# Lecture (chapter 6): Introduction to inferential statistics: Sampling and the sampling distribution

Ernesto F. L. Amaral

October 1–3, 2018
Advanced Methods of Social Research (SOCI 420)

Source: Healey, Joseph F. 2015. ”Statistics: A Tool for Social Research.” Stamford: Cengage Learning. 10th edition. Chapter 6 (pp. 144–159).

TEXAS A&M UNIVERSITY

# Chapter learning objectives

- Explain the purpose of inferential statistics in terms of generalizing from a sample to a population
- Define and explain the basic techniques of random sampling
- Explain and define these key terms: population, sample, parameter, statistic, representative, EPSEM sampling techniques
- Differentiate between the sampling distribution, the sample, and the population
- Explain the two theorems presented

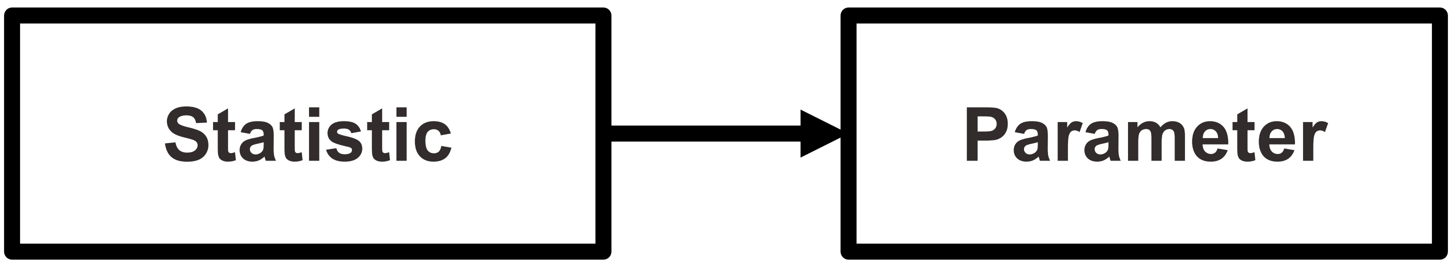
# Basic logic and terminology

- **Problem**
- The populations we wish to study are almost always so large that we are unable to gather information from every case

- **Solution**
- We choose a sample – a carefully chosen subset of the population – and use information gathered from the cases in the sample to generalize to the population

# Basic logic and terminology

- **Statistics** are mathematical characteristics of samples
- **Parameters** are mathematical characteristics of populations
- **Statistics** are used to estimate **parameters**

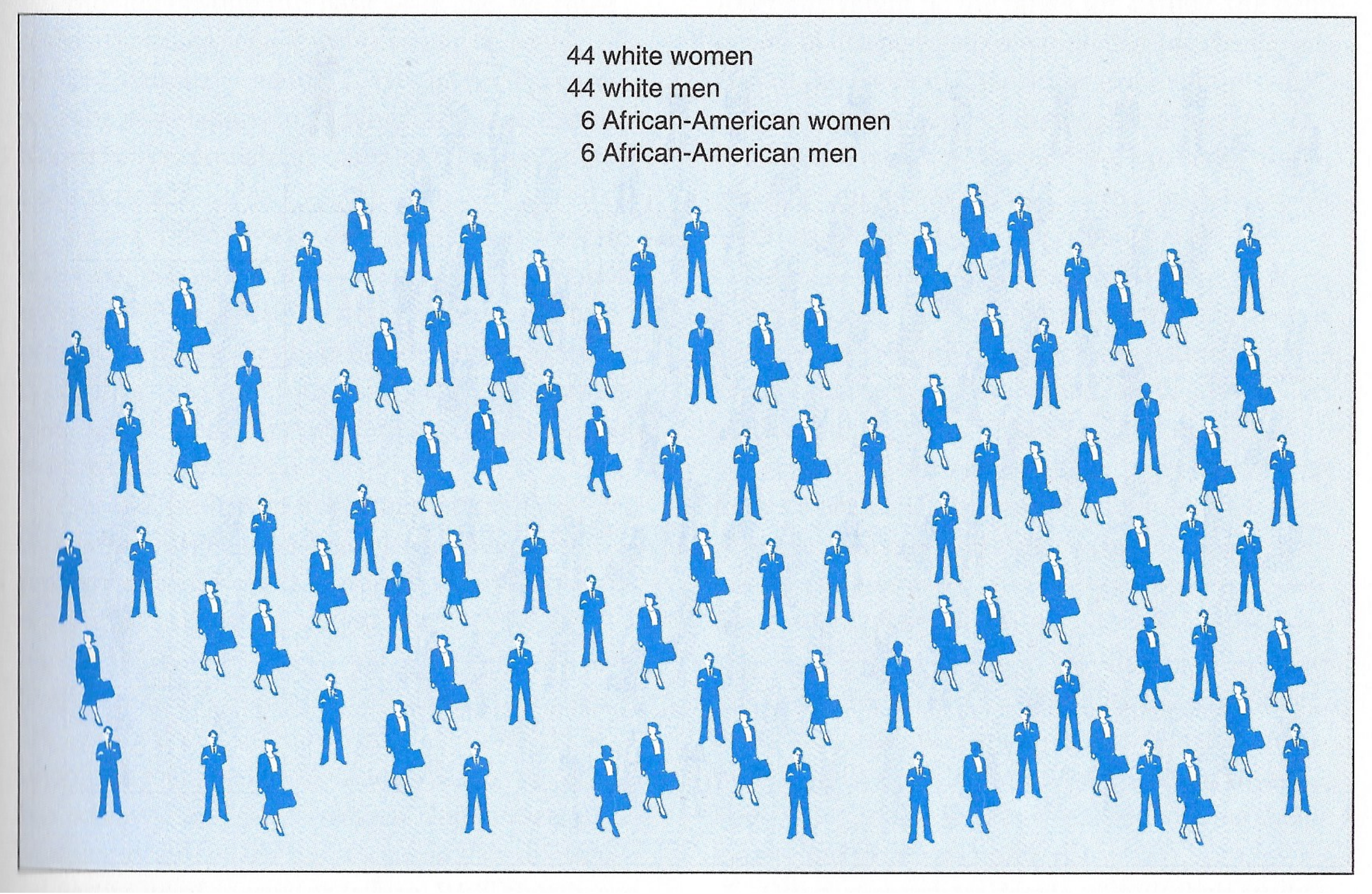


# Samples

- Must be representative of the population
  - Representative: The sample has the same characteristics as the population

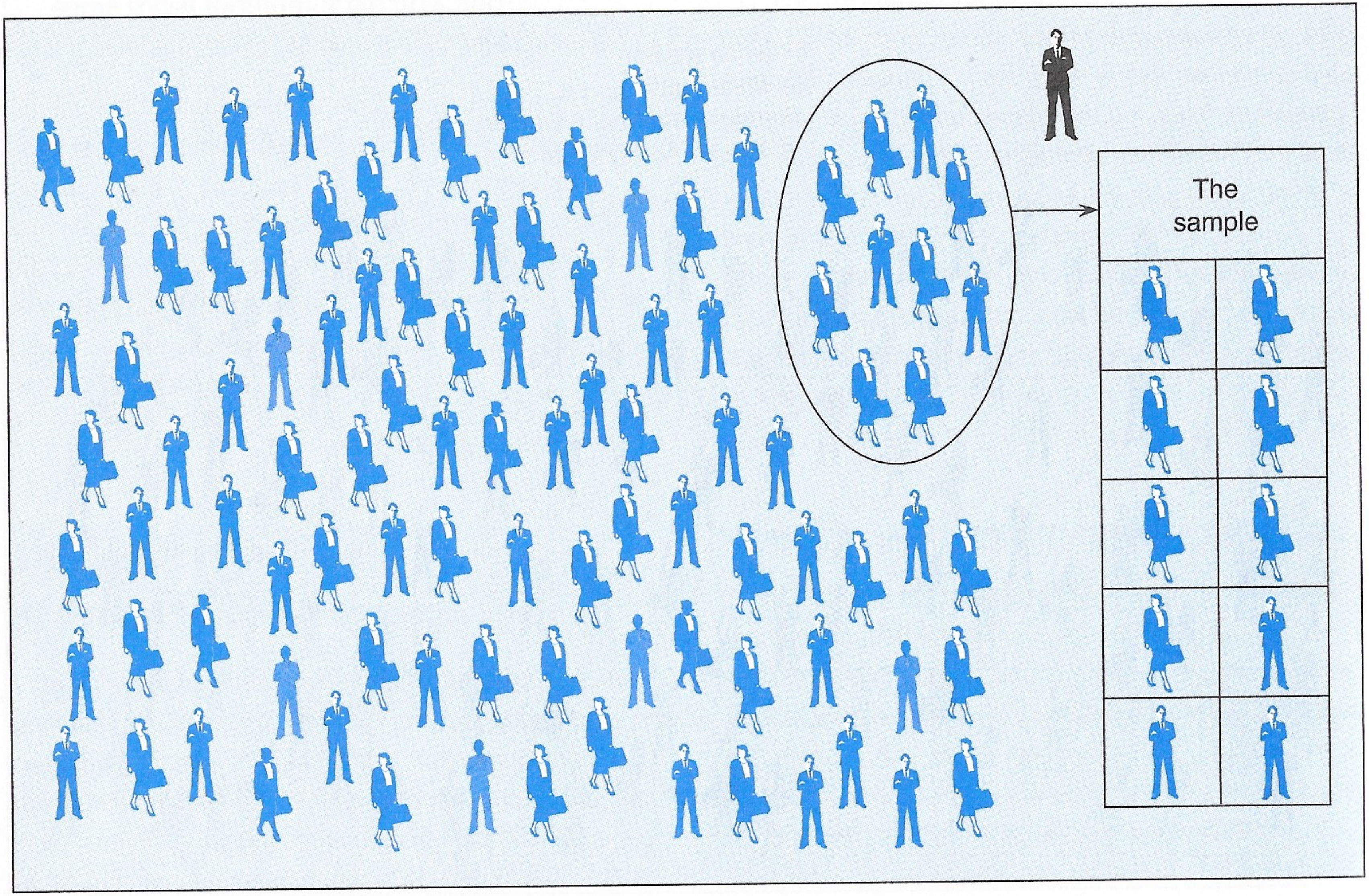
- How can we ensure samples are representative?
  - Samples drawn according to the rule of **EPSEM** (**e**qual **p**robability of **s**election **m**ethod)
  - If every case in the population has the same chance of being selected, the sample is likely to be representative

# A population of 100 people




Source: Babbie 2001, p.183.

# Nonprobability sampling




Source: Babbie 2001, p.184.

# EPSEM sampling techniques

1. Simple random sampling

2. Systematic sampling

3. Stratified sampling

4. Cluster sampling

# 1. Simple random sampling

- To begin, we need
  - A list of the population
- A method for selecting cases from the population so each case has the same probability of being selected
  - The principle of EPSEM
  - A sample selected this way is very likely to be representative of the population
  - Variable in population should have a normal distribution or *N*>30

# Example

- You want to know what percent of students at a large university work during the semester
- Draw a sample of 500 from a list of all students (N=20,000)
- Assume the list is available from the Registrar
- How can you draw names so every student has the same chance of being selected?

# Example

- Each student has a unique, 6 digit ID number that ranges from 000001 to 999999
- Use a table of random numbers or a computer program to select 500 ID numbers with 6 digits each
- Each time a randomly selected 6 digit number matches the ID of a student, that student is selected for the sample
- Continue until 500 names are selected

# Example

- **Stata**

```
set obs 500
generate student = runiformint(1,999999)
sum student
    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
     student |        500    482562.6    283480.9       3652     997200
```

- **Excel**
  - Use RANDBETWEEN function
  - Returns a random number between those you specify
  - Drag the function to 500 cells

=RANDBETWEEN(1,999999)

# Example

- Disregard duplicate numbers

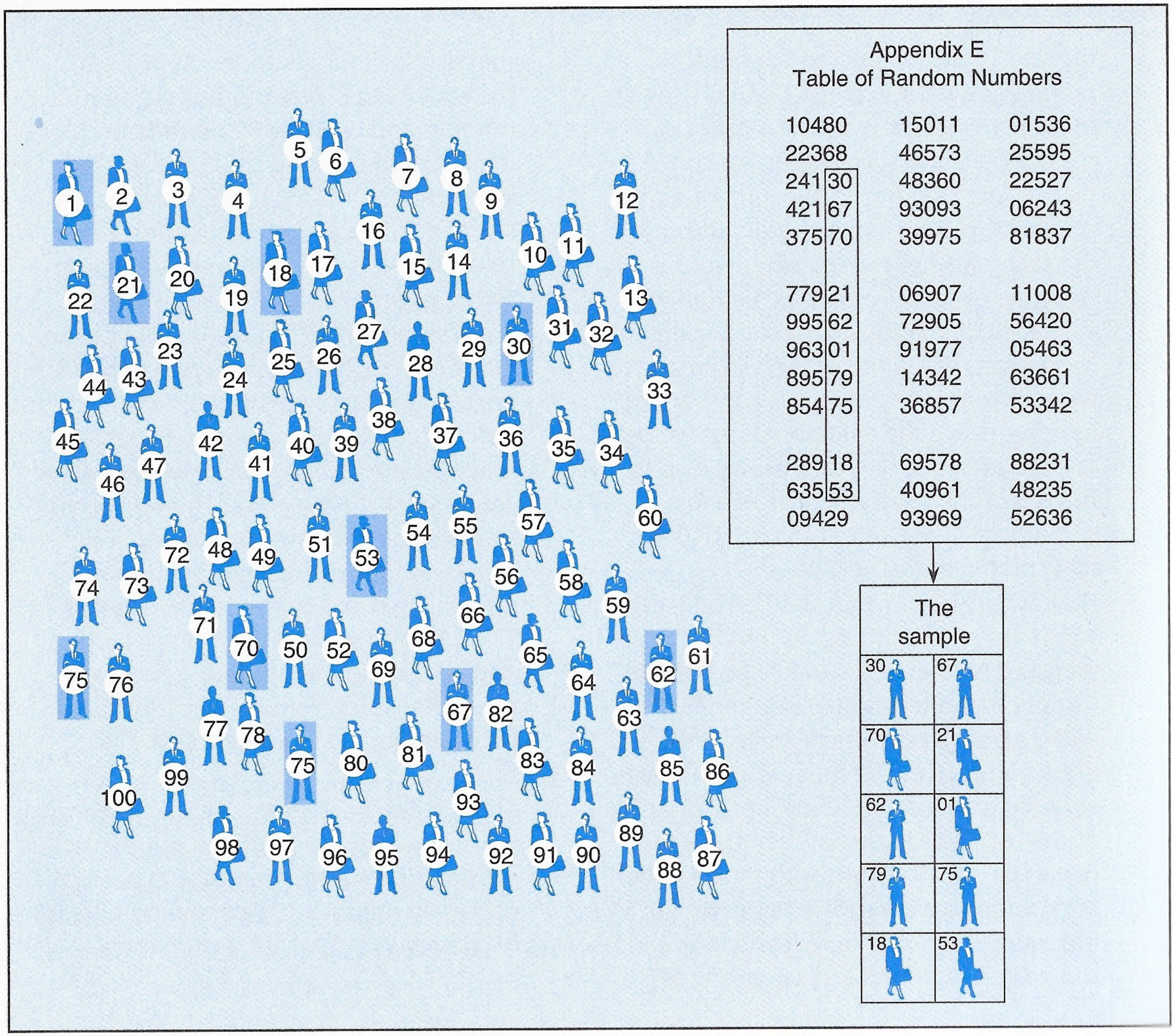
- Ignore cases in which no student ID matches the randomly selected number

- After questioning each of these 500 students, you find that 368 (74%) work during the semester

# Applying logic and terminology

- In the previous example:
- **Population:** All 20,000 students
- **Sample:** 500 students selected and interviewed
- **Statistic:** 74% (percentage of sample that held a job during the semester)
- **Parameter:** Percentage of all students in the population who held a job

# Simple random sample




Source: Babbie 2001, p.200.

# 2. Systematic sampling

- Useful for large populations
- Randomly select the first case then select every *k*th case
- **Sampling interval**
  - Distance between elements selected in the sample
  - Population size divided by sample size
- **Sampling ratio**
  - Proportion of selected elements in the population
  - Sample size divided by population size
- Can be problematic if the list of cases is not truly random or demonstrates some patterning

**Source: Babbie 2001, p.197–198.**

# Example

- If a list contained 10,000 elements and we want a sample of 1,000
- Sampling interval
  - Population size / sample size = 10,000 / 1,000 = 10
  - We would select every 10th element for our sample
- Sampling ratio
  - Sample size / population size = 1,000 / 10,000 = 1/10
  - Proportion of selected elements in population
- Select the first element at random

Source: Babbie 2001, p.197–198.

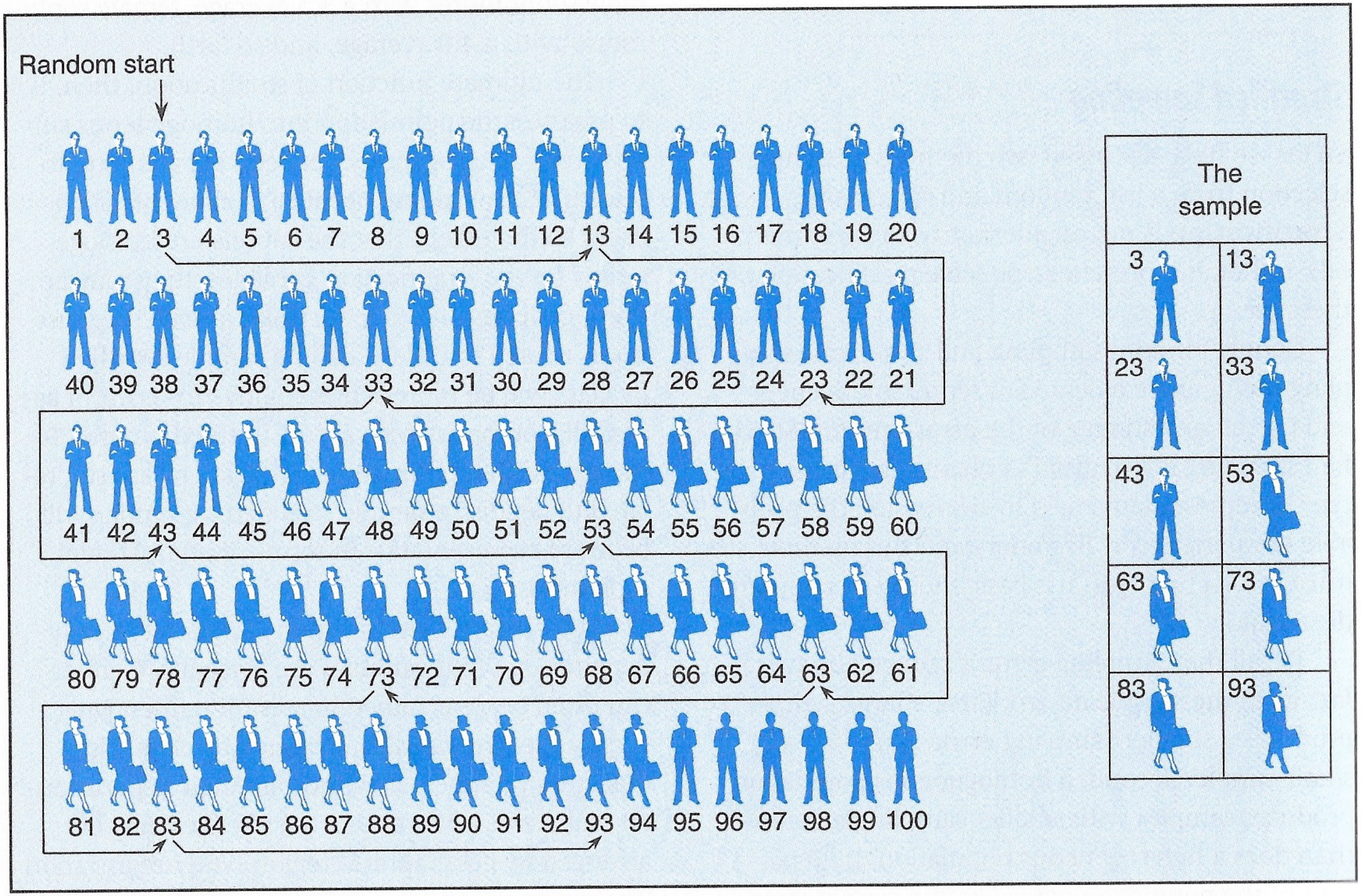
# 3. Stratified sampling

- It guarantees the sample will be representative on the selected (stratifying) variables
  - Stratification variables relate to research interests
- First, divide the population list into subsets, according to some relevant variable
  - **Homogeneity within subsets**
    - E.g., only women in a subset; only men in another subset
  - **Heterogeneity between subsets**
    - E.g., subset of women is different than subset of men
- Second, sample from the subsets
  - Select the number of cases from each subset proportional to the population

# Example

- If you want a sample of 1,000 students
  - That would be representative to the population of students by sex and GPA
- You need to know the population composition
  - E.g., women with a 4.0 average compose 15 percent of the student population
- Your sample should follow that composition
  - In a sample of 1,000 students, you would select 150 women with a 4.0 average

# Stratified, systematic sample




**Source: Babbie 2001, p.202.**

# 4. Cluster sampling

- Select groups (or clusters) of cases rather than single cases
  - **Heterogeneity within subsets**
    - E.g., each subset has both women and men, following same proportional distribution as population
  - **Homogeneity between subsets**
    - E.g., all subsets with both women and men should be similar
- Clusters are often geographically based
  - For example, cities or voting districts
- Sampling often proceeds in stages
  - Multi-stage cluster sampling
  - Less representative than simple random sampling

# Stratified vs. cluster sampling

- **Stratified**
  - Homogeneity within subsets
  - Heterogeneity between subsets
  - Select cases from each subset

| Subset of women | Subset of men |
| --- | --- |

- **Cluster**
  - Heterogeneity within subsets (groups, clusters, areas)
  - Homogeneity between subsets
  - Select groups (e.g., area 1) rather than single cases

| Area 1: women & men | Area 2: women & men |
| --- | --- |

# The sampling distribution

- The single most important concept in inferential statistics
- Sampling distribution is the probabilistic distribution of a statistic for all possible samples of a given size (*N*)
- The sampling distribution is a theoretical concept

# The sampling distribution

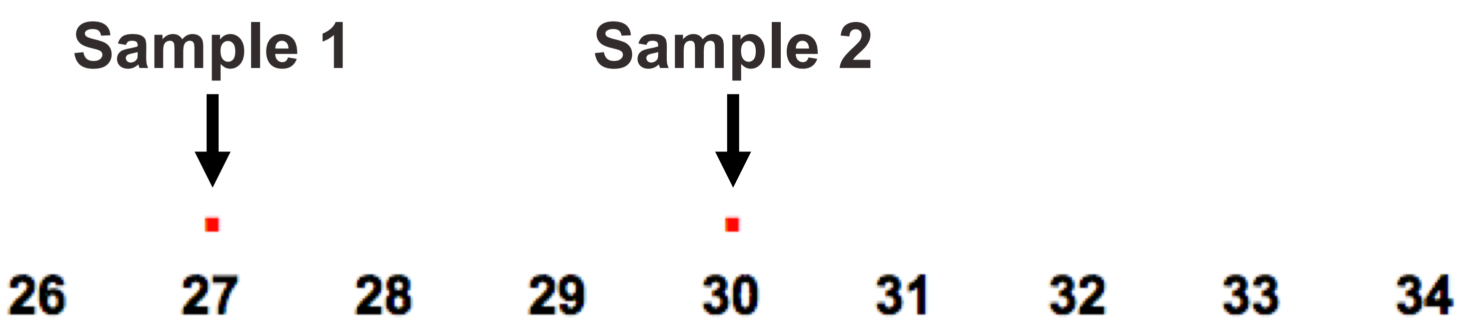
- Every application of inferential statistics involves three different distributions
  - Population: empirical; unknown
  - Sampling distribution: theoretical; known
  - Sample: empirical; known

- Information from the sample is linked to the population via the sampling distribution

# Example

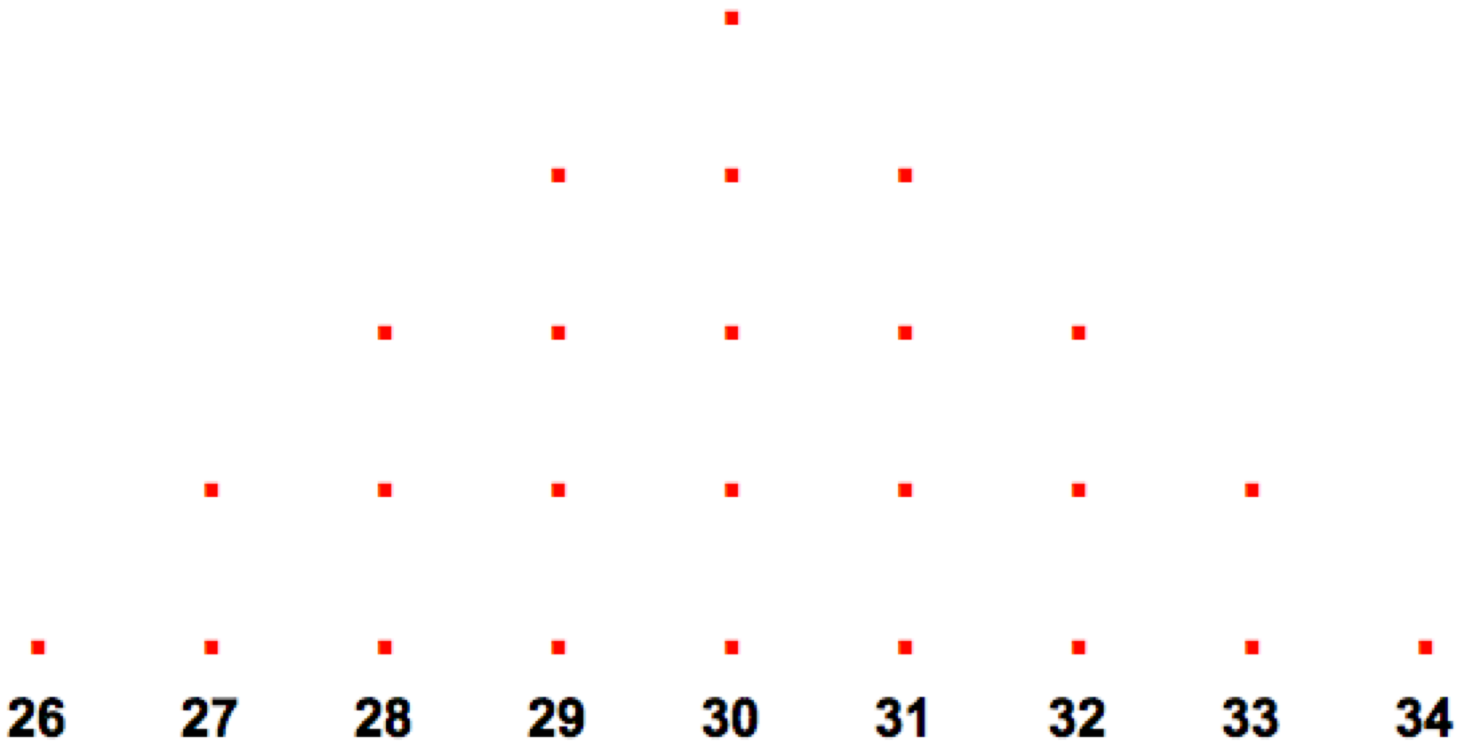
- Suppose we want to gather information on the age of a community of 10,000 individuals
  - Sample 1: N=100 people, plot sample's mean of 27
  - Replace people in the sample back to the population

  - Sample 2: N=100 people, plot sample's mean of 30
  - Replace people in the sample back to the population

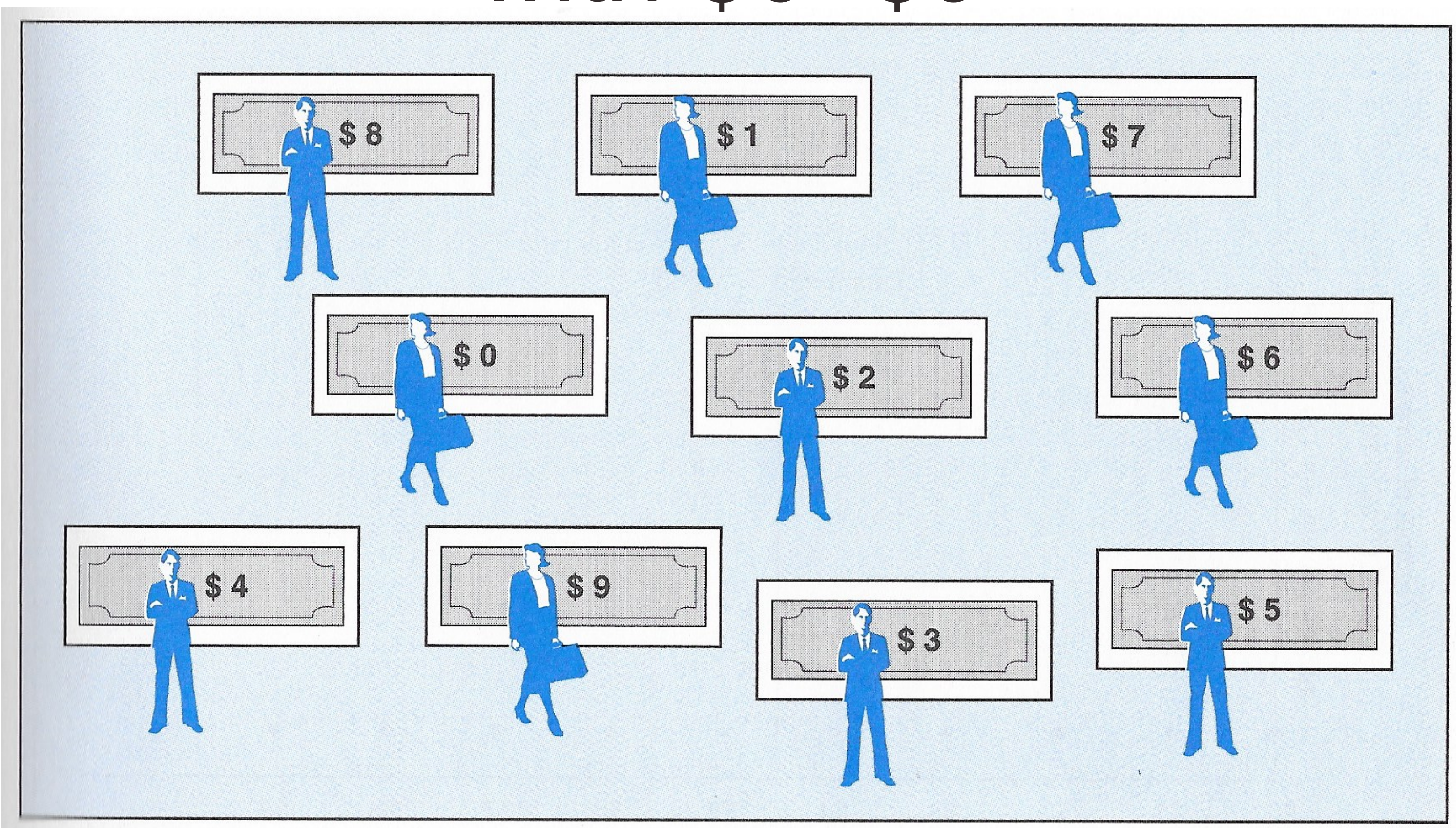


# Example

- We repeat this procedure
  - Sampling and replacing
  - Until we have exhausted every possible combination of 100 people from the population of 10,000

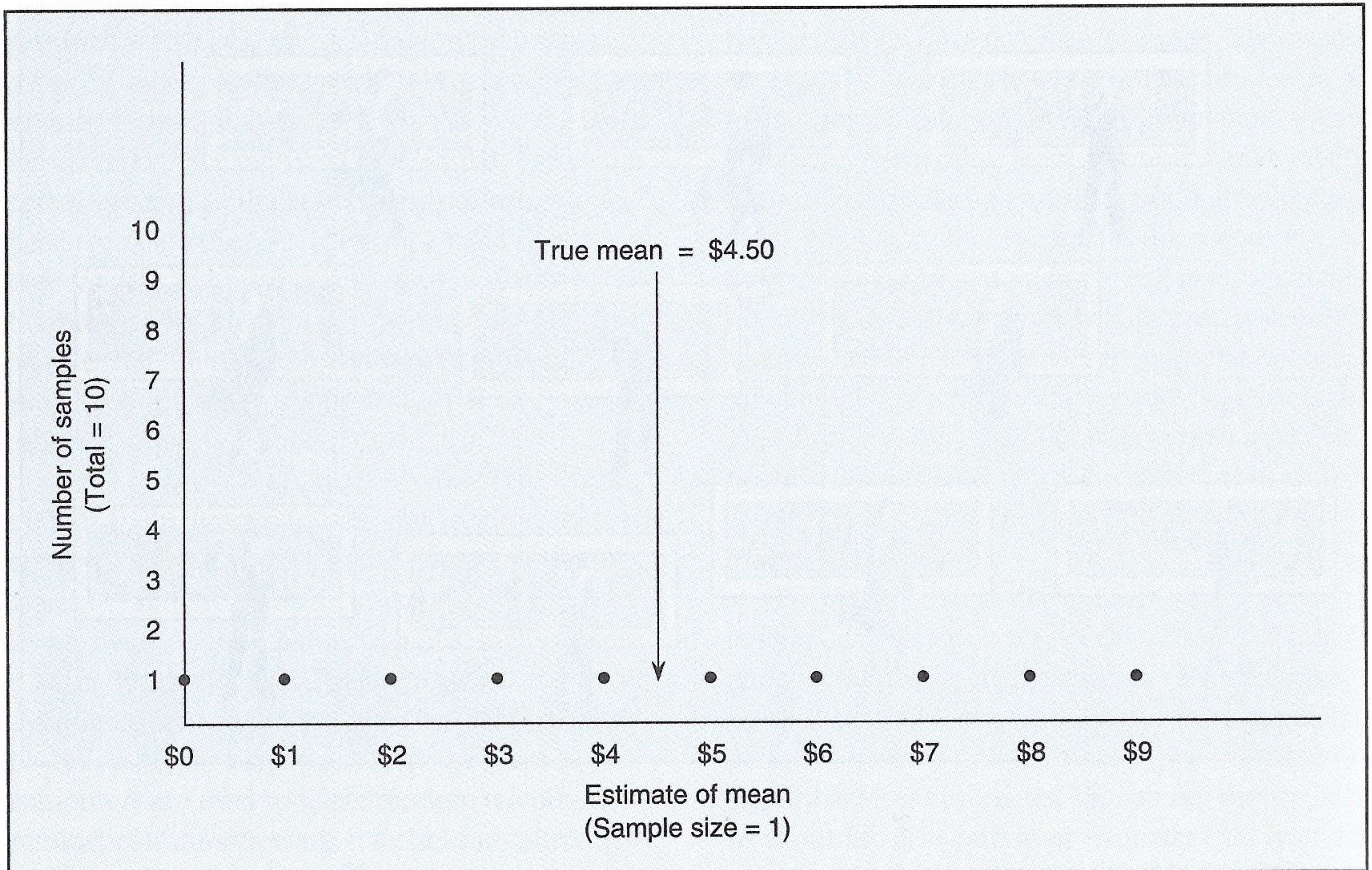


# Another example: A population of 10 people with \$0–\$9



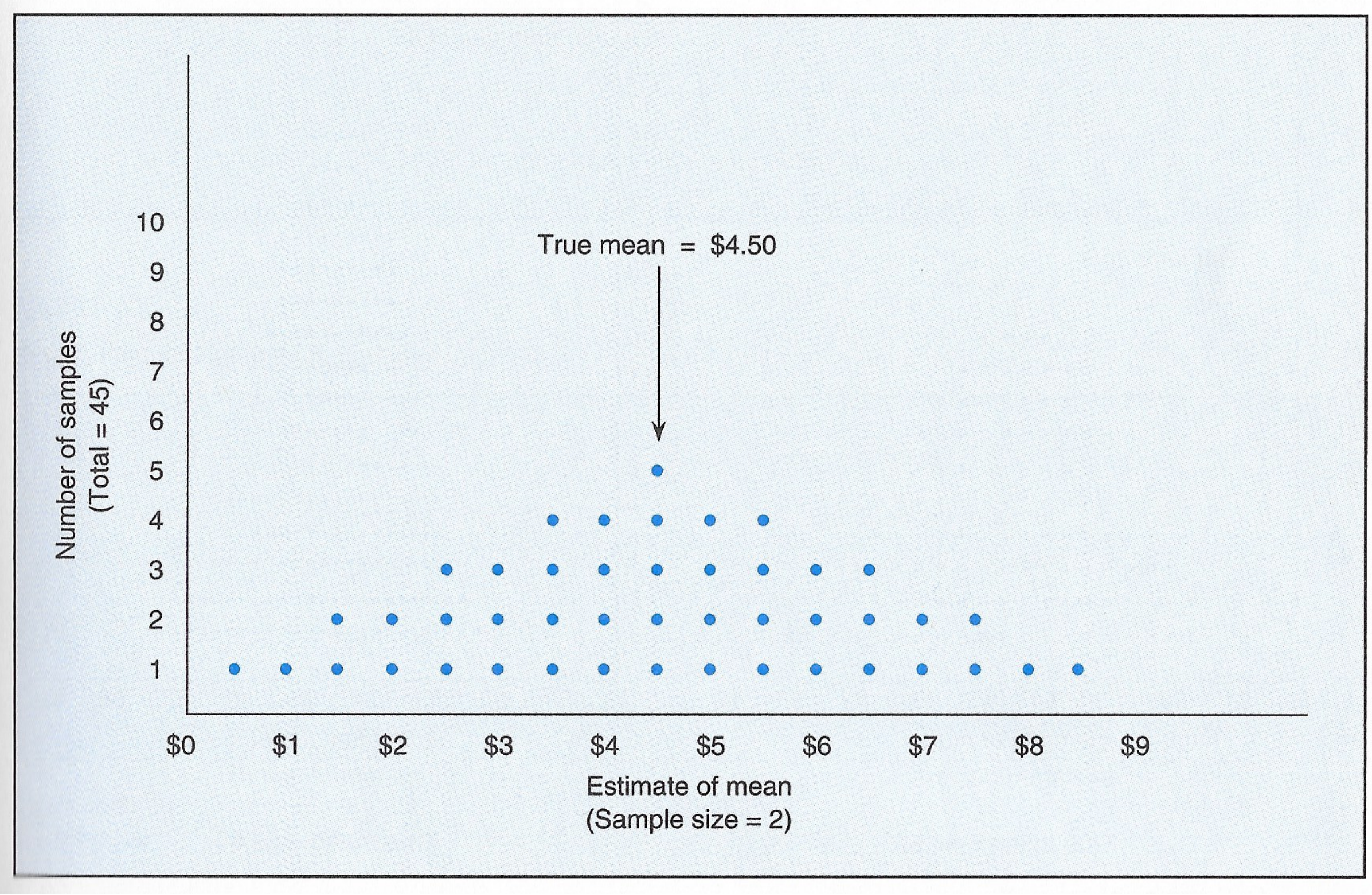

Source: Babbie 2001, p.187.

# The sampling distribution (*N*=1)



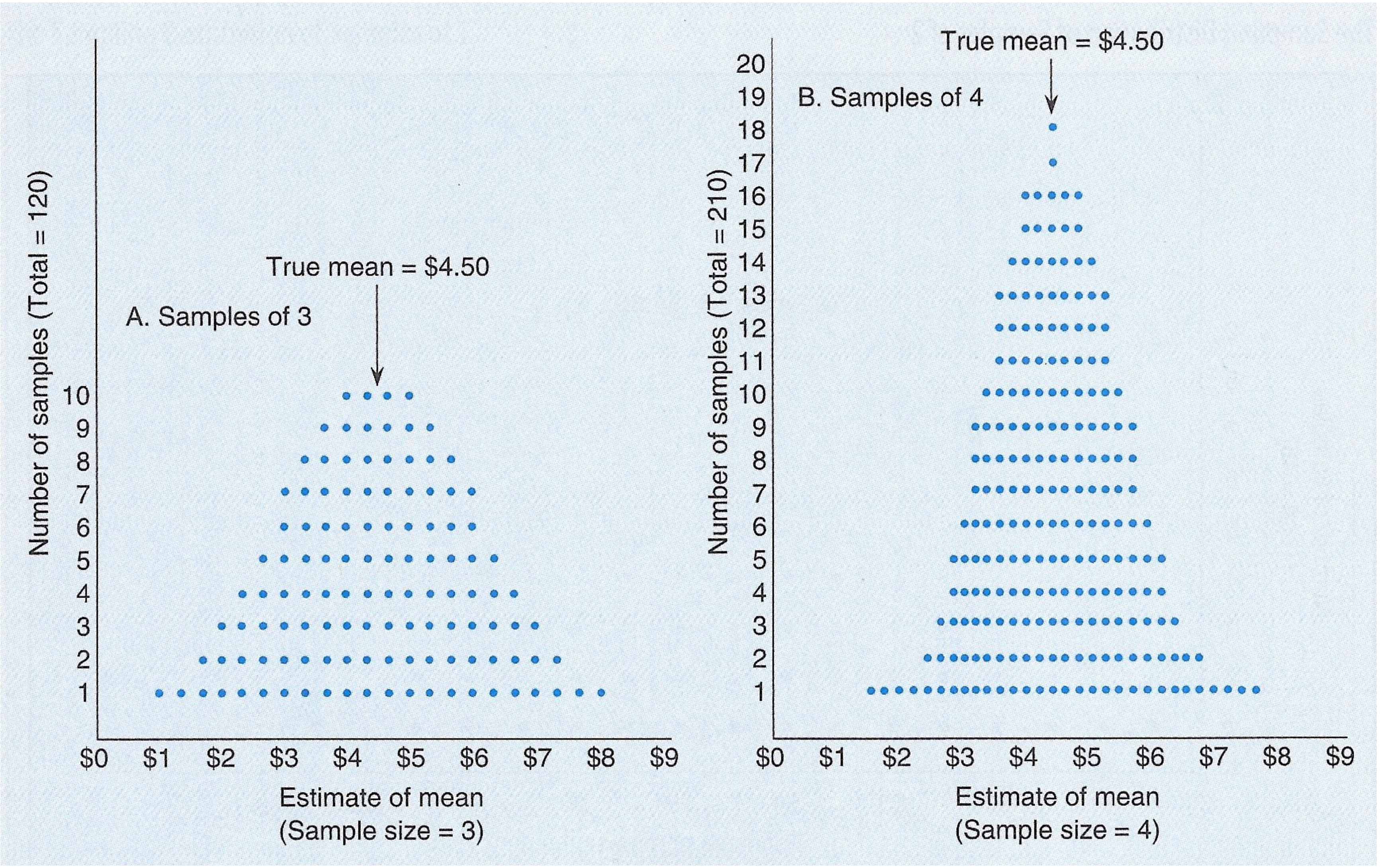

Source: Babbie 2001, p.188.

# The sampling distribution (*N*=2)



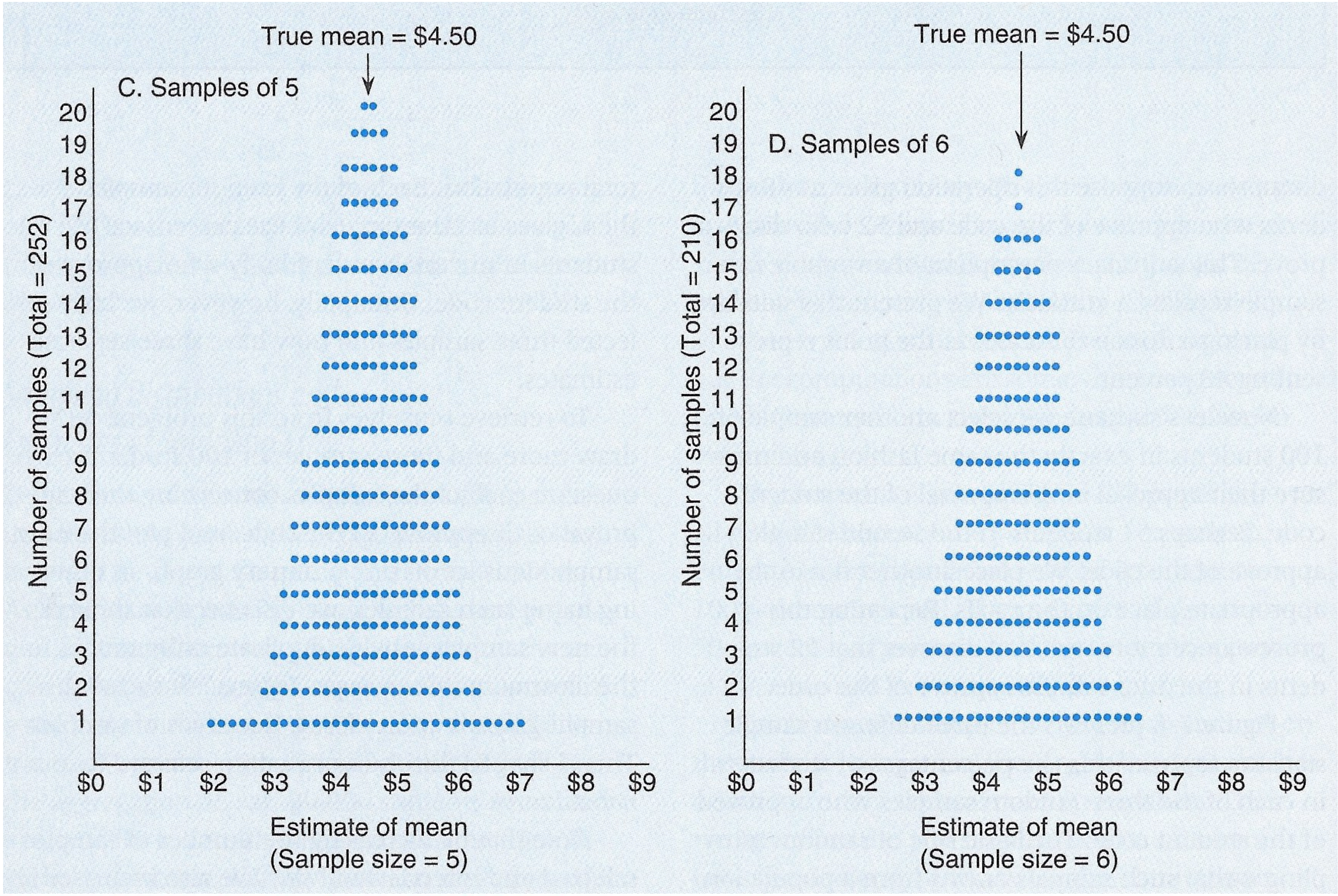

**Source: Babbie 2001, p.189.**

# The sampling distribution




**Source: Babbie 2001, p.190.**

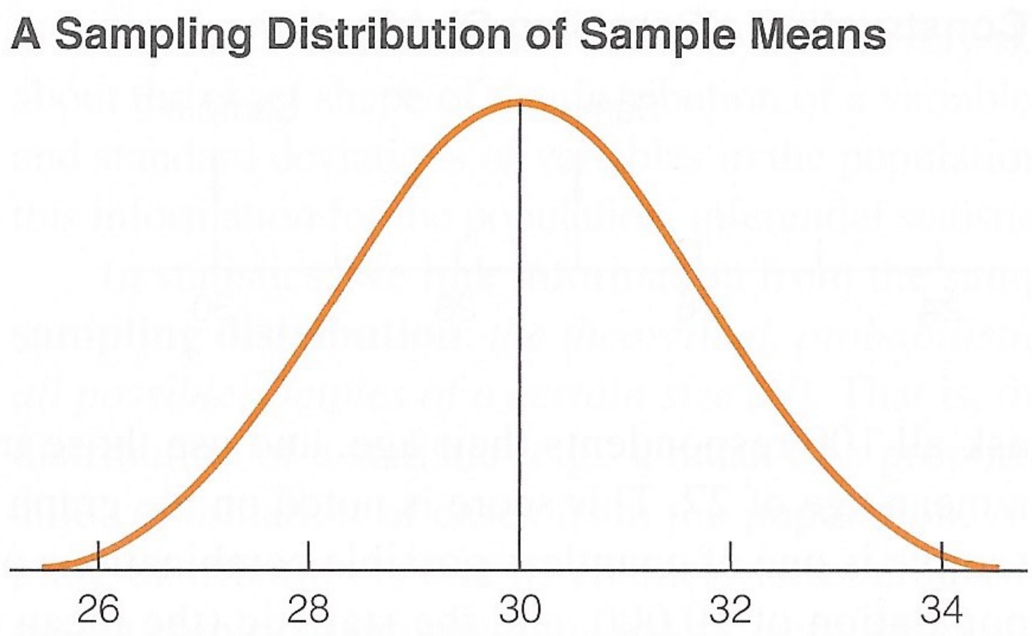
# The sampling distribution




**Source: Babbie 2001, p.190.**

# Properties of sampling distribution

- It has a mean ($\mu_{\bar{X}}$) equal to the population mean ($\mu$)
- It has a standard deviation (standard error, $\sigma_{\bar{X}}$) equal to the population standard deviation ($\sigma$) divided by the square root of *N*
- It has a normal distribution




Source: Healey 2015, p.152.

# First theorem

- Tells us the shape of the sampling distribution and defines its mean and standard deviation

- If repeated random samples of size *N* are drawn from a **normal population** with mean $\mu$ and standard deviation $\sigma$
  - Then, the sampling distribution of sample means will **have a normal distribution** with...
  - A mean: $\mu_{\bar{X}} = \mu$
  - A standard error of the mean: $\sigma_{\bar{X}} = \sigma/\sqrt{N}$

# First theorem

- Begin with a characteristic that is normally distributed across a population (IQ, height)
- Take an infinite number of equally sized random samples from that population
- The sampling distribution of sample means will be normal

# Central limit theorem

- If repeated random samples of size *N* are drawn from **any population** with mean $\mu$ and standard deviation $\sigma$
  - Then, as *N* becomes large, the sampling distribution of sample means will **approach normality** with...
  - A mean: $\mu_{\bar{X}} = \mu$
  - A standard error of the mean: $\sigma_{\bar{X}} = \sigma/\sqrt{N}$
- This is true for any variable, even those that are not normally distributed in the population
  - As sample size grows larger, the sampling distribution of sample means will become normal in shape

# Central limit theorem

- The importance of the central limit theorem is that it removes the constraint of normality in the population
  - Applies to large samples ($N \geq 100$)

- If the sample is small ($N < 100$)
  - We must have information on the normality of the population before we can assume the sampling distribution is normal

# Additional considerations

- The sampling distribution is normal
  - We can estimate areas under the curve (Appendix A)
  - Or in Stata: **`display normal(z)`**
- We do not know the value of the population mean ($\mu$)
  - But the mean of the sampling distribution ($\mu_{\bar{X}}$) is the same value as $\mu$
- We do not know the value of the population standard deviation ($\sigma$)
  - But the standard deviation of the sampling distribution ($\sigma_{\bar{X}}$) is equal to $\sigma$ divided by the square root of $N$

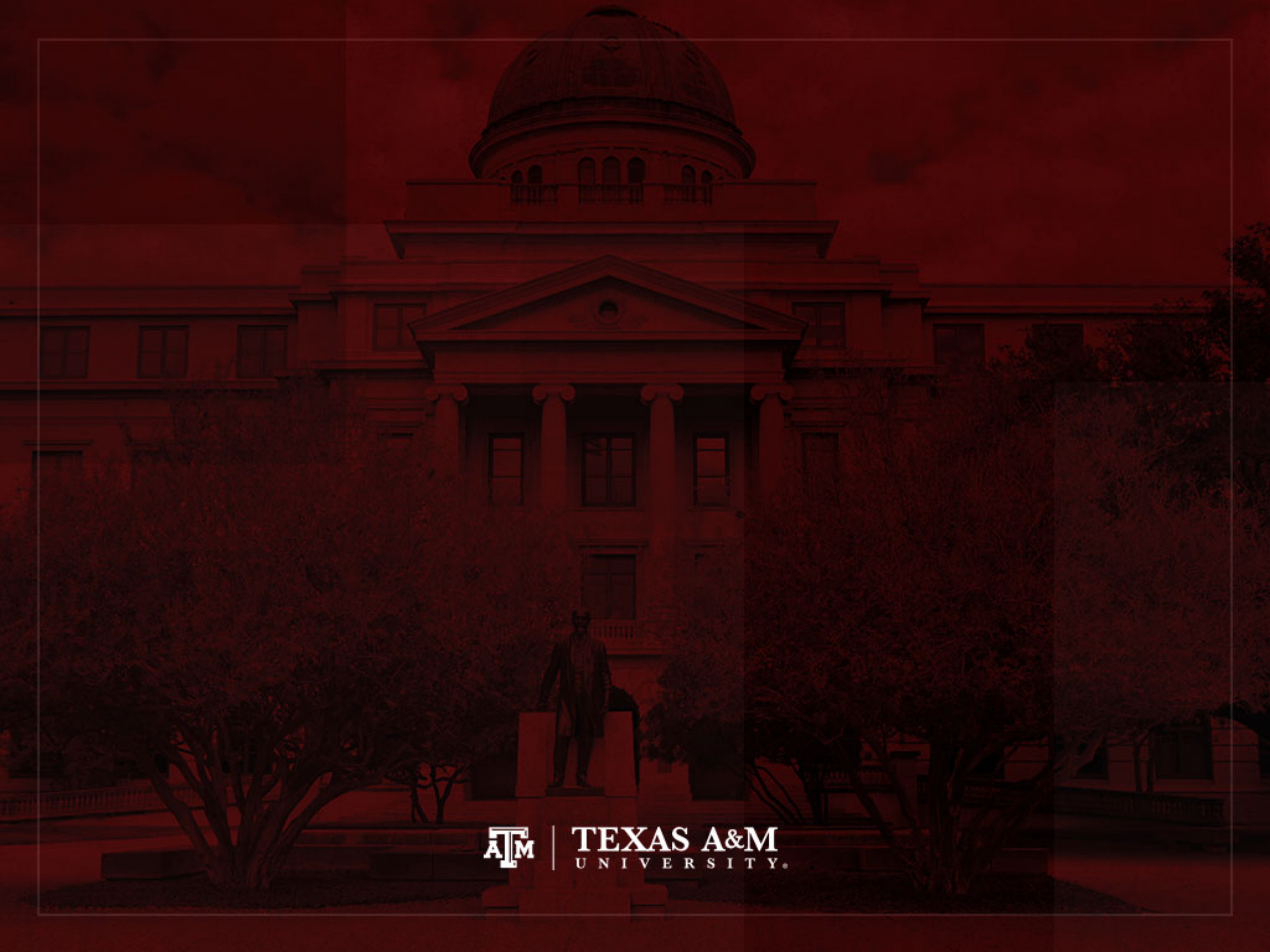
# Symbols

| Distribution | Shape | Mean | Standard deviation | Proportion |
|---|---|---|---|---|
| Samples | Varies | $\bar{X}$ | $s$ | $P_s$ |
| Populations | Varies | $\mu$ | $\sigma$ | $P_u$ |
| Sampling distributions | Normal | $\mu_{\bar{X}}$ | | |
| of means | | $\mu_{\bar{X}}$ | $\sigma_{\bar{X}} = \sigma/\sqrt{N}$ | |
| of proportions | | $\mu_p$ | $\sigma_p$ | |

Source: Healey 2015, p.157.

# Sampling distribution

- It is the distribution of a statistic (e.g., proportion, mean) for all possible outcomes of a certain size
- In inferential statistics, the sample distribution links the sample with the population
- It has a normal shape
- Central tendency and dispersion
  - Mean is the same as the population mean
  - Standard deviation is referred as standard error
    - It is the population standard deviation divided by the square root of *N*
    - We have to take into account the complex survey design to estimate the standard error (`svyset` command in Stata)


ATM
TEXAS A&M
UNIVERSITY