Lecture (chapter 3): Measures of central tendency

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Advanced Methods of Social Research (SOCI 420)

Source: Healey, Joseph F. 2015. "Statistics: A Tool for Social Research." Stamford: Cengage Learning. 10th edition. Chapter 3 (pp. 66–90).

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Chapter learning objectives

- Explain the purposes of measures of central tendency and interpret the information they convey
- Calculate, explain, compare, and contrast the mode, median, and mean
- Explain the mathematical characteristics of the mean
- Select an appropriate measure of central tendency according to level of measurement and skew

Measures of central tendency

- Univariate descriptive statistics
  - Summarize information about the most typical, central, or common score of a variable
- Mode, median, and mean are different statistics and have same value only in certain situations
  - Mode: most common score
  - Median: score of the middle case
  - Mean: average score
- They vary in terms of
  - Level-of-measurement considerations
  - How they define central tendency

Mode

- The most common score
- Can be used with variables at all three levels of measurement
- Most often used with nominal-level variables

Finding the mode

- Count the number of times each score occurred
- The score that occurs most often is the mode
- If the variable is presented in a frequency distribution, the mode is the largest category
- If the variable is presented in a line chart, the mode is the highest peak

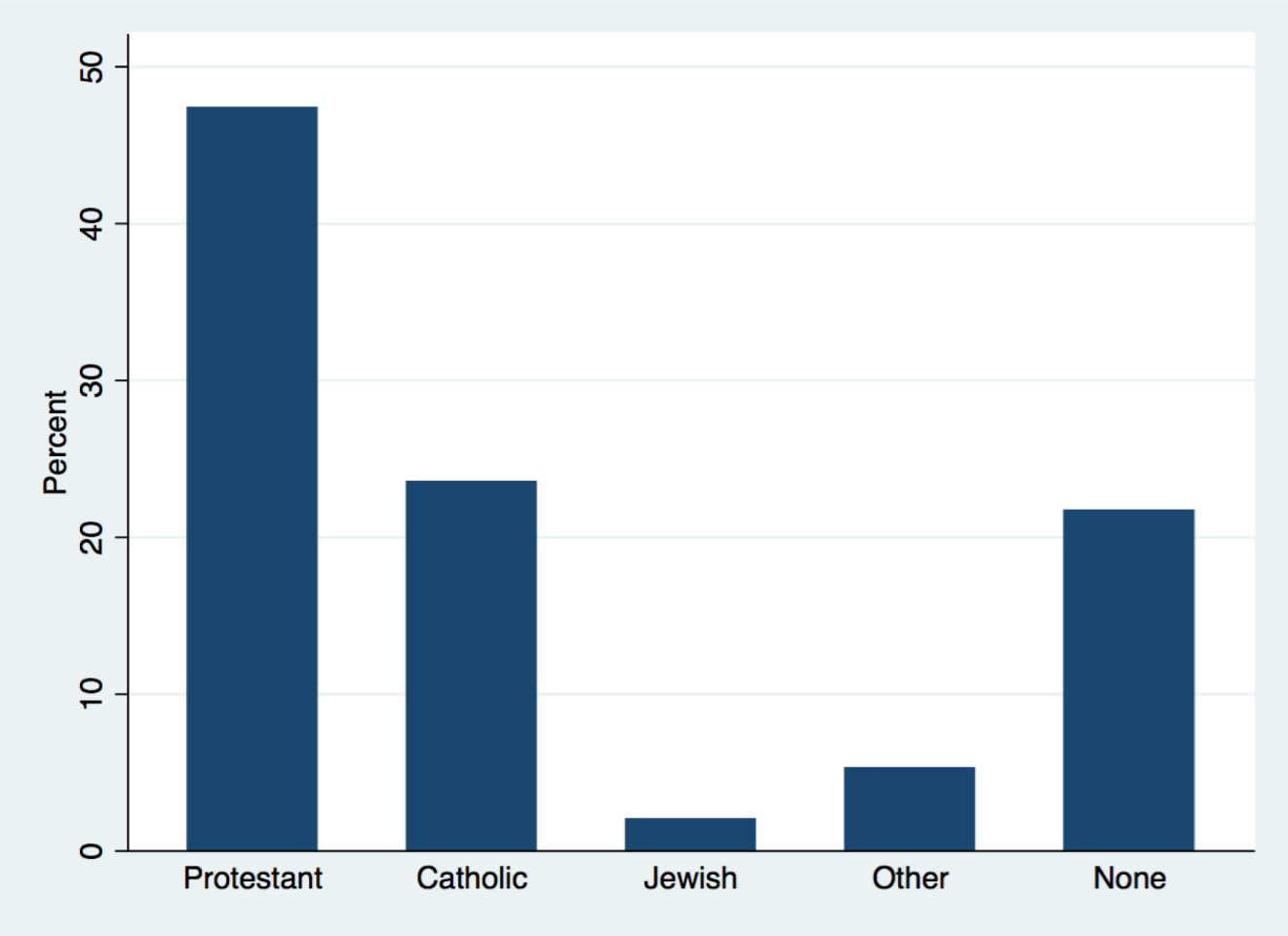
Example of mode

Top ten U.S. cities visited by overseas travelers, 2010

City	Number of visitors
Boston	1,186,000
Chicago	1,134,000
Las Vegas	2,425,000
Los Angeles	3,348,000
Miami	3,111,000
New York City	8,462,000
Oahu / Honolulu	1,634,000
Orlando	2,750,000
San Francisco	2,636,000
Washington, D.C.	1,740,000

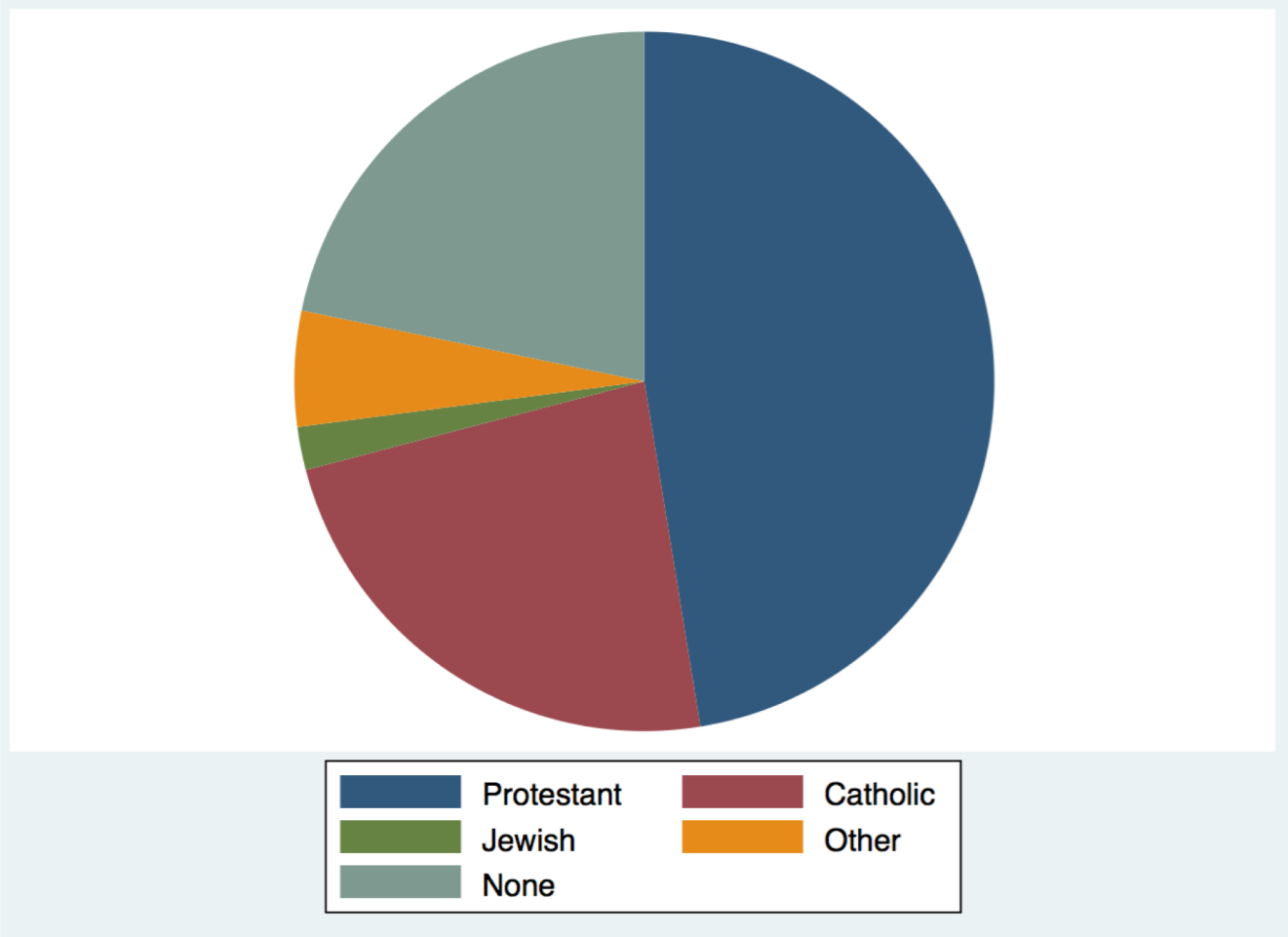
Source: Healey 2015, p.67.

Religious preference, U.S. adult population, 2016



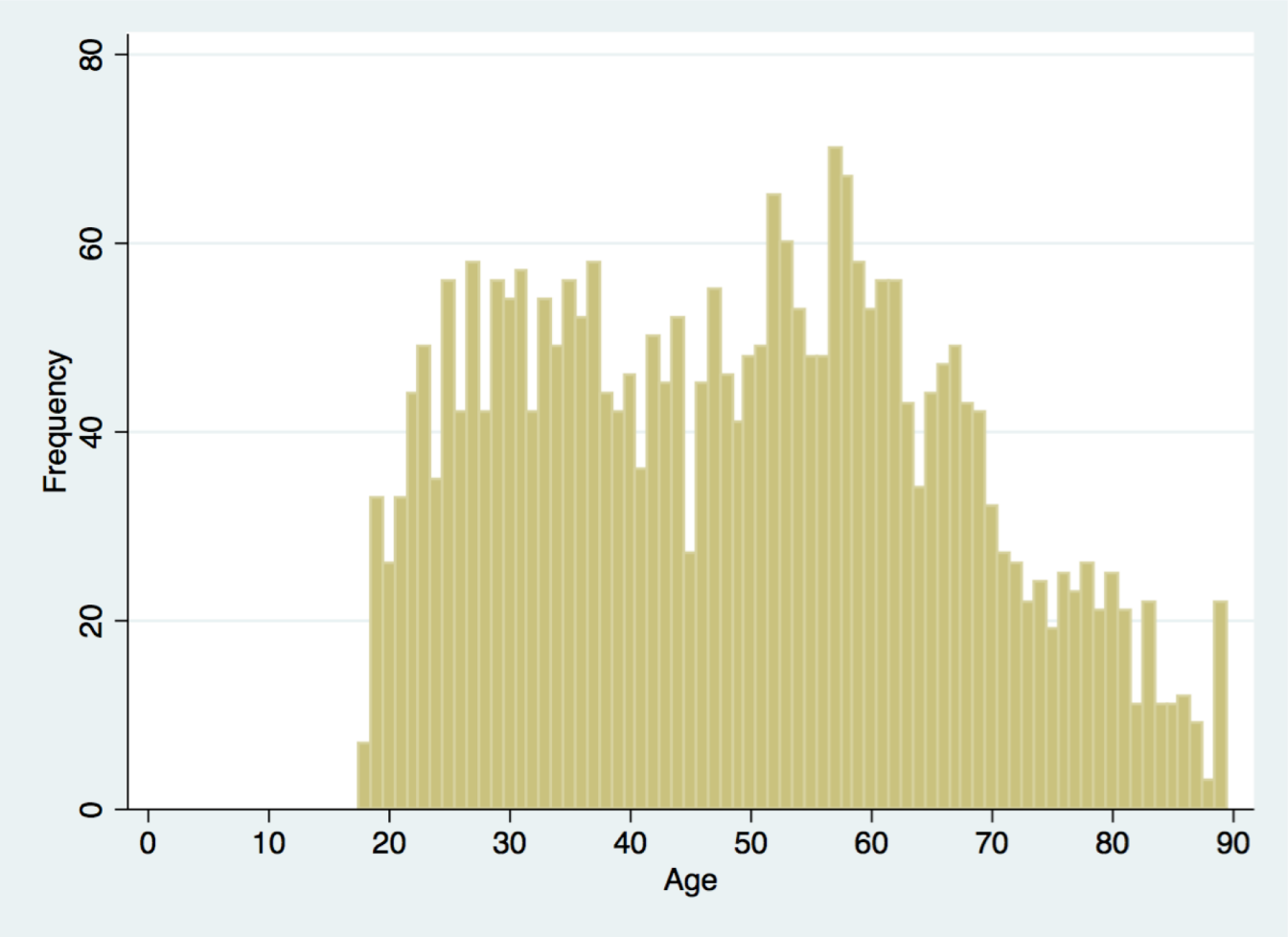
Source: 2016 General Social Survey.

Religious preference, U.S. adult population, 2016



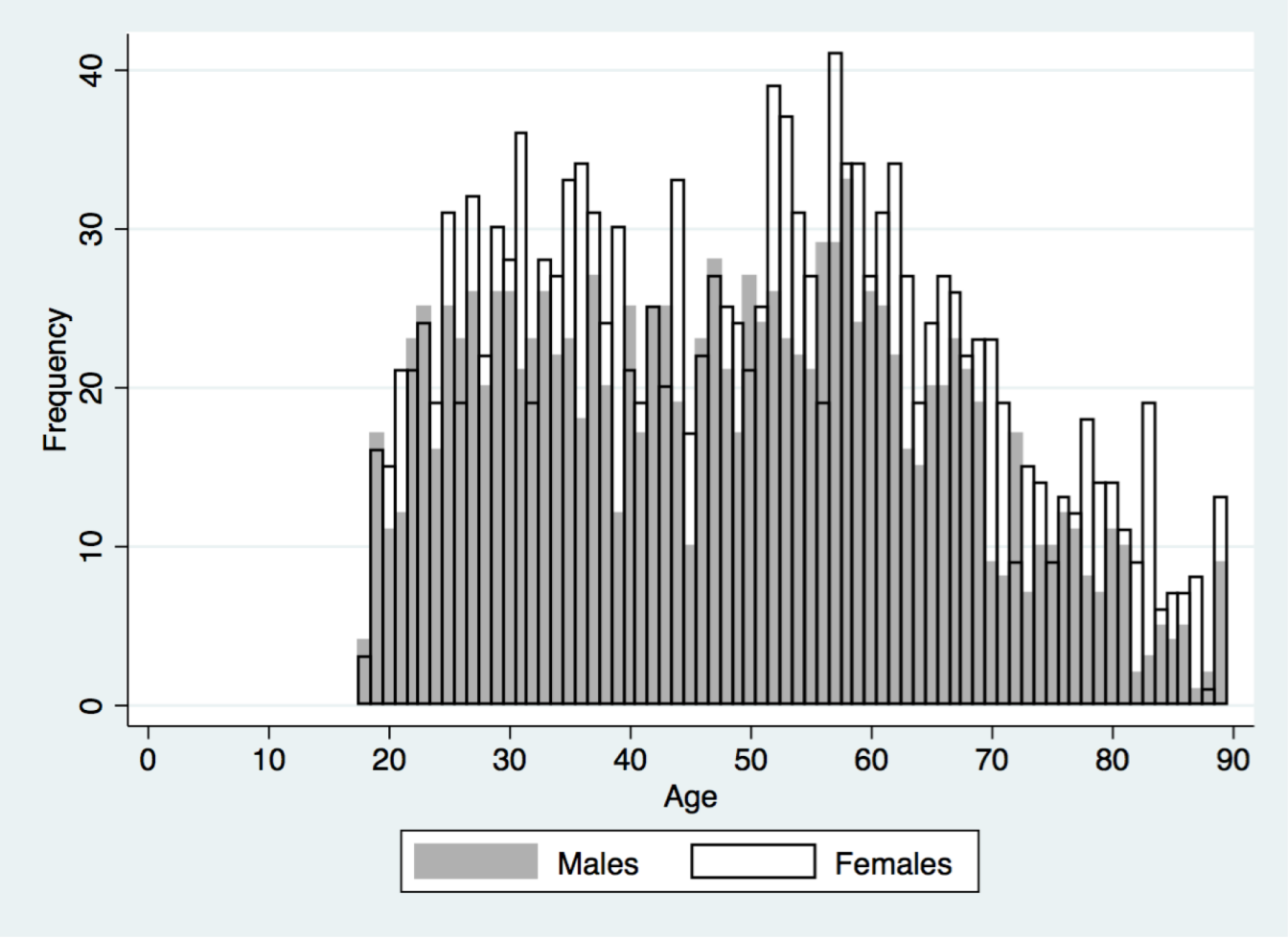
Source: 2016 General Social Survey.

Age distribution, U.S. adult population, 2016

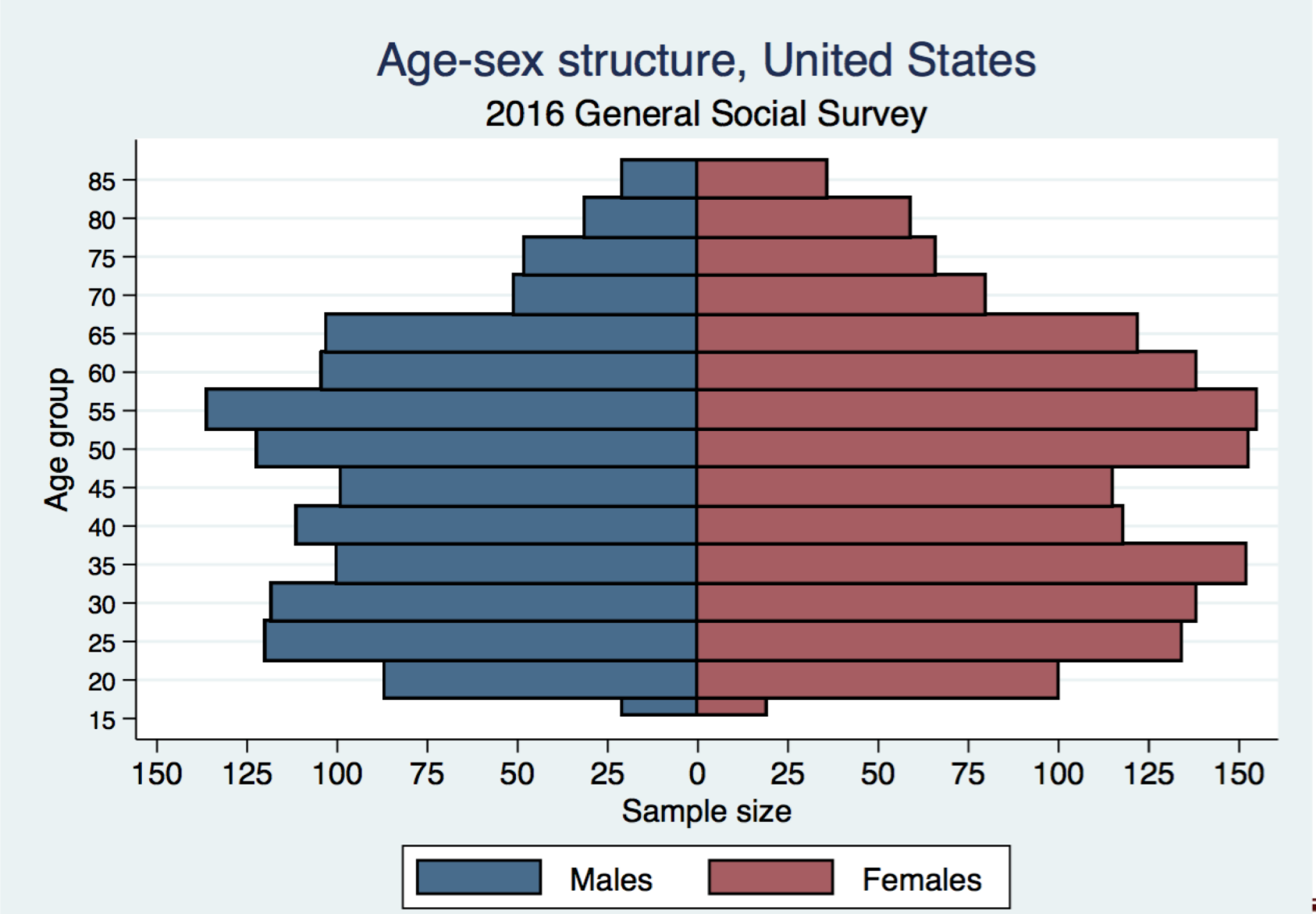


Source: 2016 General Social Survey.

Age distribution by sex, U.S. adult population, 2016



Source: 2016 General Social Survey.


Age-sex structure, United States
2016 General Social Survey
Age group
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
150
125
100
75
50
25
0
25
50
75
100
125
150
Sample size
Males
Females

Limitations of mode

- Some distributions have no mode
- Some distributions have multiple modes

Distributions of scores on two tests

Score (% correct)	Test A Frequency of scores	Test B Frequency of scores
97	14	22
91	14	3
90	14	4
86	14	22
77	14	3
60	14	22
55	14	22
Total	**98**	**98**

Source: Healey 2015, p.68.

Limitations of mode

- The mode of an ordinal or interval-ratio level variable may not be central to the whole distribution

A distribution of test scores

Score (% correct)	Frequency
93	8
68	3
67	4
66	2
62	7
Total	**24**

Source: Healey 2015, p.68.

Median

- The median (*Md*) is the exact center of distribution of scores
- The score of the middle case
- It can be used with ordinal-level or interval-ratio-level variables
- It cannot be used for nominal-level variables

Finding the median

- Arrange the cases from low to high
  - Or from high to low
- Locate the middle case
- If the number of cases (*N*) is odd
  - The median is the score of the middle case
- If the number of cases (*N*) is even
  - The median is the average of the scores of the two middle cases

Example of median

Finding the median with seven cases (*N* is odd)

Case	Score	
A	10	
B	10	
C	8	
D	7	← Median = *Md*
E	5	
F	4	
G	2	

Source: Healey 2015, p.69.

Example of median

Finding the median with eight cases (*N* is even)

Case	Score	
A	10	
B	10	
C	8	
D	7	
		← Median = *Md* = (7+5) / 2 = 6
E	5	
F	4	
G	2	
H	1	

Source: Healey 2015, p.69.

Other measures of position

- Percentiles
  - Point below which a specific percentage of cases fall
- Deciles
  - Divides distribution into tenths (10, 20, 30, ..., 90)
- Quartiles
  - Divides distribution into quarters (25, 50, 75)
- The median falls at the 50th percentile or the 5th decile or the 2nd quartile

Manual calculation

- Arrange scores in order from low to high
- Multiply the number of cases (*N*) by the proportional value of the percentile
  - For example: the 75th percentile would be 0.75
- The resultant value marks the order number of the case that falls at the percentile

Examples of manual calculation

- In a sample of 70 test grades we want to find the 4th decile (or 40th percentile)
  - 70 x 0.40 = 28
  - The 28th case is the 40th percentile

- In a sample of 70 test grades we want to find the 3rd quartile (or 75th percentile)
  - 70 x 0.75 = 52.5, rounding to 53
  - The 53rd case is the 75th percentile

Example: 2016 GSS in Stata

- 75% of the population is younger than 60 years

`sum age [aweight=wtssall], d`

age of respondent

	Percentiles	Smallest		
1%	**19**	**18**		
5%	**21**	**18**		
10%	**24**	**18**	Obs	**2,857**
25%	**33**	**18**	Sum of Wgt.	**2,855.4791**
50%	**47**		Mean	**47.56141**
		Largest	Std. Dev.	**17.58891**
75%	**60**	**89**		
90%	**72**	**89**	Variance	**309.3698**
95%	**78**	**89**	Skewness	**.2328772**
99%	**86**	**89**	Kurtosis	**2.161393**

Example: 2016 GSS in Stata

- The ”centile” command allows us to estimate any percentile, but weights are not allowed

```
centile age, centile(37)
```

- 37% of the sample is younger than 41 years

Variable	Obs	Percentile	Centile	Binom. Interp. [95% Conf.	Interval]
age	**2,857**	**37**	**41**	**40**	**42**

Mean

- The average score
- Requires variables measured at the interval-ratio level, but is often used with ordinal-level variables
- Cannot be used for nominal-level variables
- The mean (arithmetic average) is by far the most commonly used measure of central tendency

Finding the mean

- Add all of the scores and then divide by the number of scores (*N*)
- The mathematical formula for the mean is

$$\bar{X} = \frac{\sum(X_i)}{N}$$

where $\bar{X}$ = the mean

$\sum(X_i)$ = the summation of the scores

N = the number of cases

Examples of mean, 2016 GSS

Mean income by sex

table sex [aweight=wtssall], c(mean conrinc)

Sex	Mean income
Male	41,282.78
Female	28,109.34
Overall	**34,649.30**

Mean income by race/ethnicity

table raceeth [aweight=wtssall], c(mean conrinc)

Race/ethnicity	Mean income
Non-Hispanic white	38,845.62
Non-Hispanic black	23,243.04
Hispanic	23,128.92
Other	50,156.35
Overall	**34,649.30**

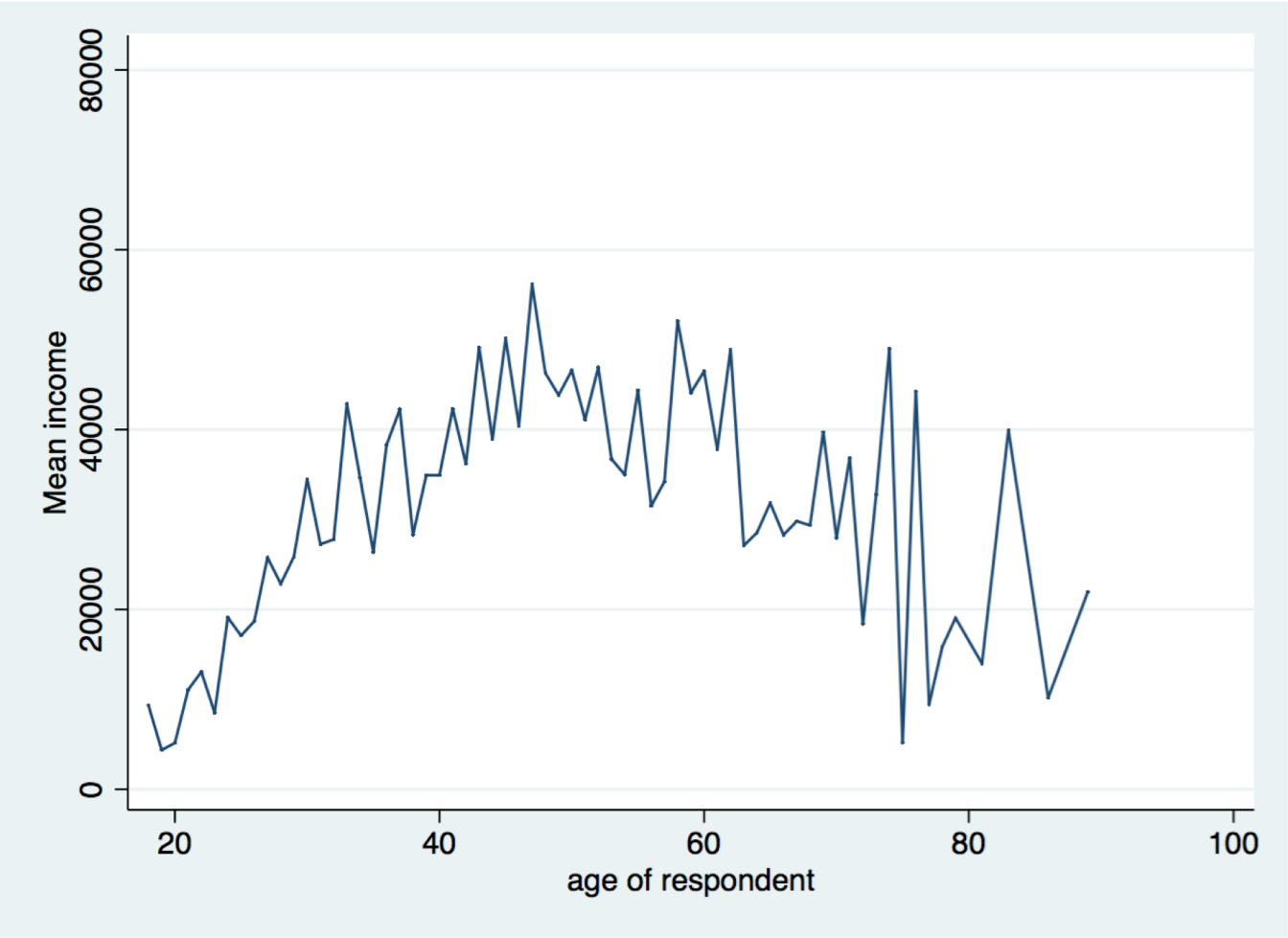
Mean income by age-group

table agegr1 [aweight=wtssall], c(mean conrinc)

Age group	Mean income
18–24	11,214.16
25–44	32,863.93
45–64	42,552.21
65–89	30,848.29
Overall	**34,649.30**

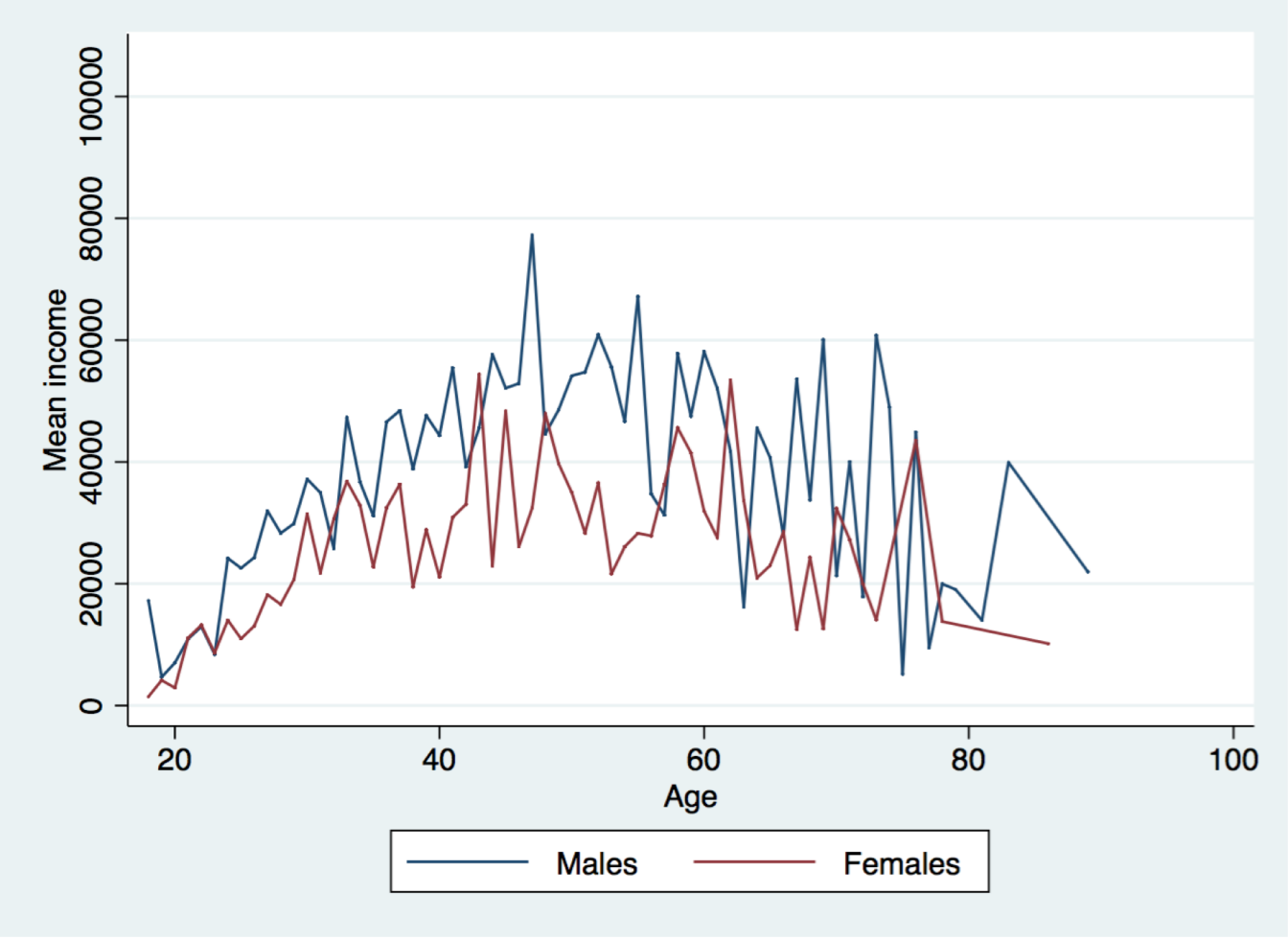
Source: 2016 General Social Survey.

Mean income by age, U.S. adult population, 2016



Source: 2016 General Social Survey.

Mean income by age and sex, U.S. adult population, 2016



Source: 2016 General Social Survey.

Three characteristics of the mean

- Mean balances all the scores in a distribution
  - All scores cancel out around the mean

$$\sum (X_i - \bar{X}) = 0$$

- Mean minimizes the variation of the scores, "least squares principle"

$$\sum (X_i - \bar{X})^2 = minimum$$

- Mean is affected by all scores
  - All scores are used in the calculation of the mean
  - It can be misleading if the distribution has "outliers"

Mean balances all the scores

- A demonstration showing that all scores cancel out around the mean

X_i	$X_i - \overline{X}$
65	65 – 78 = –13
73	73 – 78 = –5
77	77 – 78 = –1
85	85 – 78 = 7
90	90 – 78 = 12
$\sum(X_i)$ = 390 $\overline{X}$ = 390 / 5 = 78	$\sum(X_i - \overline{X})$ = 0

Source: Healey 2015, p.74.

Mean minimizes variation

- A demonstration showing that the mean is the point of minimized variation
  - If we performed these operations with any number other than the mean (e.g., 77), the result would be a sum greater than 388

X_i	$X_i - \bar{X}$	$(X_i - \bar{X})^2$	$(X_i - 77)^2$
65	65 – 78 = –13	$(-13)^2 = 169$	$(65 - 77)^2 = (-12)^2 = 144$
73	73 – 78 = –5	$(-5)^2 = 25$	$(73 - 77)^2 = (-4)^2 = 16$
77	77 – 78 = –1	$(-1)^2 = 1$	$(77 - 77)^2 = (0)^2 = 0$
85	85 – 78 = 7	$(7)^2 = 49$	$(85 - 77)^2 = (8)^2 = 64$
90	90 – 78 = 12	$(12)^2 = 144$	$(90 - 77)^2 = (13)^2 = 169$
$\sum(X_i) = 390$ $\bar{X} = 78$	$\sum(X_i - \bar{X}) = 0$	$\sum(X_i - \bar{X})^2 = 388$	$\sum(X_i - 77)^2 = 393$

Source: Healey 2015, p.75.

Mean is affected by all scores

- A demonstration showing that the mean is affected by every score

Scores	Measures of central tendency	Scores	Measures of central tendency	Scores	Measures of central tendency
15	Mean = 25	15	Mean = 718	0	Mean = 22
20		20		20	
25	Median = 25	25	Median = 25	25	Median = 25
30		30		30	
35		3500		35	

Source: Healey 2015, p.76.

Mean is affected by all scores

- **Strength**
- The mean uses all the available information from the variable

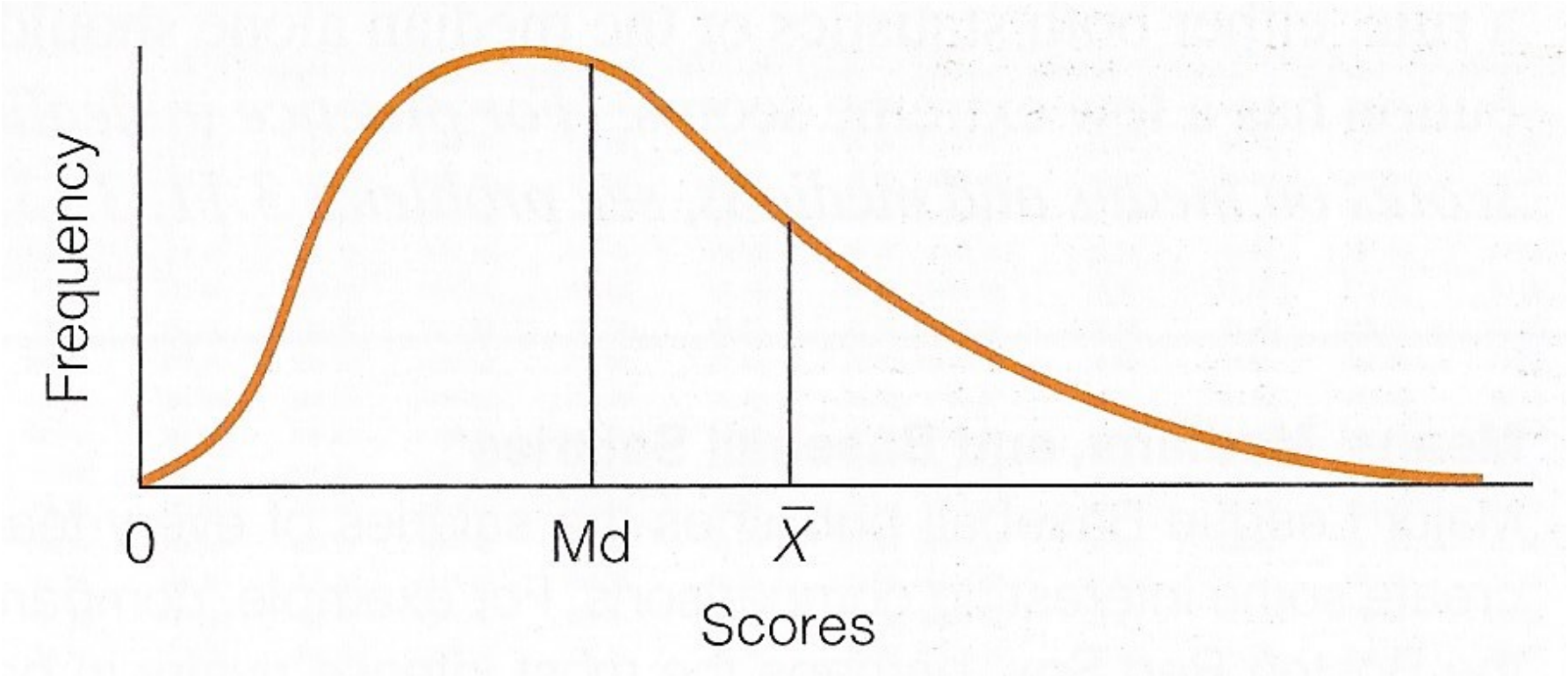
- **Weaknesses**
- The mean is affected by every score
- If there are some very high or low scores
  - Extreme scores: ”outliers”
  - The mean may be misleading
  - This is the case of skewed distributions

Skewed distributions

- When a distribution has a few very high or low scores, the mean will be pulled in the direction of the extreme scores
- For a positive skew
  - The mean will be greater than the median
- For a negative skew
  - The mean will be less than the median
- When an interval-ratio-level variable has a pronounced skew, the median may be the more trustworthy measure of central tendency

Positively skewed distribution

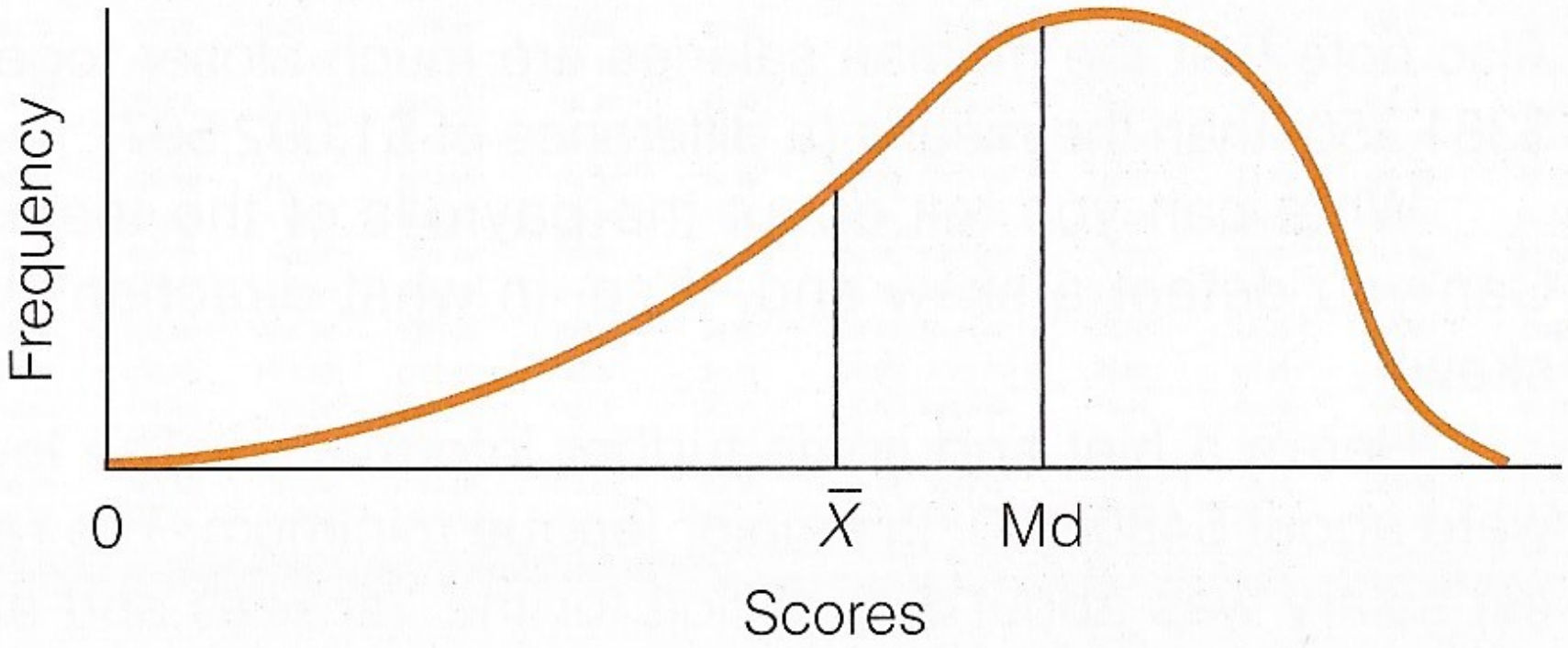
- The mean is greater in value than the median



Source: Healey 2015, p.77.

Negatively skewed distribution

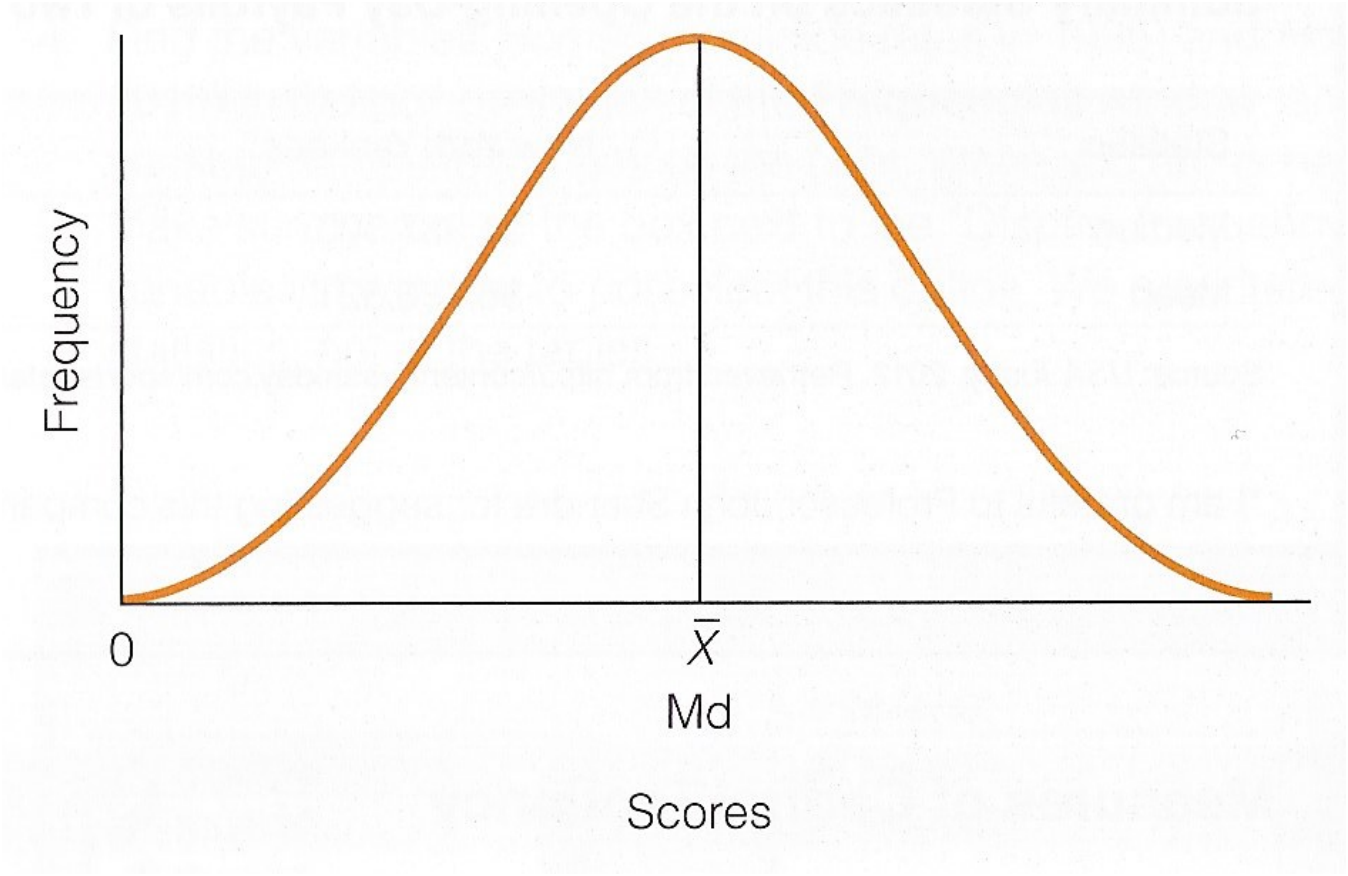
- The mean is less than the median



Source: Healey 2015, p.77.

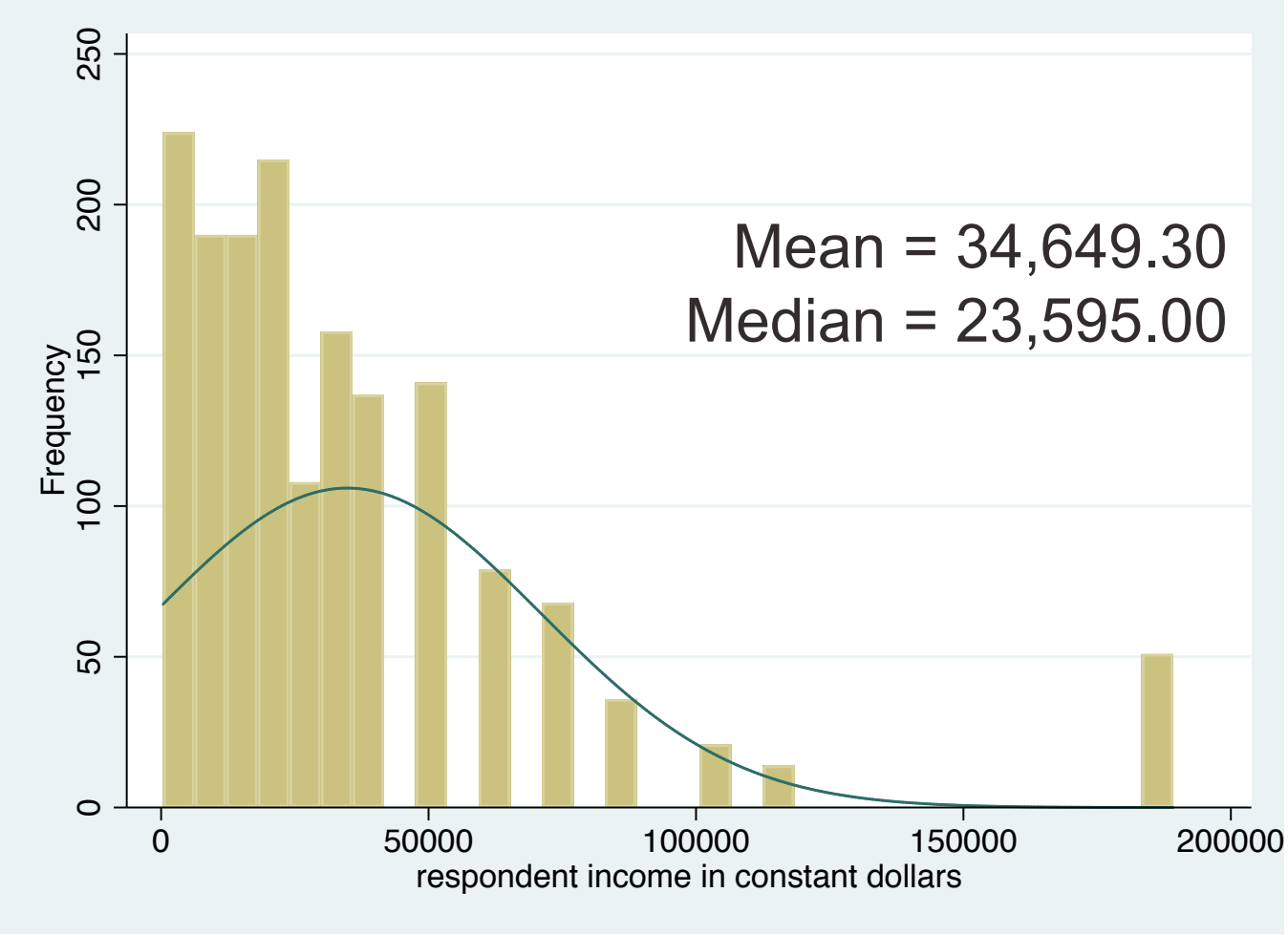
Symmetrical distribution

- The mean and median are equal



Source: Healey 2015, p.77.

Income distribution, U.S. adult population, 2016



Source: 2016 General Social Survey.

Level of measurement

- Relationship between level of measurement and measures of central tendency

Measure of central tendency	Level of measurement		
	Nominal	**Ordinal**	**Interval-ratio**
Mode	**YES**	Yes	Yes
Median	No	**YES**	Yes
Mean	No	Yes (?)	**YES**

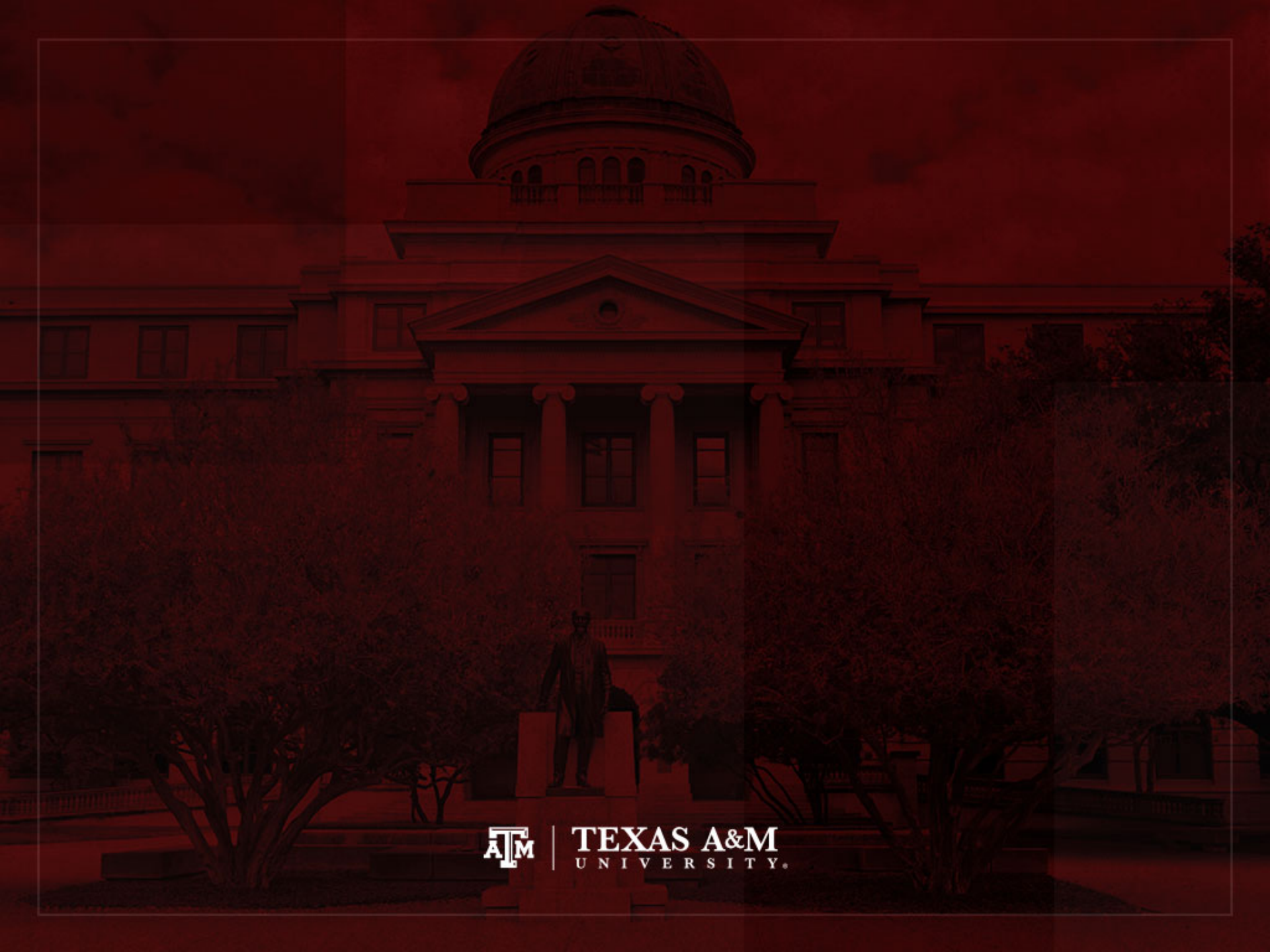
  - **YES**: most appropriate measure for each level
  - Yes: measure is also permitted
  - Yes (?): mean is often used with ordinal-level variables, but this practice violates level-of-measurement guidelines
  - No: cannot be computed for that level

Source: Healey 2015, p.80.

Summary to choose measure

Use the mode when:	1. The variable is measured at the nominal level. 2. You want a quick and easy measure for ordinal- and interval-ratio-level variables. 3. You want to report the most common score.
Use the median when:	1. The variable is measured at the ordinal level. 2. An interval-ratio variable is badly skewed. 3. You want to report the central score. The median always lies at the exact center of the distribution.
Use the mean when:	1. The variable is measured at the interval-ratio level (except when the variable is badly skewed). 2. You want to report the typical score. The mean is the statistics that exactly balances all of the scores. 3. You anticipate additional statistical analysis.

Source: Healey 2015, p.81.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