Lecture 23:
Elaborating bivariate tables

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Advanced Methods of Social Research (SOCI 420)

Source: Healey, Joseph F. 2015. ”Statistics: A Tool for Social Research.” Stamford: Cengage Learning. 10th edition. Chapter 14 (pp. 380–404).

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Chapter learning objectives

- Explain the purpose of multivariate analysis in terms of observing the effect of a control variable
- Construct and interpret partial tables
- Compute and interpret partial measures of association
- Recognize and interpret direct, spurious or intervening, and interactive relationships
- Compute and interpret partial gamma
- Explain limitations of elaborating bivariate tables

Controlling for a third variable

- Social science research projects are multivariate
- One way to conduct multivariate analysis is to observe the effect of third variables, one at a time, on a bivariate correlation
- The elaboration technique extends the analysis of bivariate tables and associations

Partial tables

- We observe how a control variable (*Z*) affects the relationship between *X* and *Y*

- To control for a third variable, the bivariate relationship is reconstructed for each value of the control variable

- Tables that display the relationship between *X* and *Y* for each value of *Z* (a third variable) are called partial tables

Focus on three basic patterns

- Direct relationships
- Spurious or intervening relationships
- Interaction

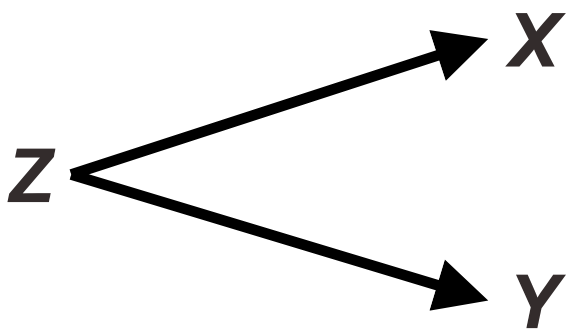
Direct relationships

- In a direct relationship, the control variable has little effect on the relationship between *X* and *Y*
- The column percentages and Gammas in the partial tables are about the same as the bivariate table
- This outcome supports the argument that *X* causes *Y*
- Also referred to as replication

$$X \longrightarrow Y$$

Spurious relationships

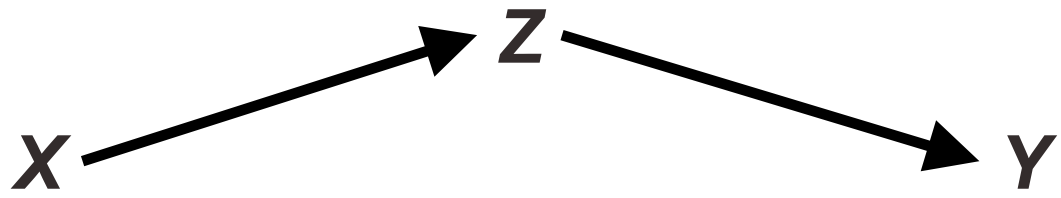
- In a spurious relationship, *X* and *Y* are not related, both are caused by *Z*
- In a spurious relationship, the Gammas in the partial tables are dramatically lower than the gamma in the bivariate table, perhaps even falling to zero
- Also referred to as explanation



Intervening relationships

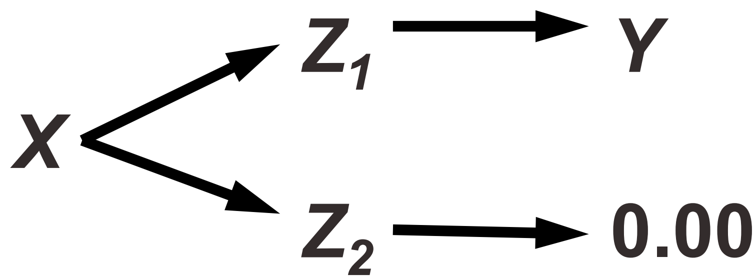
- In an intervening relationship, *X* and *Y* are not directly related to each other but are linked by *Z*, which “intervenes” between the two

- Also referred to as interpretation

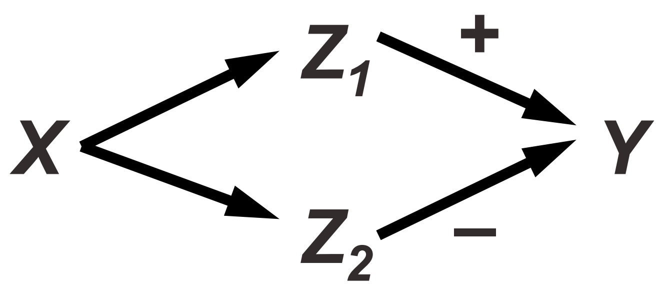


Interaction

- Interaction occurs when the association between *X* and *Y* changes across the categories of *Z*
  - *X* and *Y* could only be related for some categories of *Z*



  - *X* and *Y* could have a positive association for one category of *Z* and a negative association for others



Summary

- Possible results when controlling for third variables

Compared with Bivariate Table, Partial Tables Show	Pattern	Implications for Further Analysis	Likely Next Step	Theoretical Implications
Same relationship between *X* and *Y* (e.g., gammas for partial tables are no more than ±0.10 different from the bivariate gamma)	Direct relationship (replication)	Disregard *Z*	Analyze another *Z* variable	Theory that *X* causes *Y* is supported
Weaker relationship between *X* and *Y* (e.g., gammas for partial tables are all at least 0.10 weaker than the bivariate gamma)	Spurious relationship (explanation)	Incorporate *Z*	Focus on relationship between *Z* and *Y*	Theory that *X* causes *Y* is not supported
	Intervening relationship (interpretation)	Incorporate *Z*	Focus on relationships among *X*, *Y*, and *Z*	Theory that *X* causes *Y* is partially supported but must be revised to take *Z* into account
Mixed (e.g., there is a difference of at least ±0.10 between gammas for the partial tables and between the gammas for partial tables and the bivariate gamma)	Interaction (specification)	Incorporate *Z*	Analyze subgroups (categories of *Z*) separately	Theory that *X* causes *Y* is partially supported but must be revised to take *Z* into account

Source: Healey 2015, p.389.

Partial Gamma

- Partial Gamma indicates the overall strength of association between *X* and *Y* after the effects of the control variable (*Z*) have been removed
  - Compare Partial Gamma (G_p) to the Gamma (G) for the bivariate table to see if the relationship has changed

$$G_p = \frac{\sum N_s - \sum N_d}{\sum N_s + \sum N_d}$$

  - N_s is the number of pairs of cases ranked in the **same order** across all partial tables
  - N_d is the number of pairs of cases ranked in **different order** across all partial tables

Example 1

- Association between
  - Number of memberships in student organizations
    - *X*, independent variable
  - Satisfaction with college
    - *Y*, dependent variable

Satisfaction with College by Number of Memberships in Student Organizations

	Memberships (*X*)		
Satisfaction (*Y*)	*None*	*At Least One*	TOTALS
Low	57 *(54.3%)*	56 *(33.9%)*	113
High	48 *(45.7%)*	109 *(66.1%)*	157
TOTALS	105 *(100.0%)*	165 *(100.0%)*	270

Gamma = +0.40

Source: Healey 2015, p.381.

Interpretation

- Comparing the conditional distributions of *Y* (the column percentages), we find a positive relationship
  - This direction is confirmed by the sign of Gamma (+0.40)

- College students with at least one membership in a student organization are more likely than students with no memberships to have high satisfaction with college

GPA as a control variable

- Associations remain positive

Satisfaction by Membership, Controlling for GPA

A. High GPA

Satisfaction (*Y*)	Memberships (*X*) None	Memberships (*X*) At Least One	TOTALS
Low	29 (*54.7%*)	28 (*34.1%*)	57
High	24 (*45.3%*)	54 (*65.9%*)	78
TOTALS	53 (*100.0%*)	82 (*100.0%*)	135

Gamma = 0.40

B. Low GPA

Satisfaction (*Y*)	Memberships (*X*) None	Memberships (*X*) At Least One	TOTALS
Low	28 (*53.8%*)	28 (*33.7%*)	56
High	24 (*46.2%*)	55 (*66.3%*)	79
TOTALS	52 (*100.0%*)	83 (*100.0%*)	135

Gamma = 0.39

Source: Healey 2015, p.383.

Association still positive

- The relationship between integration and satisfaction is the same in the partial tables as it was in the bivariate table
  - This is evidence of a **direct relationship**

High GPA	Low GPA
N_s = (29)(54) = 1566	N_s = (28)(55) = 1540
N_d = (28)(24) = 672	N_d = (28)(24) = 672

$$G_p = \frac{\sum N_s - \sum N_d}{\sum N_s + \sum N_d} = \frac{(1566 + 1540) - (672 + 672)}{(1566 + 1540) + (672 + 672)} = 0.40$$

Source: Healey 2015, p.390.

Class standing as a control

- There is no more association
  - Upperclass students: seniors and juniors
  - Underclass students: sophomores and freshmen

Satisfaction by Membership, Controlling for Class

A. Upperclass Students

Satisfaction (*Y*)	Memberships (*X*): *None*	Memberships (*X*): *At Least One*	TOTALS
Low	8 (*25.0%*)	32 (*24.8%*)	40
High	24 (*75.0%*)	97 (*75.2%*)	121
TOTALS	32 (*100.0%*)	129 (*100.0%*)	161

Gamma = 0.01

B. Underclass Students

Satisfaction (*Y*)	Memberships (*X*): *None*	Memberships (*X*): *At Least One*	TOTALS
Low	49 (*67.1%*)	24 (*66.7%*)	73
High	24 (*32.9%*)	12 (*33.3%*)	36
TOTALS	73 (*100.0%*)	36 (*100.0%*)	109

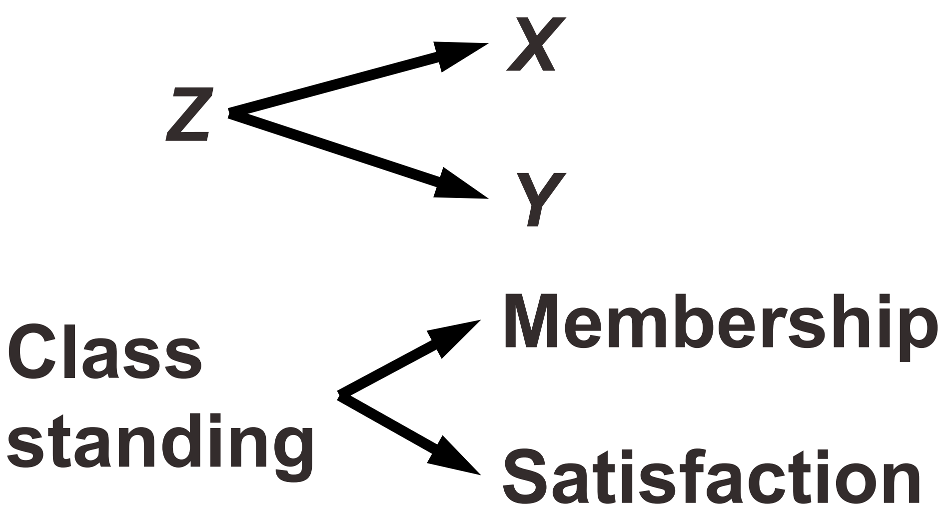
Gamma = 0.01

Source: Healey 2015, p.385.

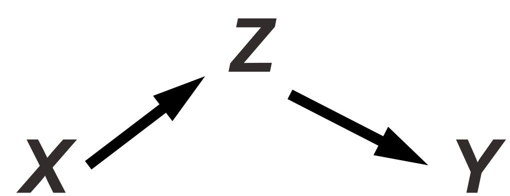
Association disappears

- The original bivariate relationship between memberships and satisfaction disappears in the partial tables
  - When the association disappears, we have either a spurious or an intervening relationship

Spurious



Intervening



Spurious relationship

- Decision about whether the association is spurious or intervening is based on
  - Temporal (timing) or theoretical grounds
- A spurious relationship makes more sense
  - Class standing likely predicts the number of memberships, and not the other way around
    - Partial Gamma supports our conclusion (reduced to zero)

Upperclass	Underclass
N_s = (8)(97) = 776	N_s = (49)(12) = 588
N_d = (32)(24) = 768	N_d = (24)(24) = 576

$$G_p = \frac{\sum N_s - \sum N_d}{\sum N_s + \sum N_d} = \frac{(776 + 588) - (768 + 576)}{(776 + 588) + (768 + 576)} = 0.01$$

Source: Healey 2015, p.390.

Example 2

- Relationship for 50 immigrants between
  - Length of residence: *X*, independent variable
  - English fluency: *Y*, dependent variable

	Length of Residence		
English Fluency	*Less Than Five Years (Low)*	*More Than Five Years (High)*	TOTALS
Low	20	10	30
High	5	15	20
TOTALS	25	25	50

- Gamma = +0.67
  - Strong and positive association
  - As length of residence increases, English fluency also increases

Source: Healey 2015, p.398, problem 14.1.

Sex as a control variable

- Associations remain positive
- $G_m = 0.78$
- $G_f = 0.65$

A. Males

English Fluency	Length of Residence: *Less Than Five Years (Low)*	*More Than Five Years (High)*	TOTALS
Low	10	5	15
High	2	8	10
TOTALS	12	13	25

B. Females

English Fluency	Length of Residence: *Less Than Five Years (Low)*	*More Than Five Years (High)*	TOTALS
Low	10	5	15
High	3	7	10
TOTALS	13	12	25

Source: Healey 2015, p.398, problem 14.1.

Partial Gamma

$$G_m = \frac{N_s - N_d}{N_s + N_d} = \frac{80 - 10}{80 + 10} = 0.78$$

$$G_f = \frac{N_s - N_d}{N_s + N_d} = \frac{70 - 15}{70 + 15} = 0.65$$

$$G_p = \frac{\sum N_s - \sum N_d}{\sum N_s + \sum N_d} = \frac{(80 + 70) - (10 + 15)}{(80 + 70) + (10 + 15)} = 0.71$$

Source: Healey 2015, p.398, problem 14.1.

Sex has no effect

- While the two Gammas for the partial tables (0.78 and 0.65) differ slightly
  - They both indicate a strong and positive association between length of residence and English fluency

- Comparing Partial Gamma (0.71) to the original Gamma (0.67), we find little difference

- We have evidence of a **direct relationship**
  - Controlling for sex does not affect the association between length of residence and English fluency for immigrants

Source: Healey 2015, p.398, problem 14.1.

Example 3

- Relationship for 78 juvenile males between
  - Academic record: *X*, independent variable
  - Delinquency: *Y*, dependent variable

Delinquency by Academic Record

	Academic Record		
Delinquency	*Poor*	*Good*	TOTALS
Low	13 (*27.1%*)	20 (*66.7%*)	33 (*42.3%*)
High	35 (*72.9%*)	10 (*33.3%*)	45 (*57.7%*)
TOTALS	48 (*100.0%*)	30 (*100.0%*)	78 (*100.0%*)

Gamma = −0.69

- Gamma = –0.69
  - Juvenile males with better academic records have lower delinquency

Source: Healey 2015, p.392.

Area of residence as a control

- Associations differ across partial tables

Delinquency by Academic Record, Controlling for Residence

A. Urban

Delinquency	Academic Record: Poor	Academic Record: Good	TOTALS
Low	10 (*27.8%*)	3 (*30.0%*)	13 (*28.3%*)
High	26 (*72.2%*)	7 (*70.0%*)	33 (*71.7%*)
TOTALS	36 (*100.0%*)	10 (*100.0%*)	46 (*100.0%*)

Gamma = −0.05

B. Nonurban

Delinquency	Academic Record: Poor	Academic Record: Good	TOTALS
Low	3 (*25.0%*)	17 (*85.0%*)	20 (*62.5%*)
High	9 (*75.0%*)	3 (*15.0%*)	12 (*37.5%*)
TOTALS	12 (*100.0%*)	20 (*100.0%*)	32 (*100.0%*)

Gamma = −0.89

Source: Healey 2015, p.392.

Interpretation

- Gamma for urban areas is –0.05
  - No association between academic record and delinquency
- Gamma for nonurban areas is –0.89
  - Strong and negative association between academic record and delinquency
- Associations between *X* and *Y* differ across partial tables
  - This is an indication of **interaction**

Origin of control variables

- Control variables are based on theory

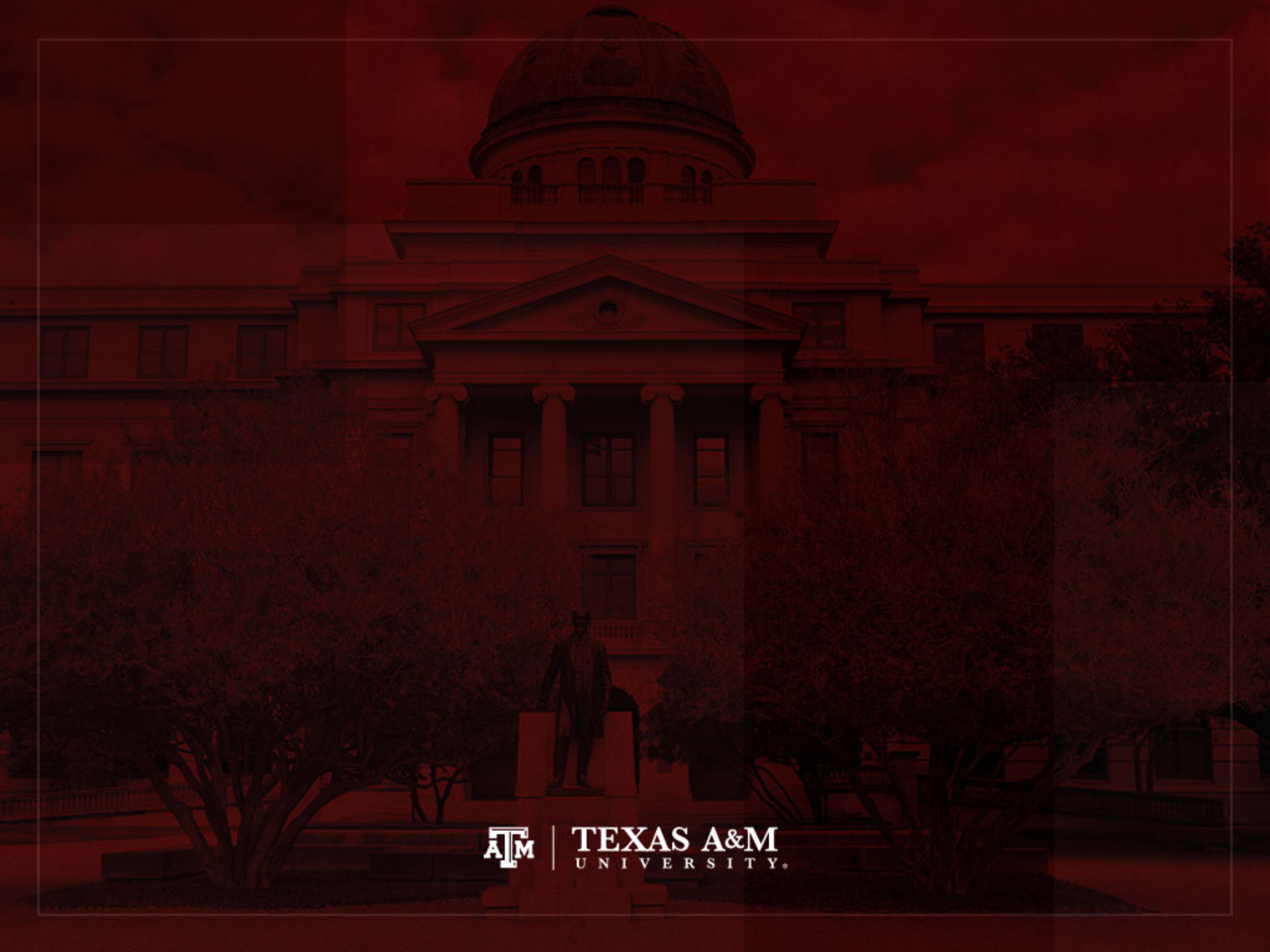
- Research projects are anchored in theory, so control variables come mainly from theory

- Understanding a spurious relationship (explanation) or an intervening relationship (interpretation) cannot be based on statistical grounds or inspecting the partial tables

Limitations of partial tables

- Basic limitation: Sample size
  - Greater the number of partial tables, the more likely to run out of cells or have small cells

- Potential solutions
  - Reduce number of cells by collapsing categories (recoding)
  - Use very large samples
  - Use techniques appropriate for interval-ratio level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