Period fertility

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Demographic Methods (SOCI 320)

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Period fertility

- Introduction
- Period measures
- Period age-specific fertility
- Period *NRR*, *GRR*, and *TFR*
- Age-standardized rates
- Tempo and quantum
- Princeton indices

Introduction

- There are several types of fertility analysis
  - **Period (cross-sectional) perspective:** based on a particular point or period of time
  - **Cohort analysis:** based on fertility patterns of a group (cohort) of women who go through childbearing years at the same time
  - **Micro analysis:** fertility analysis of persons
  - **Macro analysis:** fertility analysis of groups, e.g., countries

Concepts of fertility

- Fertility
  - Actual production of male and female births
- Reproduction
  - Actual production of female births
- Fecundity
  - Biological capability of producing live births

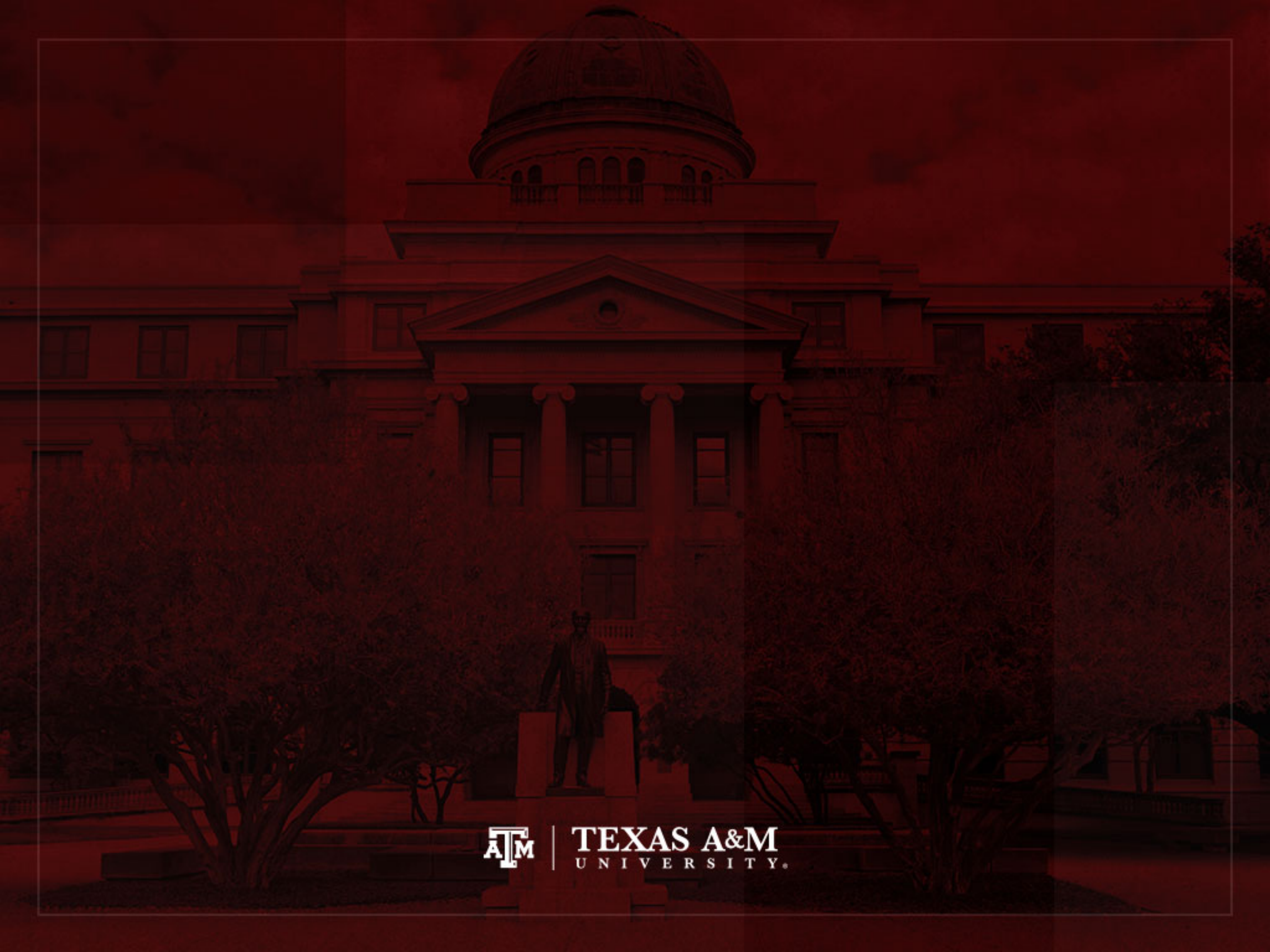
Fertility terms

- Fertility: actual production of births
- Infertility: childlessness either voluntary or involuntary
- Fecundity: ability to reproduce
  - Subfecund: definitely sterile, probably sterile, semifecund, and fecundity indeterminate
- Infecundity: sterility
- Menarche: beginning of the female reproductive period (first menstrual flow)
- Menopause: end of reproductive period (termination of menstruation)
- Postpartum: period of infecundability following a pregnancy; a function of the duration and intensity of lactation

Childbearing years

- Women in age group 15–49: these are the main ages when women are able to give birth

- Sometimes the age group of 15–44 is used, especially in developed countries, because so few births occur to women ages 45–49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Period measures

- We will now discuss how to calculate versions of the demographic measures based on period data
- The concepts of these measures are fundamentally cohort concepts
  - They describe features of the life course of individuals
- When we move to period versions of our measures, the concepts do not change
  - But the kinds of data do change

Motivation

- The motivation for calculating measures with period data is timeliness
  - We cannot determine a cohort's *NRR* until the last member of the cohort has completed childbearing
  - We cannot determine a full cohort lifetable until the last member of a cohort has died
- The most recent cohort *NRR*s and lifetables are not very relevant as a description of today's and tomorrow's childbearing and mortality
  - Cohort measures are out of date long before they are complete

Assumptions

- To estimate fertility and mortality before we have the full story required making assumptions
- Due to uncertainty, demographers adopt a neutral assumption
  - Today's rates will stay the same, rather than going up or down
  - More complicated forecasts can be calculated, but this assumption provides a baseline
  - Even though events prove this assumption wrong, it allows comparisons from place to place and time to time

Game of pretend

- When we calculate a period measure, we pretend that age-specific rates we see today for different age groups continue unchanged into the future
- We are creating an imaginary cohort whose life experience is pieced together from the experiences of different people found at different ages in one period of time

Example in Lexis diagram

- Period marked off by the vertical bar on the left is the current time (year 2000)
- Dashed horizontal lines single out the age group 20–25
  - Events of birth and death and counts of person-years in the small rectangle determine the period age-specific rates
  - These rates carry forward in time unchanged...

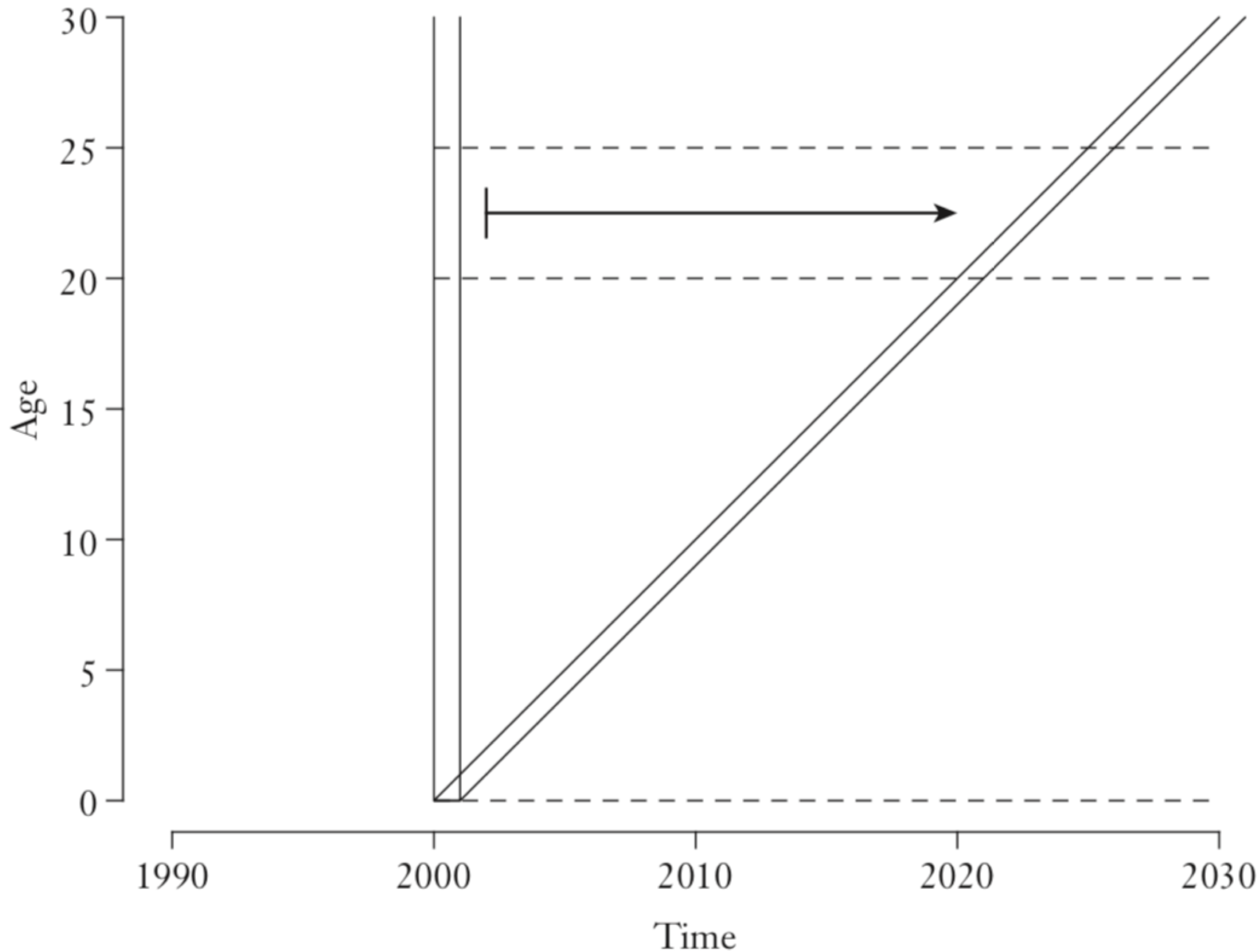


Figure 6.1 From period to cohort on a Lexis diagram

Source: Wachter 2014, p.127.

Synthetic cohort

- We call this imaginary cohort the synthetic cohort
  - *syn*: “together”
  - *thetic*: “pieced”
  - *synthetic*: “pieced together”
- Age-specific cohort rates of the synthetic cohort are the age-specific period rates of the period population
- The concept of a synthetic cohort is central to demography

Calculating period measure

- Take the formula for the cohort version of the measure
- Replace all the cohort age-specific rates with period-age specific rates
- Then evaluate the formula...

Examples

- What would a cohort have for an *NRR* if the period's age-specific rates persisted forever?
  - Period *NRR*
- What would a cohort have for a lifetable if the period's age-specific rates persisted forever?
  - Period lifetable
- The concepts are still cohort concepts
  - What changes is the source of data

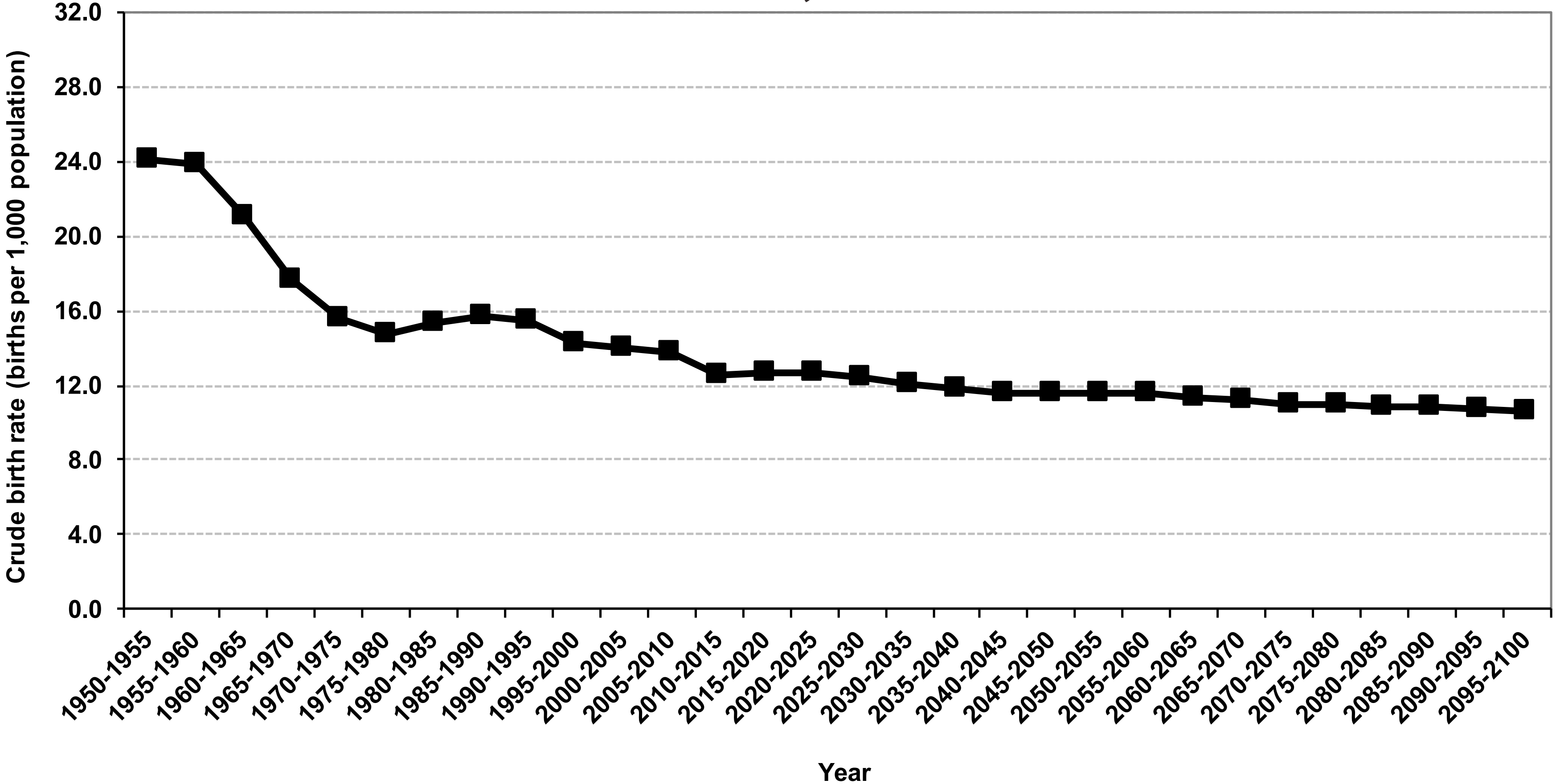
Crude birth rate (*CBR*)

- *CBR* is the number of live births (*B*) in a year divided by the total midyear population (*K*)

$$CBR = B / K * 1{,}000$$

  - It is usually multiplied by 1,000 to reduce decimals
  - It does not take into account which people in the population were at risk of having births
  - It ignores age structure of the population, which can affect the number of live births in a year

Crude birth rates, United States, 1950–2100



Source: United Nations, World Population Prospects 2017
https://esa.un.org/unpd/wpp/Download/Standard/Population/
(medium variant).

General fertility rate (*GFR*)

- *GFR* is the total number of births in a year (*B*) divided by the number of women in childbearing ages (${}_{30}K^{f}_{15}$)

$$GFR = B \,/\, {}_{30}K^{f}_{15} * 1{,}000$$

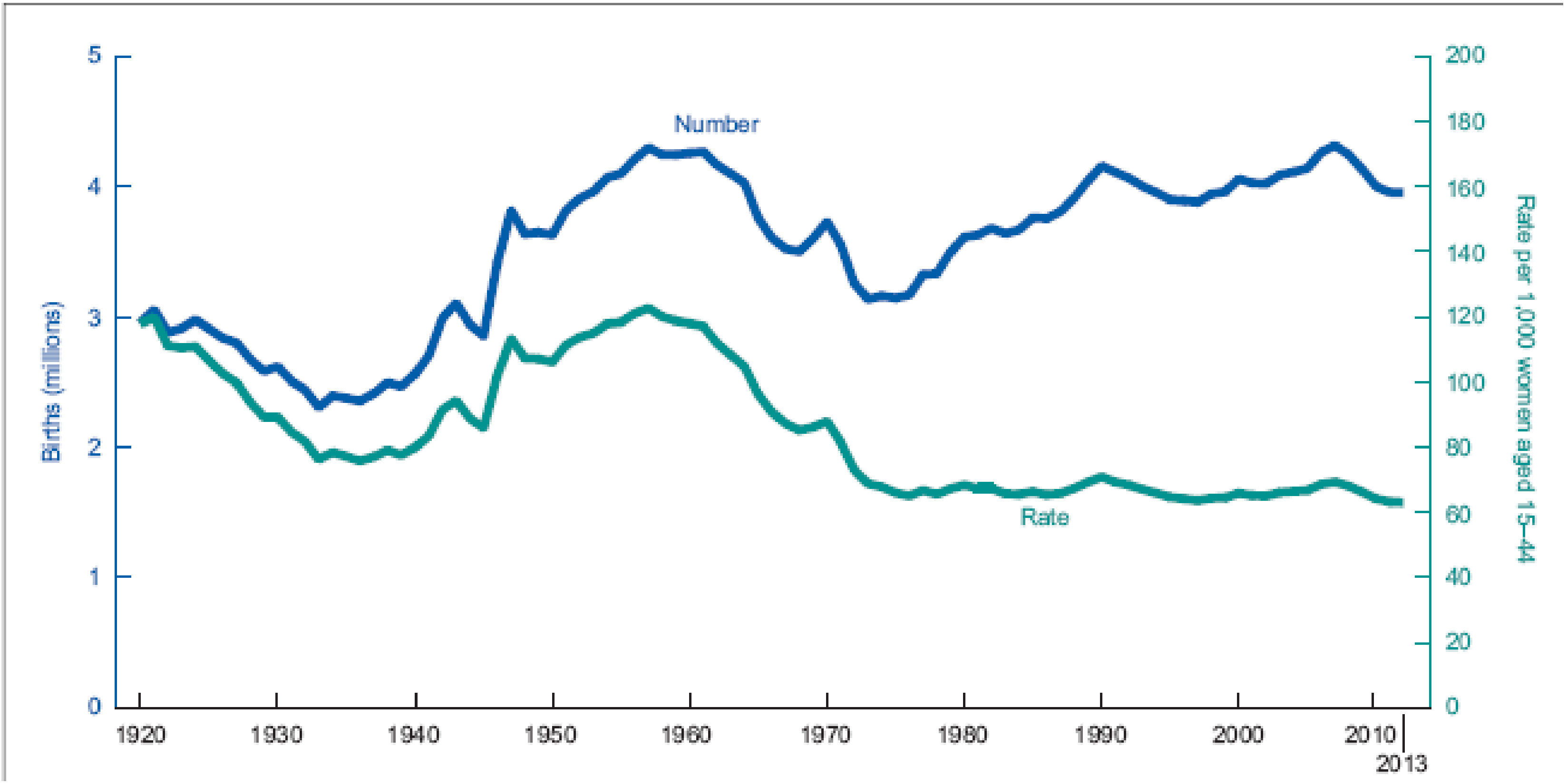
  - It is sometimes called “the fertility rate”
  - It uses information about age and sex structure
  - It usually equals to about 4.5 times the *CBR*

$$GFR = CBR * 4.5$$

If only data for *CBR* is available

Live births and *GFR*

Live Births and General Fertility Rates,* 1920 to 2013



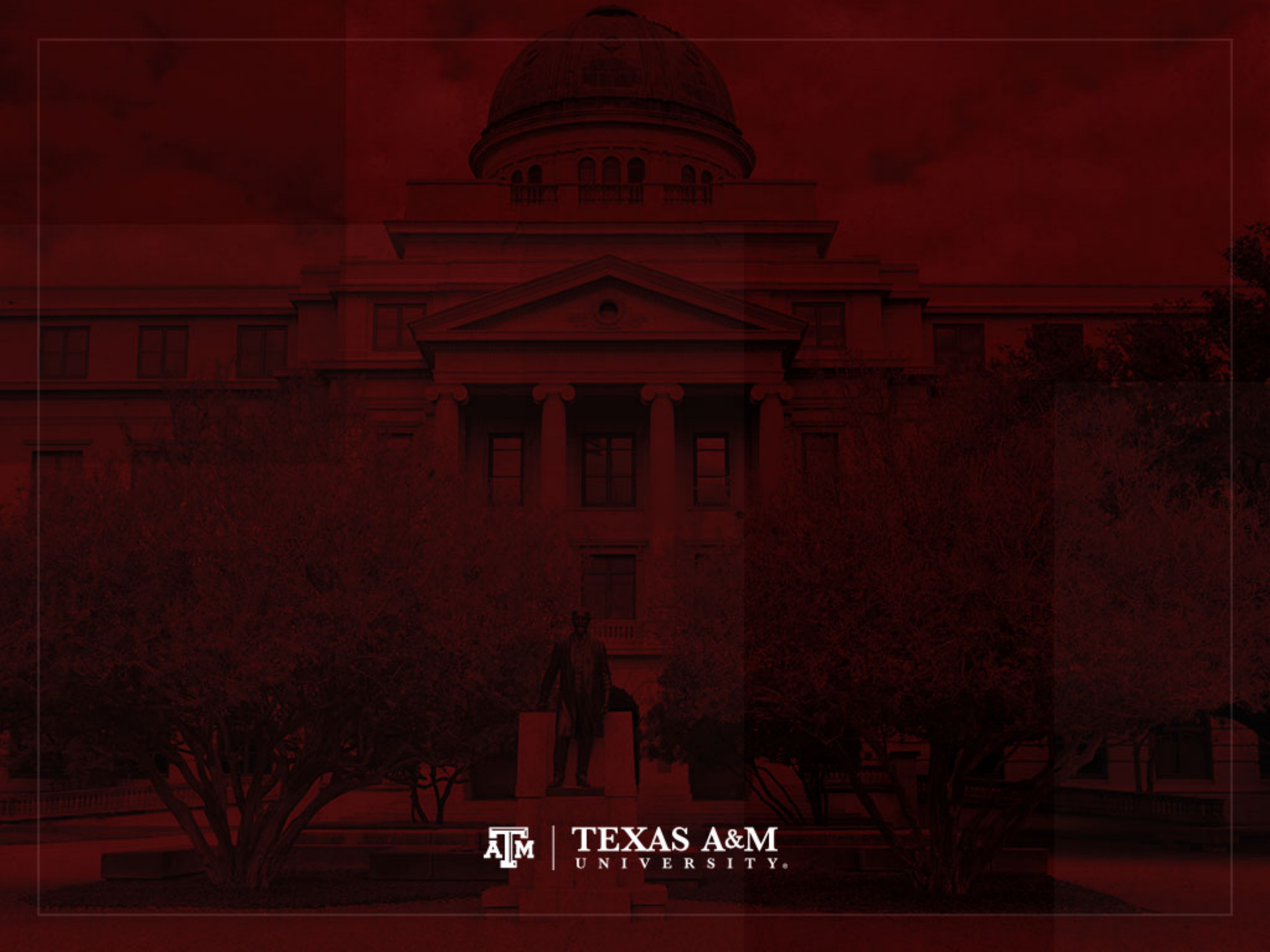
*The denominator of the General Fertility Rates is women aged 15-44.
Source: Martin, Hamilton, and Osterman, 2015: 3.

Child-woman ratio (*CWR*)

- *CWR* is the ratio of young children (0–4) enumerated in the census to the number of women of childbearing ages (15–49)

$$CWR = {}_{4}K_{0} \,/\, {}_{35}K^{f}_{15} * 1{,}000$$

  - It provides an index of fertility that is conceptually similar to *GFR*, but it relies only on census data
  - It uses an older upper limit on women's age, because some of the children (0–4) will have been born up to five years prior to the census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Period age-specific fertility

- Cohort age-specific fertility ($_nf_x$)
  - Numerator: count of babies born to the cohort between ages *x* a *x+n*
  - Denominator: cohort person-years lived
- Period age-specific fertility ($_nF_x$)
  - Numerator: count of babies born in the period to population members between ages *x* and *x+n*
  - Denominator: period person-years lived by people between ages *x* and *x+n*

Period age-specific rates

- For a specific period
  - ${}_nB_x$: count of births
  - ${}_nD_x$: count of deaths
  - *PPYL*: person-years lived for women (or men) aged x to $x+n$ in the period
- Period age-specific fertility rate (*ASFR*)

$${}_nF_x = {}_nB_x \,/\, PPYL$$

- Period age-specific mortality rate (${}_nM_x$)

$${}_nM_x = {}_nD_x \,/\, PPYL$$

n and *T*

- Period rates usually have different *n* and *T*
  - The width *n* of the age group is not generally the same as the duration *T* of the period
  - Often *n*=5 and *T*=1

- In Leslie matrices, age-group width (*n*) and length of projection step (*T*) have to be the same

Period and cohort rates

- Period rates (${}_nF_x$)
  - Events in the numerator and the person-years in the denominator are being counted inside the rectangle in the Lexis diagram with height *n* and base *T*
- Cohort rates (${}_nf_x$)
  - Events and person-years are counted inside a parallelogram with diagonal sides on the Lexis diagram
- Thus, ${}_nF_x$ is only approximately equal to the cohort ${}_nf_x$ for the cohort born *x* years before the period

Denominator

- Our usual estimate of period person-years lived (*PPYL*) is the mid-period population times the period length (${}_nK_x$ * T)
  - With a period 1 year in length, *PPYL* has the same numerical value as the mid-year population
    - But units of person-years instead of units of people
  - With a period 10 years in length, *PPYL* would be about 10 times the mid-period population
    - But there would be about 10 times as many babies born over 10 years for the numerator
    - Thus, ${}_nF_x$ would be roughly the same

ASFR

- Age-specific fertility rate (*ASFR*) is the number of births (*B*) occurring in a year to mothers aged x to $x+n$ (${}_{n}B_{x}$) per 1,000 women (${}_{n}K^{f}_{x}$) of that age

$${}_{n}ASFR_{x} = {}_{n}F_{x} = {}_{n}B_{x} / {}_{n}K^{f}_{x} * 1,000$$

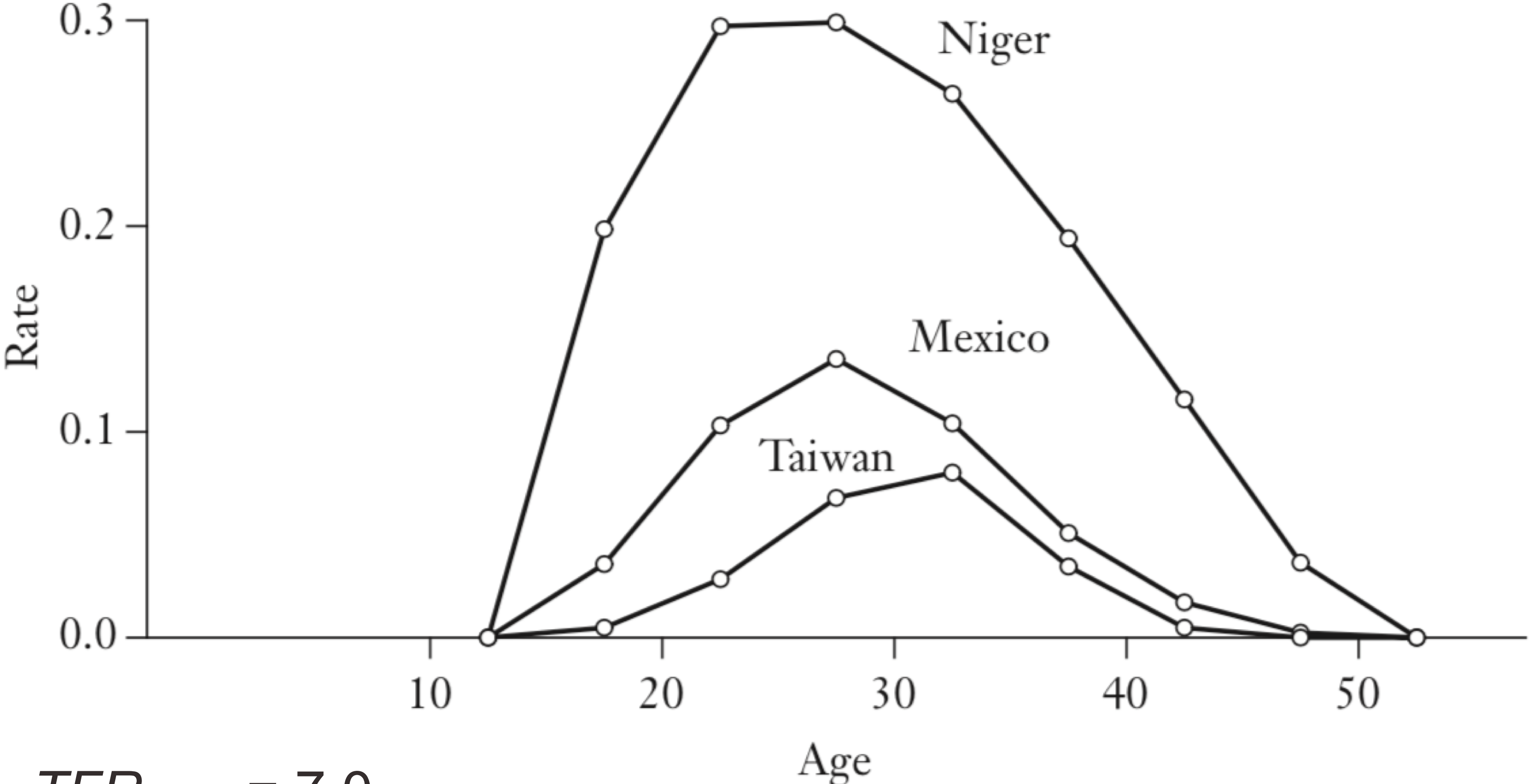
  - ${}_{n}ASFR_{x}$ means *ASFR* for age group x to $x+n$
  - It provides births rates of women according to their ages
  - It requires comparisons of fertility be done on an age-by-age basis
  - It is usually calculated in five-year age groups

5-year age groups

- *ASFR* ($_nF_x$) are usually calculated for women in each of the seven 5-year age groups
  - 15–19, 20–24, 25–29, 30–34, 35–39, 40–44, 45–49
  - Sometimes 35 single-year age groups are used

- Fertility schedule (age curve of fertility)
  - Set of values $_nF_x$ for all reproductive age groups
  - Fertility schedules have typical shapes
  - Seven plotted $_nF_x$ usually have an inverted U shape...

Age-specific fertility schedules, 2013



$TFR_{Niger} = 7.0$
$TFR_{Mexico} = 2.2$
$TFR_{Taiwan} = 1.1$

Source: Wachter 2014, p.129.

Other rates use ${}_nF_x$

- Period age-specific rates are the building blocks for the main period measures of fertility
  - *NRR*, *GRR*, *TFR*

- In the next example for fertility (${}_nF_x$) and mortality (${}_nM_x$) rates
  - Mid-year count (${}_nK_x$) is the estimate of *PPYL* for one-year period

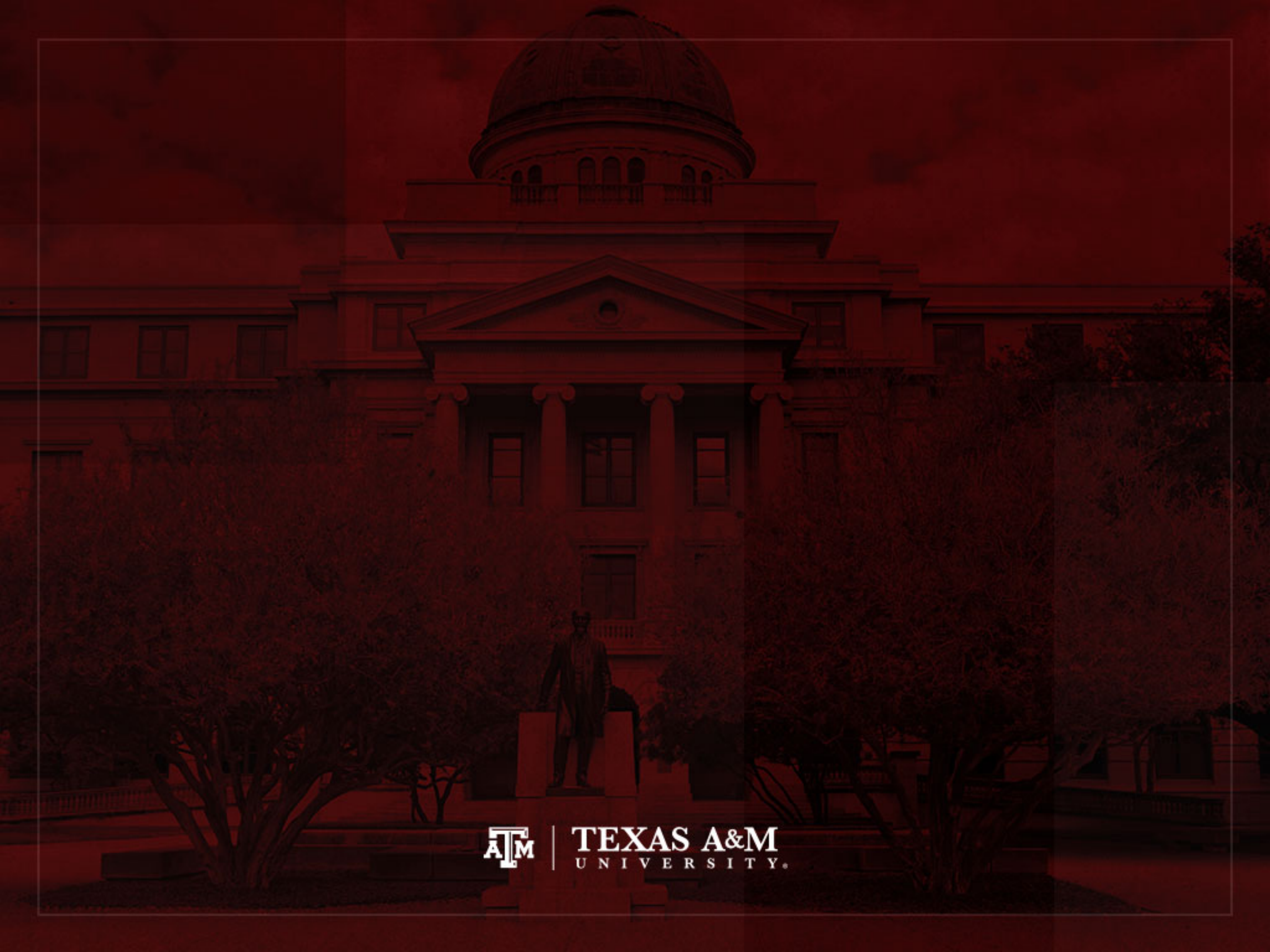
Table 6.1 Age-specific rates for India, 2000

x	${}_nB_x$	${}_nK_x$	${}_nF_x$	${}_nD_x$	${}_nK_x$	${}_nM_x$	${}_nL_x$
15	2,430	48,407	0.050	65	48,407	0.00134	4,442
20	9,258	42,371	0.218	115	42,371	0.00270	4,398
25	7,128	39,708	0.180	97	39,708	0.00245	4,341
30	3,593	36,036	0.100	112	36,036	0.00311	4,281
35	1,632	31,880	0.051	88	31,880	0.00276	4,219
40	627	27,725	0.023	104	27,725	0.00376	4,151
45	183	23,125	0.008	98	23,125	0.00424	4,069

Source: Author's calculations from WPP (2003).

$l_0 = 1,000$

Source: Wachter 2014, p.130.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Period *NRR*, *GRR*, and *TFR*

- The inputs for calculating a period *NRR* and other period fertility measures are
  - Period ${}_nF_x$ values
  - ${}_nL_x$ values from the period lifetable
  - Both computed by pretending that the age-specific rates of the period continue unchanged forever

Net reproduction ratio (*NRR*)

- The formula for the period *NRR* is exactly the same as the formula for the cohort *NRR*, except
- Period age-specific fertility rates (${}_nF_x$) replace cohort age-specific fertility rates (${}_nf_x$)
- Period lifetable (${}_nL_x$) values replace cohort lifetable (${}_nL_x$) values

$$NRR = \sum {}_nF_x \, {}_nL_x \, f_{\text{fab}} / \ell_0$$

Table 6.2 Calculating the NRR, India, 2000

x	${}_5F_x$	${}_5L_x$	Babies	$x + n/2$	Product
15	0.050	4,442	222	17.5	3,885
20	0.218	4,398	959	22.5	21,578
25	0.180	4,341	781	27.5	21,477
30	0.100	4,281	428	32.5	13,910
35	0.051	4,219	215	37.5	8,063
40	0.023	4,151	95	42.5	4,037
45	0.008	4,069	33	47.5	1,567
Sum	0.630		2,733		74,518

- $NRR = \Sigma\, {}_nF_x\, {}_nL_x\, f_{fab} / l_0$
- $NRR = 2{,}733 * 0.4886 / 1{,}000 = 1.335$

Source: Wachter 2014, p.131.

We use ${}_nL_x$, not *PPYL*

- For the period *NRR*, person-years are ${}_nL_x$ values from the period lifetable
  - Not Period Person-Years Lived (*PPYL*)
  - We use *PPYL* values to find the age-specific rates ${}_nF_x$ and ${}_nM_x$
  - From that point on, age-specific rates are all we need
  - We use ${}_nF_x$ and ${}_nM_x$ to compute a period lifetable, which is a cohort lifetable for the synthetic cohort
  - Every further computation uses quantities for the synthetic cohort from ${}_nL_x$ (not *PPYL*)

Eliminating age structure effect

- If we made the mistake of using *PPYL* instead of ${}_nL_x$
  - Our answers would depend on how many people are at various ages in the period population

- Dividing births and deaths by *PPYL* gets rid of the effect of current age structure
  - It lets us construct period measures that express the life experience implied by the age-specific rates

Other notations for *NRR*

- It considers the factor of mortality among mothers from the time of births of their daughters
  - Based on the concept of population replacement

$$NRR = \sum({}_nASFR^f_x * {}_nL_x / 5l_0 * n)$$

$$NRR = \sum({}_nASFR_x * 0.4886 * {}_nL_x / 5l_0 * n)$$

  - ${}_nASFR^f_x$: female births per women in age group
  - ${}_nL_x$: total number of person-years lived in age group
  - l_0: number of people at age 0
  - ${}_nL_x / 5l_0$: proportion of people who survive from age 0 to the midpoint of each of the seven age intervals
  - n: width of the age group, usually 5

Other notations for *NRR*

- *NRR* is the age-specific birth rates using only female babies ($ASFR_f$)
  - Multiplied by the probability that a woman will survive to the midpoint of the age interval
  - The probability that a woman will survive to the midpoint of the age interval equals
    - ${}_nL_x$ (number of women surviving to the age interval x to $x+n$)
    - Divided by $5{*}l_0$ (radix multiplied by 5)

$$NRR = \Sigma\ [(ASFR^f * 5)(nLx\ /\ 5{*}l_0)] = \Sigma\ (ASFR^f * nLx\ /\ l_0)$$

More about *NRR*

- *NRR* is less than *GRR*
  - *S*ince some women die before the end of the reproductive period

- If *NRR* = 1
  - Each generation of females has the potential to replace itself (generational replacement)

Mean age at childbearing

- We can find out how young or old, on average, the mothers are at the babies' births
- Since our data are grouped into 5-year-wide intervals, we do not know the exact ages of mothers
- We can use the middle age in each age group as an approximation

Formula

- Previous result is the average age of mothers from the synthetic cohort at their babies' births
- It is called the synthetic cohort mean age at childbearing, μ (mu)

$$\mu = \sum ({}_nF_x)({}_nL_x)(x + n/2) / \sum ({}_nF_x)({}_nL_x)$$

Usual μ value

- In most countries and periods μ is around 27
- Women in countries with early marriage also tend to continue to bear children late in their reproductive span
  - Leading to average ages that are not very different from countries with late marriage but substantial fertility control

Table 6.2 Calculating the NRR, India, 2000

x	${}_5F_x$	${}_5L_x$	Babies	$x + n/2$	Product
15	0.050	4,442	222	17.5	3,885
20	0.218	4,398	959	22.5	21,578
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Sum	0.630		2,733		74,518

- Women aged 15–20
  - ${}_nF_x * {}_nL_x = 0.050 * 4{,}442 \approx 222$
  - ${}_nF_x * {}_nL_x * x + n/2 = 0.050 * 4{,}442 * 17.5 \approx 3{,}885$
- All women
  - $\Sigma\ ({}_nF_x * {}_nL_x) \approx 2{,}733$
  - $\Sigma\ ({}_nF_x * {}_nL_x * x + n/2) \approx 74{,}518$
- Mean age at childbearing: 74,518 / 2,733 ≈ 27.27

Source: Wachter 2014, p.131.

TFR and *GRR*

- Total Fertility Rate and Gross Reproduction Ratio can also be calculated from period data
- Formulas for period *TFR* and *GRR* are the same as the formulas for cohort *TFR* and *GRR*,
  - With period ${}_nF_x$ replacing cohort ${}_nf_x$
  - With the rate ${}_nF_x$ applying to n full years of risk in each age group, since *TFR* and *GRR* assume survival through all ages of childbearing

$$\text{Period } TFR = \sum ({}_nF_x)(n)$$

$$\text{Period } GRR = \sum ({}_nF_x)(n)(f_{\text{fab}})$$

Logical relationships

- *NRR* is always less than or equal to the *GRR*
  - Since mortality can only decrease the net total of daughters
- *GRR* is always less than or equal to the *TFR*
  - Since daughters are a subset of sons and daughters

Table 6.2 Calculating the NRR, India, 2000

x	${}_5F_x$	${}_5L_x$	Babies	$x + n/2$	Product
15	0.050	4,442	222	17.5	3,885
20	0.218	4,398	959	22.5	21,578
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45	0.008	4,069	33	47.5	1,567
Sum	0.630		2,733		74,518

- Every age group is 5 years wide, so we can add up ${}_nF_x$ values before multiplying by n

$$TFR = 0.630 * 5 = 3.150$$

$$GRR = 3.150 \ast 0.4886 = 1.539$$

Source: Wachter 2014, p.131.

Different age-group widths

- When age-group widths differ, we have to compute the products of ${}_nF_x$ and n row by row and add up the results to find the *TFR* and the *GRR*

Other notations for *TFR*

- The most popular measure of fertility
- Mostly cross-sectional, but also calculated for cohorts
- Definition
  - Number of births that a hypothetical group of 1,000 women would produce during their reproductive years
  - Between the ages of 15 and 49

$$TFR = \sum ({}_nASFR_x * n)$$

  - n: width of the age group, usually 5
  - *TFR* can be divided by 1,000 to obtain the average number of children born per woman

TFR assumption

- **Assumption**: current birth rates remain constant and no woman dies before reaching the end of childbearing years

- **Synthetic cohort**: *ASFR*s are used to project what would happen if all women went through their lives bearing children at the same rate as women at a given date

- It can be compared across populations, because it takes into account differences in age structure

TFR oscillations

- *TFR* can change rapidly and can signal major shifts in demographic experience
- Period *TFR* is an abstraction calculated for a synthetic cohort
- Real cohorts do not experience large swings in total fertility as indicated by period measures
- Period *TFR* is affected by changes
  - In ages of childbearing
  - In completed cohort parity through influences called "tempo effects"

Limitations

- As *TFR*, the period *NRR* is an abstraction
- It is a useful abstraction that tells us what the long-term implications of present-day fertility and mortality would be for population growth
- In the real world, age-specific fertility and mortality do not remain constant as they are assumed to do in our “game of pretend”
- The long-term implications of short-term rates expressed in the *NRR* are not telling us about the real future

Need careful interpretation

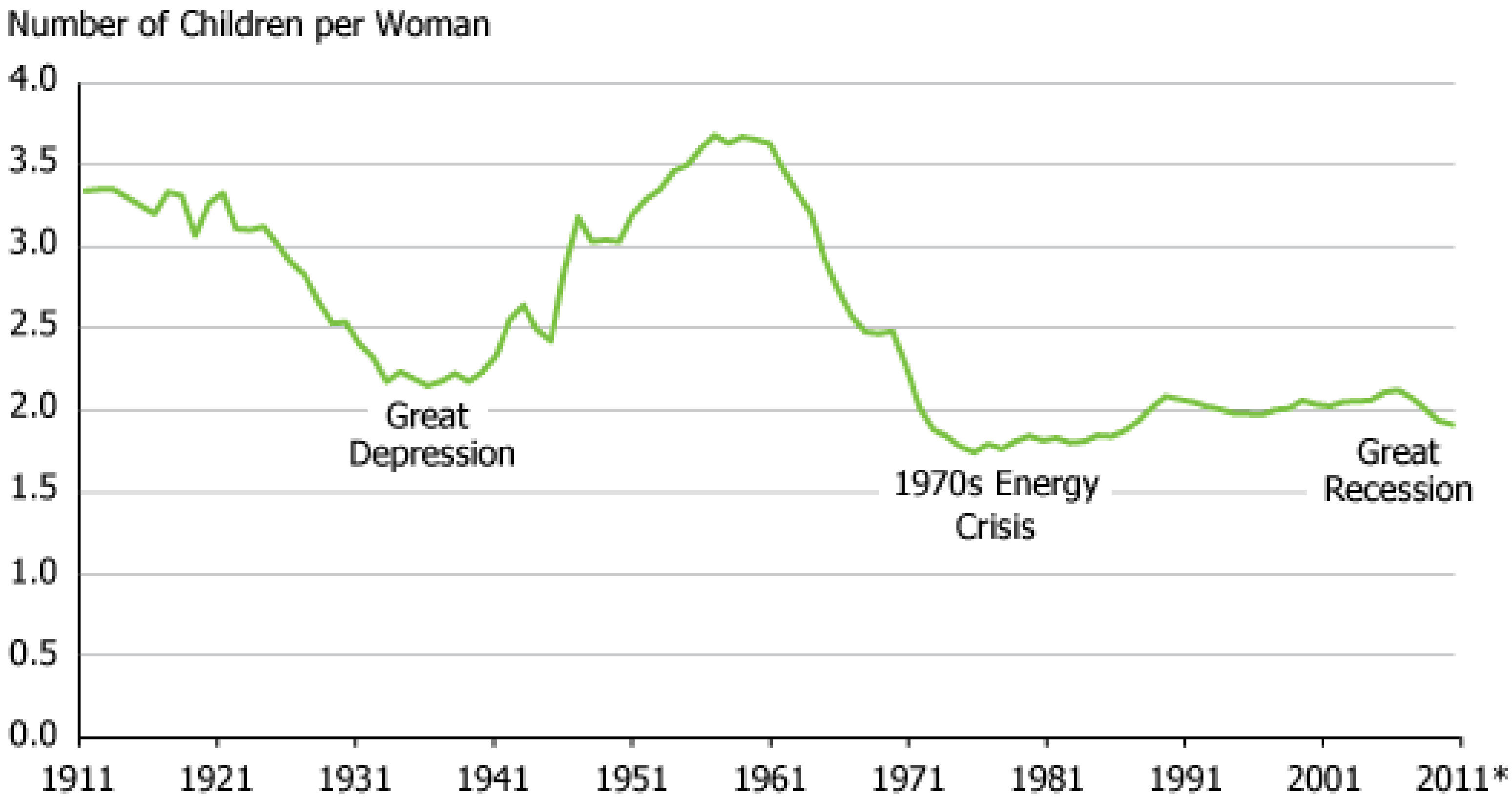
- In 2010, the period *NRR* for the United States was a little below 1
  - If the age-specific rates of 2010 continued unchanged forever, eventually the size of the U.S. population (excluding net immigration) would begin to decline
  - This doesn't mean that population growth has ceased
- In fact, the U.S. has the fastest growing population of any large developed country
  - Births exceed deaths by a substantial amount
  - Moreover, net immigration is a major factor for population growth in the U.S.

Example of *TFR*

- Even with limitations, *TFR* is one of the most widely cited measures in demography
- As reported in the Human Fertility Database (HFD), the period *TFR* in the United States
  - 2.380 in 1945
  - 3.161 in 1947 at the onset of the Baby Boom
  - 3.738 in 1957
  - 2.467 by 1968
  - 1.792 in 1984
  - Around 1.928 in 2010

TFR in the United States

Total fertility rates, United States, 1911 to 2011.



Source: Mather, 2012 (reprinted with permission of the Population Reference Bureau).

Approximation for *TFR*

- *TFR* = *CBR* * 4.5 * 30 = *GFR* * 30
  - When only *CBR* or *GFR* data are available
- Period *TFRs* are preferred over cohort *TFRs* due to their currency

Other notations for *GRR*

- *GRR* is the sum of age-specific birth rates using only female babies ($ASFR^f$), since only female babies will bear children

$$GRR = \sum({}_nASFR^f{}_x * n)$$

  - ${}_nASFR^f{}_x$: female births per women in age group x to $x+n$
  - n: width of the age group, usually 5
  - Similar to *TFR*, but it includes female births only
  - Based on the concept of population replacement

GRR interpretation

- It is the number of female children that a female just born may expect to have during her lifetime
  - *GRR*=1; women replace themselves
  - *GRR*<1; women do not replace themselves
  - *GRR*>1; next generation of women will be bigger than the present one

GRR assumption

- Current birth rates remain constant and no woman dies before reaching the end of childbearing years

Approximation to *GRR*

- Approximation to *GRR*

$$GRR = TFR * \text{female births} / \text{births}$$

$$GRR = TFR * 0.488$$

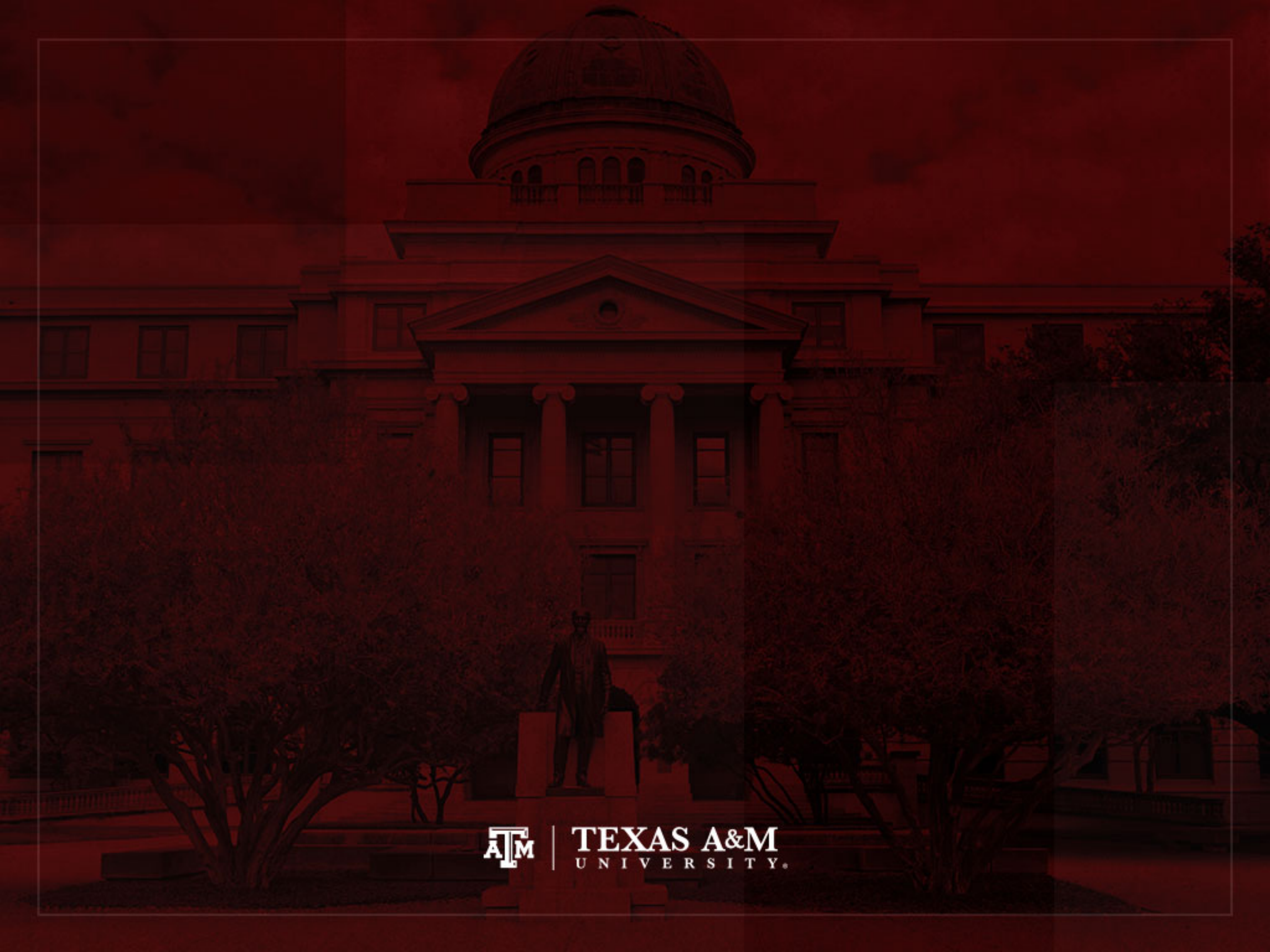
  - Constant 0.488 is based on the sex ratio at birth of most countries
  - *SRB* = 105
  - Proportion of female births (f_{fab})

$$f_{fab} = 1 – \text{proportion of male births}$$

$$f_{fab} = 1 – [105 / (105+100)]$$

$$f_{fab} = 1 – 0.512 = 0.488$$

  - If SRB ≠ 105, another constant should be used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Age-standardized rates

- We prefer *TFR*, *GRR*, and *NRR* instead of *CBR*, because they are not influenced by the age distribution of the population
  - E.g., bringing 20-year-olds into a population will raise number of births and raise a crude measure (*CBR*)
  - But will leave period *TFR*, *GRR*, and *NRR* unchanged if age-specific rates remain unchanged
- Sometimes, we can keep the simplicity of a crude rate while removing the effects of the observed age distribution
  - We can calculate age-standardized rates

Calculating standardized rates

- Age-standardized rates remove the effects of observed age distribution

- They serve for quick comparisons among contrasting countries or areas
  – Use a standard population (e.g., world in 2000)
  – Take each country, multiply its age-specific rates by the standard population counts
  – Add up the products
  – Divide by the total standard population to obtain the age-standardized rate

Table 6.3 An age-standardized birth rate

	x	n	Standard ${}_nK_x$	France ${}_nF_x$	Product (babies)
	0	15	882	0	0
W	15	5	270	0.008	2.107
O	20	5	248	0.056	13.864
M	25	5	245	0.134	32.726
E	30	5	232	0.118	27.483
N	35	5	209	0.050	10.531
	40	5	182	0.012	2.108
	45	5	164	0.000	0
	50	∞	574	0	0
M					
E	0	∞	3,051	0	0
N					

Source: United Nations World Population Prospects (2001).

- Add up standard population

  6,057 million

- Add up babies

  88.819 million

- Standardized *CBR*

  88.819 / 6,057

  = 0.014539

- Now we can get another country, apply same standard population, and compare fertility levels freed from direct effects of age structure
- For reference, observed *CBR* from France: 0.013228

Source: Wachter 2014, p.137.

Other standardized rates

- Use age-specific marriage rates
  - Apply to standard counts
  - Get an age-standardized marriage rate

- The same can be done for mortality rates...

Mortality standardized rates

- We can use age-specific mortality rates (${}_nM_x$), apply to standard counts to get an age-standardized death rate
  - Most widely used, because age structure has so much influence on the *CDR*

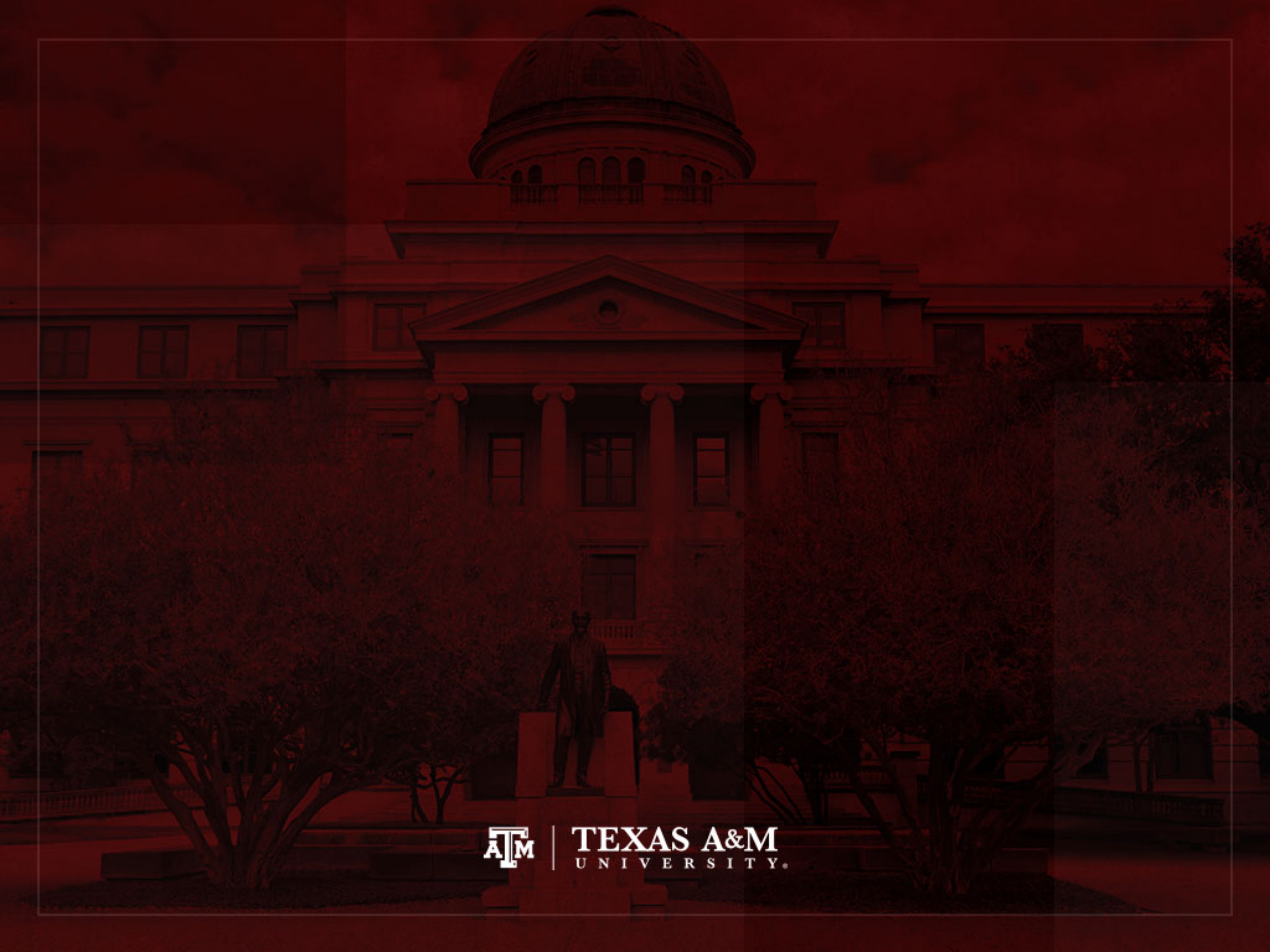
- Age-standardized rates remove false impression
  - The highest *CDR*s are not found in countries with the most severe mortality
  - The highest age-standardized death rates are found in countries with the most severe mortality

Example of *CDR*

- $CDR_{\text{U.S.}} \approx CDR_{\text{world}} \approx 9$
- The U.S. has lower age-specific mortality rates at every age than the world
- But the U.S. has a much higher percentage of its population in older, high-risk age groups
- Favorable mortality rates can be disguised by the effect of older age structure

Direct, indirect standardizations

- Previous examples are direct standardization
  - Rates come from countries under study
  - Counts of people at risk come from a standard population
- Indirect standardization
  - Rates come from a standard population
  - Counts of people at risk come from countries under study
  - They are the basis for indices of family limitation called “Princeton Indices”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Tempo and quantum

- Demographers use the word “tempo” in general to refer to the timing of births (or other events) within a person’s lifecourse

- The word “quantum” refers to the lifetime number of births (or other events)

Recent changes in fertility

- In recent years, major swings in fertility have trended to respond to chronological period influences, cutting across cohorts
- Adjustments for changing ages at childbearing are meaningful
  - We care about the cohort quantum of fertility

Changing ages of childbearing

- *TFR* is a measure of fertility standardized for the number of women at risk of childbearing

- We can take a step further and calculate a measure which is standardized for average ages of childbearing
  - *TFR*(*t*): Total Fertility Rate in the period from *t* to *t*+*T*
  - *A*(*t*): average age of childbearing implied by the age-specific fertility rates for each instant of time
  - *A*(*t*) needs to be estimated from fertility rates in short periods centered on *t*

Standardizing by childbearing age

- Begin by choosing some standard age, $A^{(s)} = 25$
  - Take a standardized sample of births, reflecting the whole set of age-specific fertility rates over time
  - Each child birth occurs at some time t to a mother at some age x
  - We shift this birth forward or backward along its mother's diagonal lifeline on the Lexis diagram by the difference $A(t) - A^{(s)}$
- Thus, a birth that takes place at mother's age x at time t is reassigned
  - To take place at mother's age $x - A(t) + A^{(s)}$
  - At time $t - A(t) + A^{(s)}$

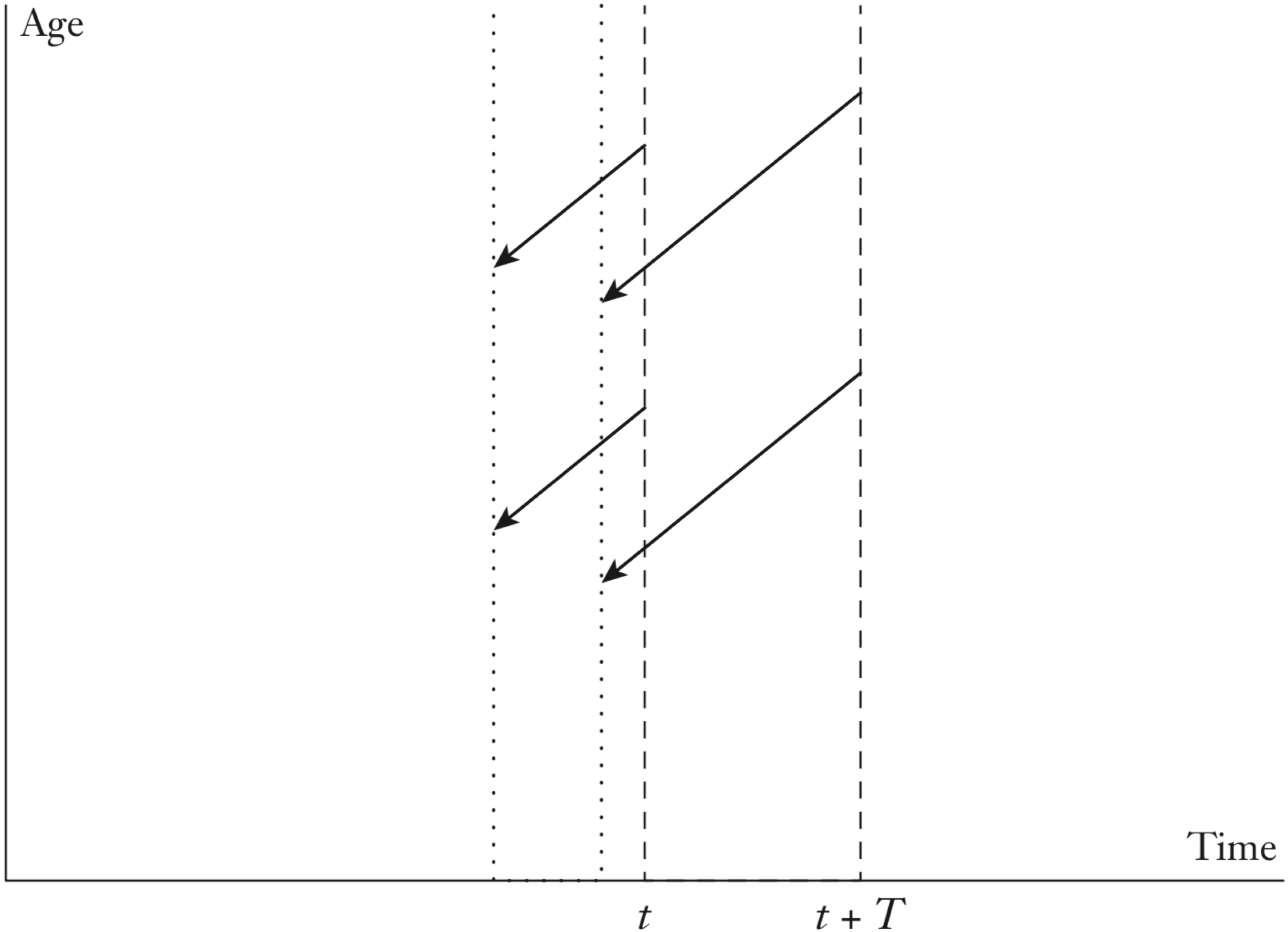


Figure 6.4 Birth age standardization for tempo adjustment

Source: Wachter 2014, p.139.

Result after diagonal shift

- As long as A(s) is sensible enough to avoid negative ages, these diagonal shifts leave every cohort TFR unchanged
  - All births originally belonging to a given cohort of mothers still belong to the same cohort of mothers
- However, the average age of the mothers for births originally at time *t* is changed by the shift and ends up equal to

$$A(t) - (A(t) - A^{(s)}) = A^{(s)}$$

  - After the shift, average ages are all constant and equal to our choice of standard age, as intended

What does the shift do to *TFR*?

- All births originally occurring in the period box between t and $t + T$ end up in a box between
  - A starting time $t - (A(t) - (A^{(s)})$
  - And an ending time $t + T - (A_{t+T} - A^{(s)})$

- The width of the new box is found by taking the difference between the new endpoints

$$T - (A_{T+t} - A_t)$$

Changes in denominator

- When we calculate the *TFR* in the shifted box
- The count of births in the numerator remains the same
- The width of box in the denominator changes
  - From T
  - To T times $1 + (A_{T+t} - A_t) / T$

Adjusted *TFR*

- The $TFR^{(s)}$ in the shifted box equals the original *TFR* divided by this factor

$$TFR^{(s)} = \frac{TFR(t)}{1 - (1/T)(A(t+T) - A(t))}$$

- Age standardization removes the effects of the thinning out in time that occurs when women are postponing births later and later in their lives
  - Trends in adjusted *TFR* are a better guide to trends in cohort fertility than period *TFR*

Example for France, 1980–1985

- $TFR = 1.878$
- Increasing average age at childbearing

  $A(1980) = 26.82$

  $A(1985) = 27.48$

- Factor in denominator

  $1 – ((A(1985) – A(1980)) / 5) = 1 – ((27.48 – 26.82) / 5)$

  $1 – (0.66 / 5) = 1 – 0.132 = 0.868$

- $TFR^{(s)} = 1.878 / 0.868 = 2.163$
  - If ages at childbearing had not been rising, level of fertility would have been above replacement

Falling ages at childbearing

- When ages at childbearing are falling
  - Births are compressed in time
  - The factor in the denominator is greater than 1
  - $TFR^{(s)}$ is smaller than the period TFR

Chosen standard age $A^{(s)}$

- The chosen standard age $A^{(s)}$ does not affect the standardized rate
  - Its effects on location of our shifted box are essentially arbitrary
  - We suppress them, always attributing $TFR^{(s)}$ to the same period as $TFR(t)$

Birth order

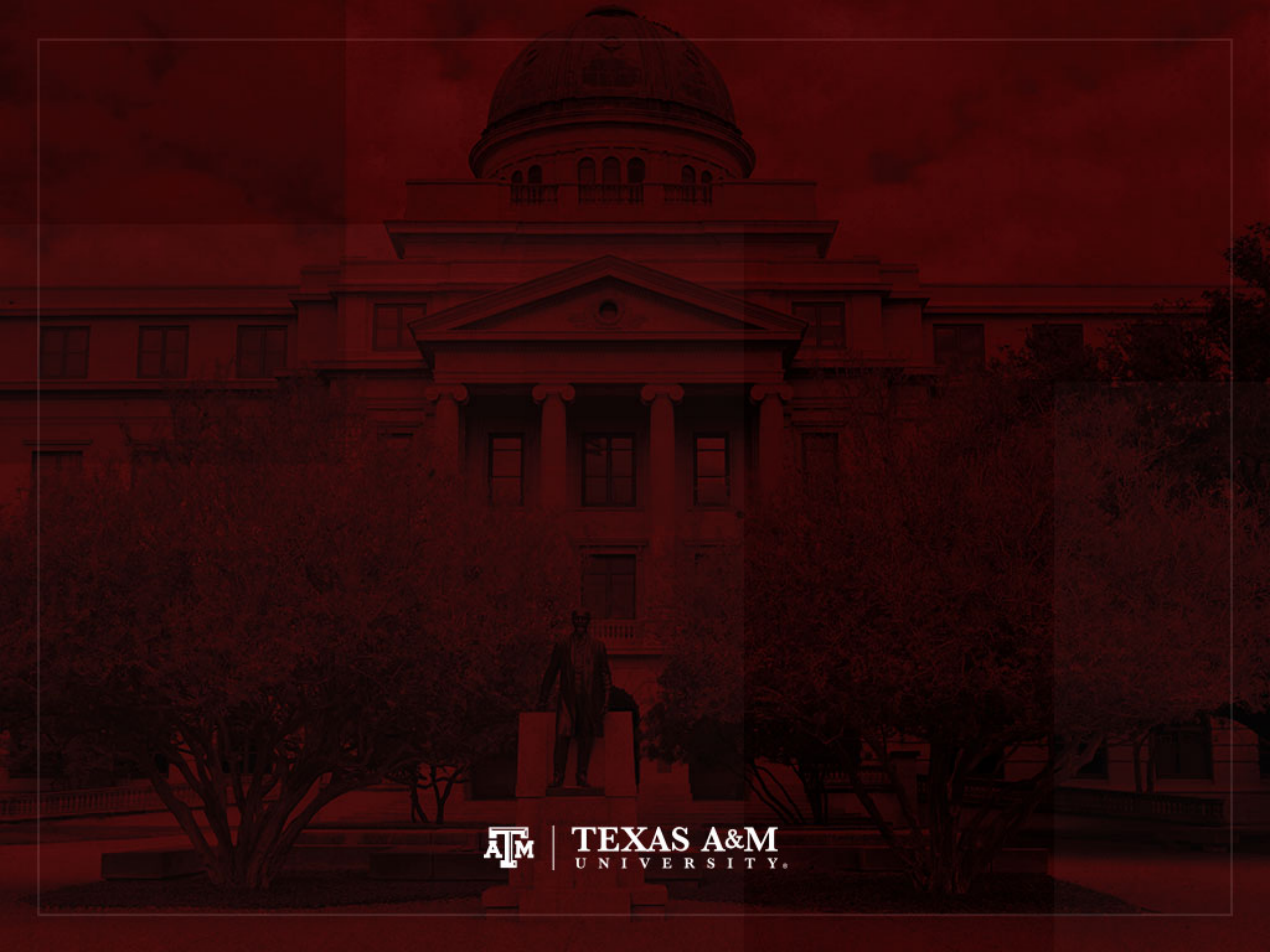
- A version of $A(t)$ can be calculated for each birth order
  - First births, second births, third births...
  - A standard age is chosen for each order
  - Births of each order are shifted to make each $A(t)$ equal each standard age
- A standardized *TFR* can be obtained by summing the standardized *TFR* values for the birth orders
  - Groups can also be defined by mother's level of education, for instance

Tempo-adjusted *TFR*

- A measure close to the standardized $TFR^{(s)}$ was introduced by Bongaarts and Feeney (1998)
  - Separate out births by birth order
  - Use 1-year-wide age groups
  - Estimate $A(t+1) - A(t)$ by half the difference between the average age in the year following t+1 and the average age in the year preceding t
  - Same denominator as $TFR^{(s)}$

Tempo and mortality

- Tempo adjustments for mortality do not make sense, because the quantum of mortality does not vary
  - Every person dies exactly once
- Mortality is entirely a matter of tempo
  - It is a matter of the timing of deaths in the life course
  - Changing ages at death reflect real changes in mortality
- Changing ages at childbirth may not reflect real changes in the total numbers of children that individuals have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Princeton indices

- The Princeton European Fertility Project proposed
  - Overall index of fertility (I_f)
  - Index of marital fertility (I_g)
  - Index of marriage (I_m)
  - Index of non-marital fertility (I_h)

Data

- Counts of births at local levels broken down by marital status of mothers
  - Birth registration systems

- Counts of women by age and marital status
  - National censuses

Applicability of Princeton indices

- Can be calculated with data widely and uniformly available at a provincial or local level across Europe since the mid-1800s
- Measure how favorable the patterns of age at marriage are to high fertility
- Separate out the effects of changing ages of marriage from changes in fertility within marriage

Hutterite rates as standard

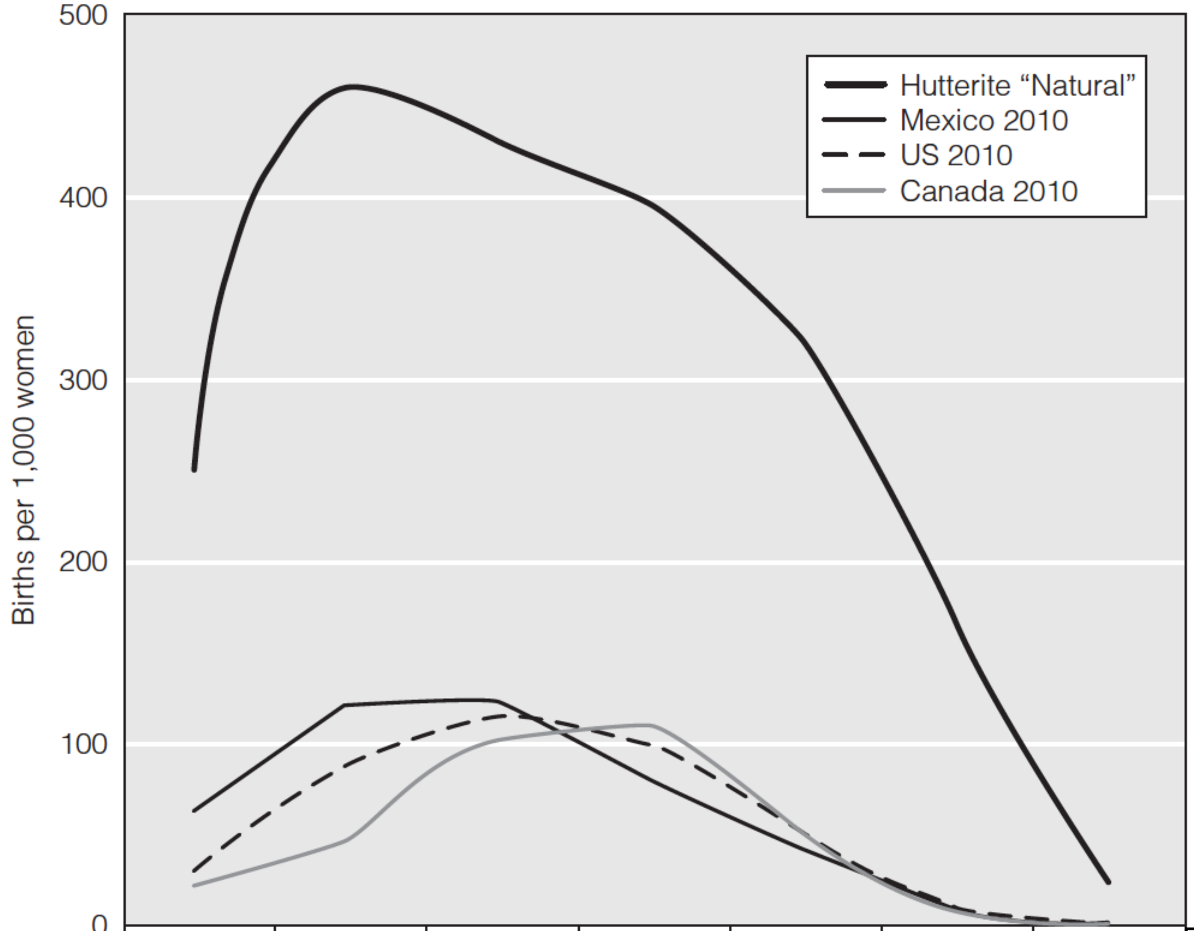
- Princeton indices are a form of indirect standardization
- Take a standard schedule of age-specific fertility rates (Hutterites)
- Compare the number of births that a population actually has in a period with the number that the population would have had if their fertility rates had been equal to the Hutterite rates

Natural fertility

- Natural fertility (Henry 1961, Coale and Trussell 1974)
  - Level of reproduction in the absence of deliberate fertility control
  - Closer to 6 or 7 live births per woman
  - 25% of completed fertility is due to genetics (same as mortality)
- Hutterites had 11 children per woman (1930s)
  - Ethnoreligious group formed in the early 16th century
  - Early age at marriage, good diet, good medical care, regularly engage in intercourse without contraception or abortion
  - Nowadays, almost all live in South Dakota, North Dakota, Montana, and Western Canada

Source: Weeks, 2015.

Age-specific fertility rates



		15–19	20–24	25–29	30–34	35–39	40–44	45–49	TFR
1930	Hutterite "Natural"	250	460	431	396	321	167	24	11.0
	Mexico 2010	63	121	123	80	42	9	2	2.2
	US 2010	31	88	115	99	50	10	1	2.0
	Canada 2010	15	47	102	110	51	8	0	1.7

Source: Weeks, 2015.

Overall index of fertility (I_f)

- Numerator
  - Births to all women observed in the actual population ($B^{overall}$)
- Denominator
  - Hypothetical total of implied births
  - Multiply actual counts of women (${}_nK^f_x$) by standard Hutterite rates (${}_5F_{x,Hutt}$)

$$I_f = B^{overall} / [\Sigma({}_5K^f_x)({}_5F_{x,Hutt})]$$

Index of marital fertility (I_g)

- Numerator
  - Births to married women in the actual population ($B^{marital}$)
- Denominator
  - Hypothetical implied births within marriage
  - Multiply actual counts of married women (${}_nK^f_{x,married}$) by standard Hutterite rates (${}_5F_{x,Hutt}$)

$$I_g = B^{marital} / [\Sigma({}_5K^f_{x,married})({}_5F_{x,Hutt})]$$

Data for Berlin, 1900

Age *x*	Hutterite Rates	Overall Women	Implied Babies	Married Women	Implied Babies
15	0.300	91,358	27,407	1,538	461
20	0.550	114,464	62,955	28,710	15,791
25	0.502	99,644	50,021	55,417	27,819
30	0.407	88,886	36,177	62,076	25,265
35	0.406	75,729	30,746	55,293	22,449
40	0.222	66,448	14,751	47,197	10,478
45	0.061	54,485	3,324	36,906	2,251
Total		591,014	225,381	287,137	104,514

- Also know: 49,638 births of which 42,186 within marriage

Source: Wachter 2014, p.143.

Calculating I_f and I_g for Berlin

- Overall index of fertility

$$I_f = B^{overall} / [\Sigma({}_5K^f_x)({}_5F_{x,Hutt})] = 49{,}638 / 225{,}381 = 0.220$$

  – Limitation overall was well advanced by 1900 in Berlin

- Index of marital fertility

$$I_g = B^{marital} / [\Sigma({}_5K^f_{x,married})({}_5F_{x,Hutt})] = 42{,}186 / 104{,}514 = 0.404$$

  – Fertility within marriage was not wholly responsible for limitation

Index of marriage (I_m)

- Measures how conducive marriage pattern is to high fertility
- Numerator
  - Take the denominator from I_g
  - Hypothetical implied births within marriage
- Denominator
  - Take the denominator from I_f
  - Hypothetical total of implied births

$$I_m = [\Sigma({}_5K^f_{x,married})({}_5F_{x,Hutt})] / [\Sigma({}_5K^f_x)({}_5F_{x,Hutt})]$$
$$= 104{,}514 / 225{,}381 = 0.464$$

  - Babies within marriage were 46.4% of overall births
  - Low proportions marrying contributed to low levels of overall fertility (0.220), compared to marital fertility (0.404)

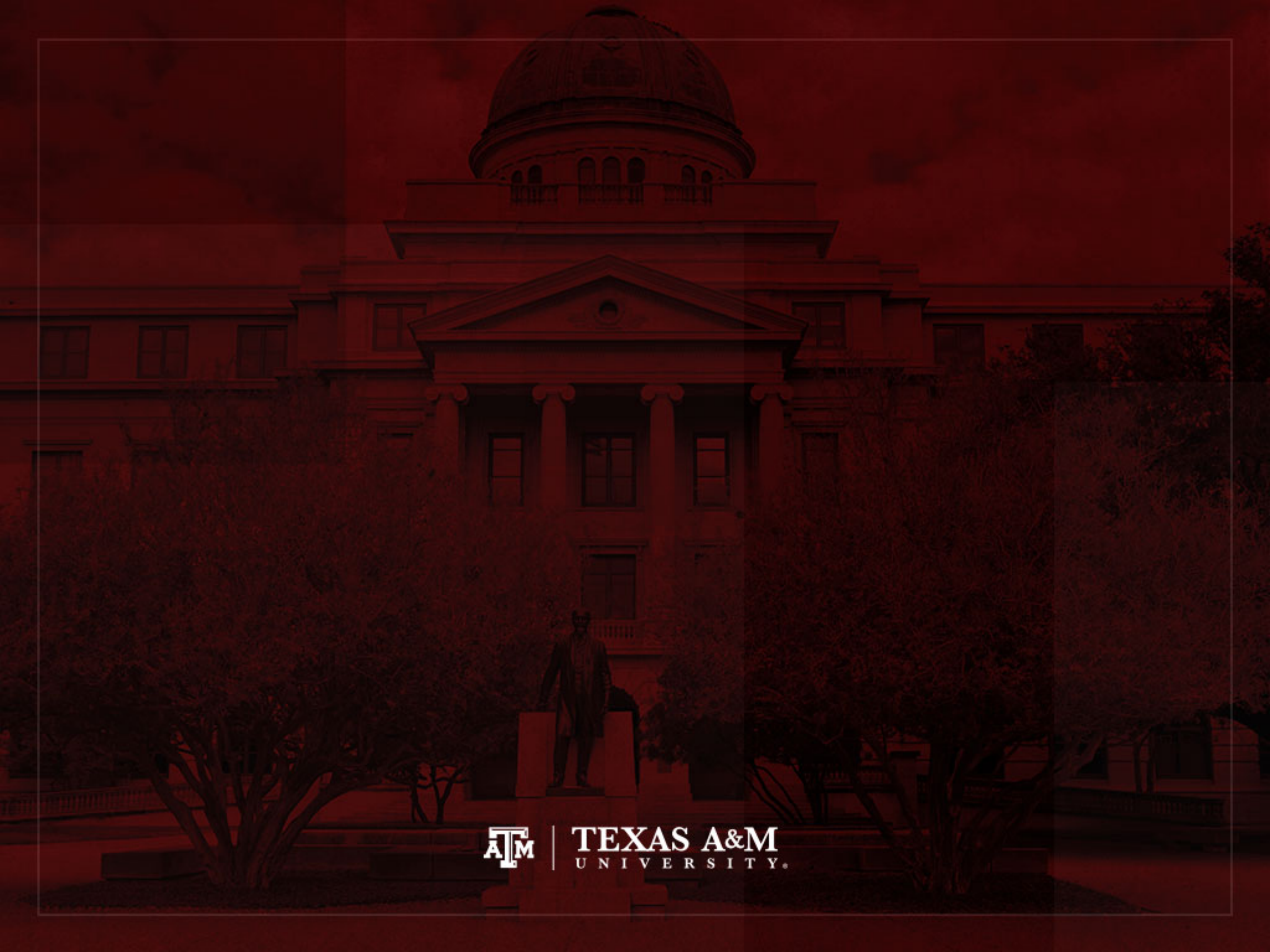
Index of non-marital fertility (I_h)

- It is rarely employed, when illegitimate fertility is a small part of overall fertility
- Numerator
  - Observed births out of wedlock
- Denominator
  - Hypothetical births that unmarried women in the population would have had at Hutterite rates
- When non-marital fertility is small, I_h can be neglected, and I_f is close to the product of I_g with I_m

$$I_f = (I_g)(I_m) + (I_h)(1 - I_m) \approx (I_g)(I_m)$$

References

Wachter KW. 2014. Essential Demographic Methods. Cambridge: Harvard University Press. Chapter 6 (pp. 125–152).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