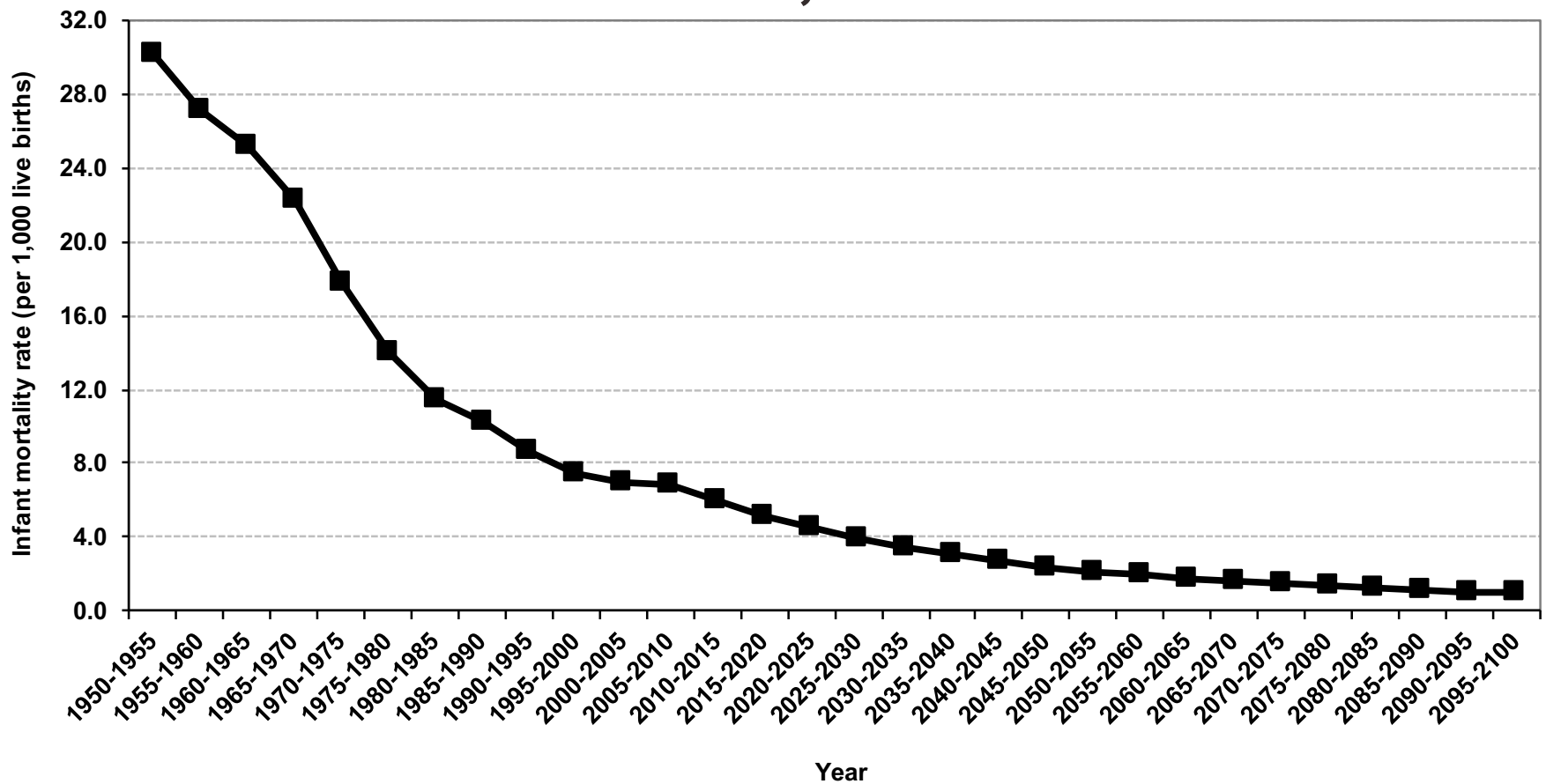


# Infant mortality rate (IMR)

$$IMR = \frac{\textit{the number of deaths under age 1 in the period}}{\textit{the number of live births in the period}}$$

- IMR is a period measure
- It uses current information from vital registration
- It can be computed for countries without reliable census or other source for a count of the population at risk by age
- Infants born by teenagers and by older mothers are at higher risk

# Infant mortality rates, United States, 1950–2100



Source: United Nations, World Population Prospects 2017  
<https://esa.un.org/unpd/wpp/Download/Standard/Population/>  
(medium variant).



# Probabilities

(Fleurence, Hollenbeak 2007)

- Probabilities describe the likelihood that an event will occur for a single individual in a given time period and range from 0 to 1
  - Do not include time in the denominator
  - Divide the number of events by the total number of people at risk in the relevant time frame
- Conversion between rates and probabilities:  
probability:  $p = 1 - e^{-rt}$   
rate:  $r = -1/t * \ln(1-p)$
- An approximation for the denominator is the population at the beginning of the period



# Ratios

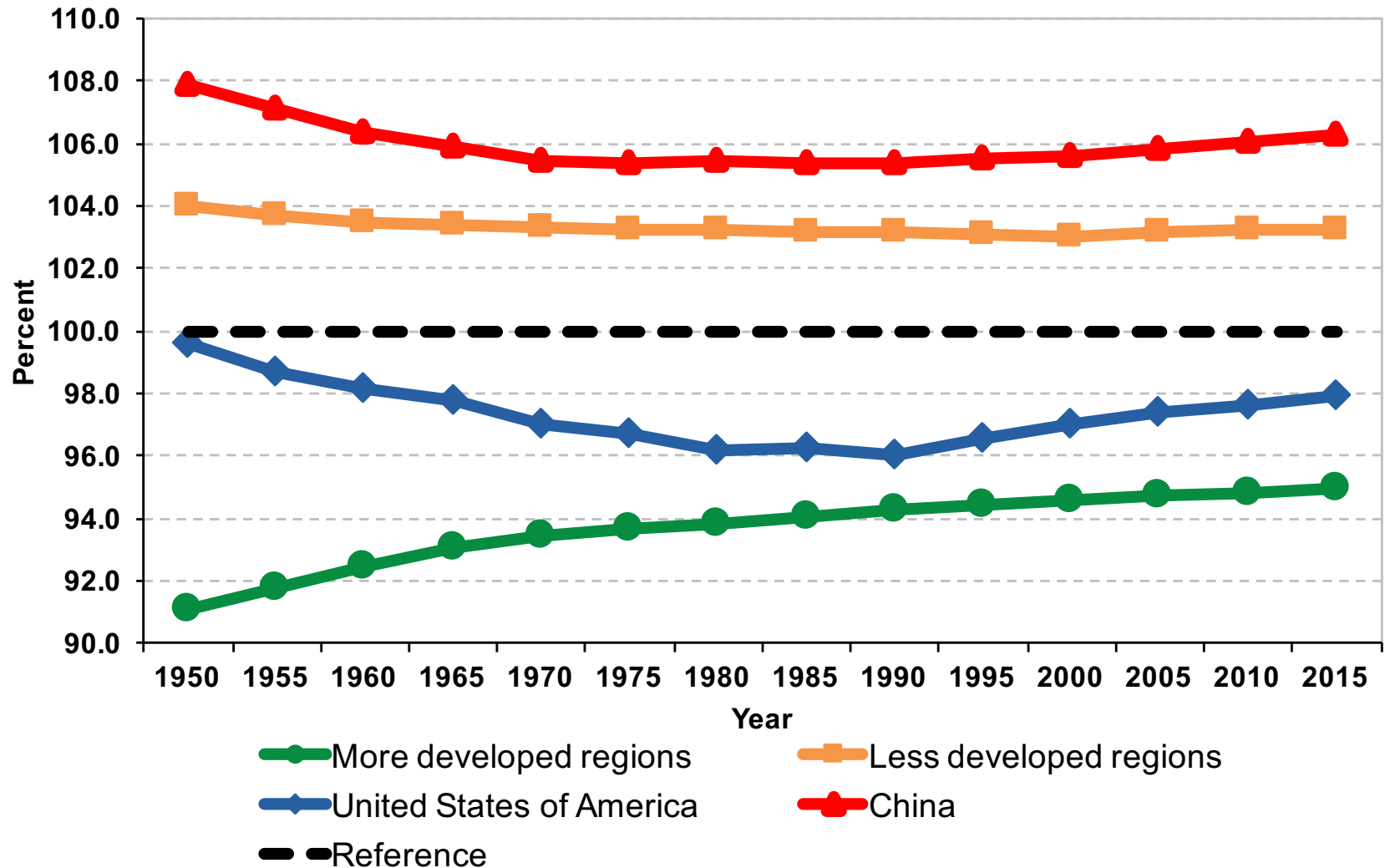
- Describe a relationship between two numbers
  - Compare the size of one group to the size of another group
  - Compare the relative sizes of categories
  - Indicate how many times the first number contains the second
  - Denominator is not at “risk” of moving to numerator
  - Optional: multiply by 100 to get percentage

$$\textit{Sex ratio} = \frac{\textit{Population of males}}{\textit{Population of females}}$$

$$\textit{Total dependency ratio} = \frac{\textit{Pop. children (0 to 14)} + \textit{Elderly pop. (65+)}}{\textit{Working age population (15 to 64)}}$$



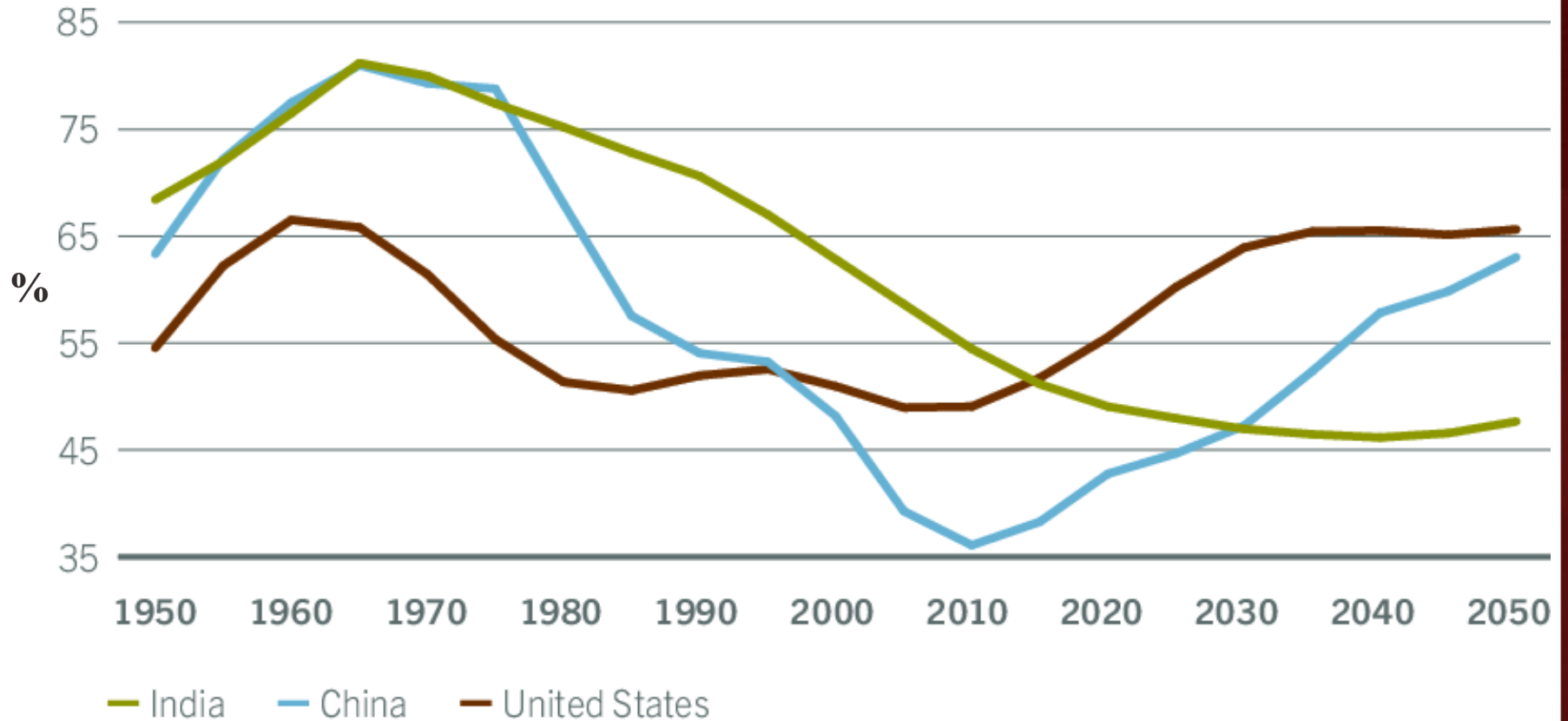
# Sex ratios, 1950–2015



Source: United Nations, World Population Prospects 2017  
<https://esa.un.org/unpd/wpp/Download/Standard/Population/>

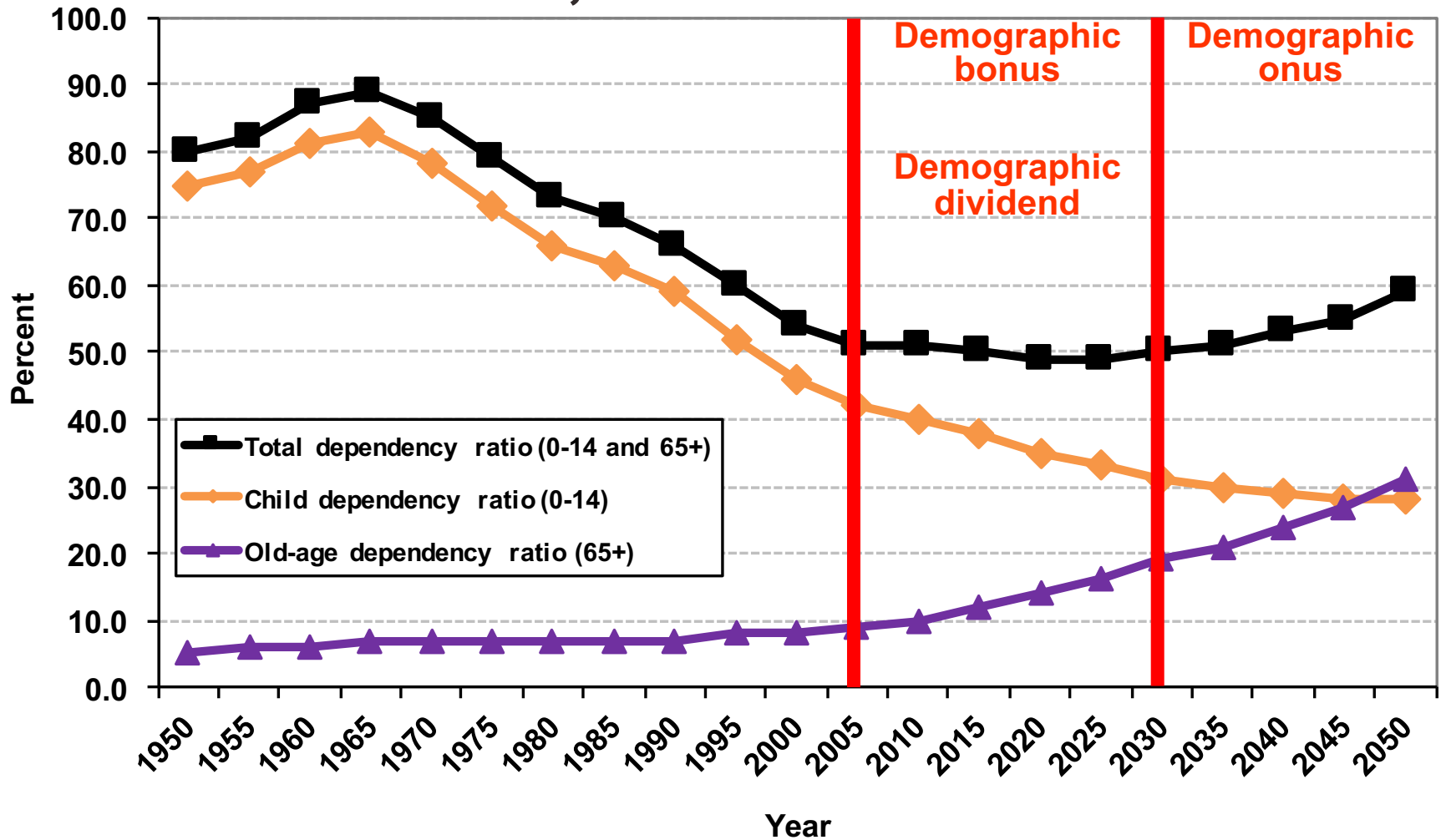


# Total dependency ratios, India, China, United States



Source: United Nations Population Division

# Dependency ratios, Brazil, 1950–2050



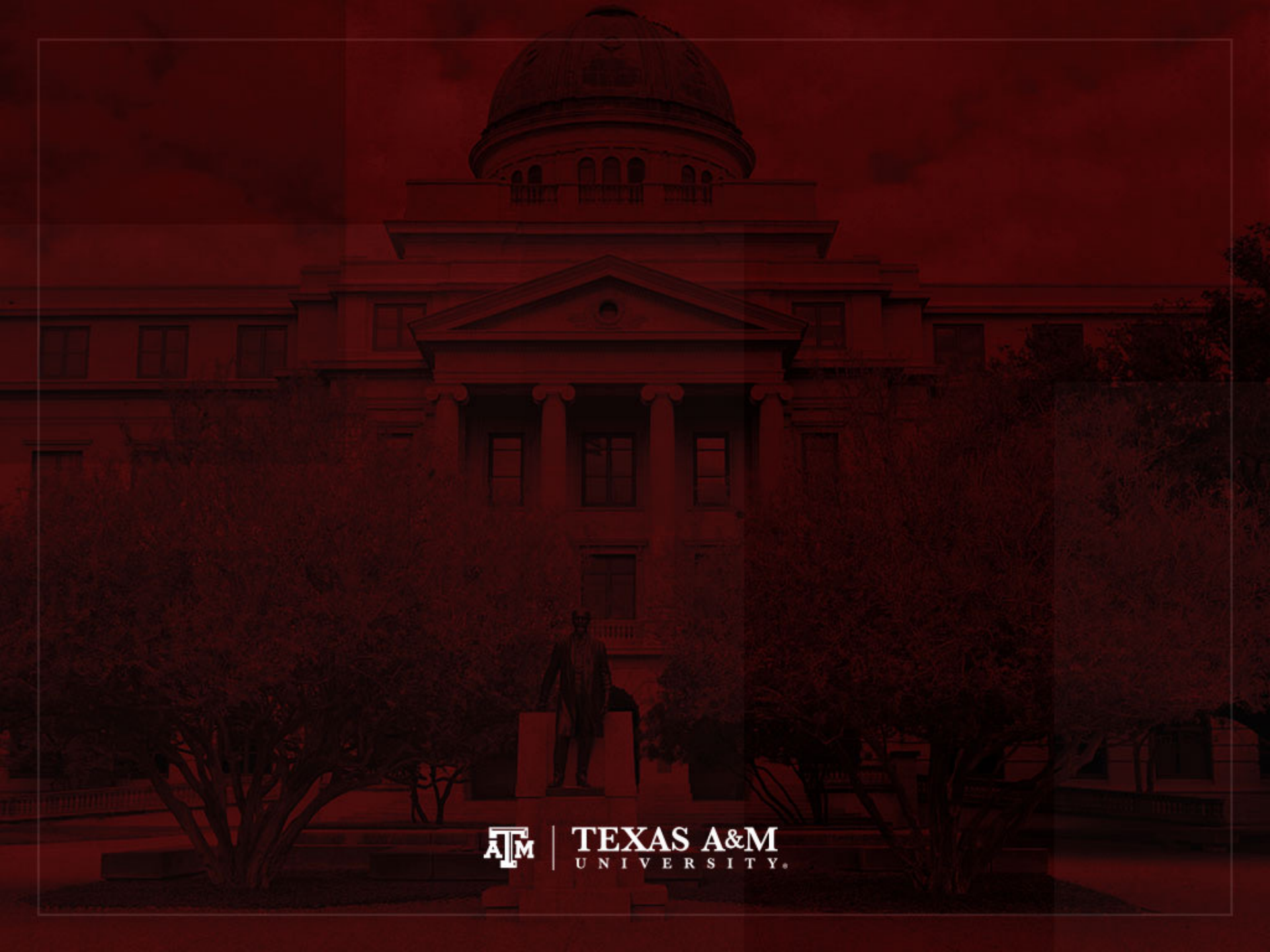
Source: United Nations - <http://esa.un.org/unpp> (medium variant).



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