

# Lecture (chapter 7): Estimation procedures

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Source: Healey, Joseph F. 2015. "Statistics: A Tool for Social Research." Stamford: Cengage Learning. 10th edition. Chapter 7 (pp. 160–184).



# Chapter learning objectives

- Explain the logic of estimation, role of the sample, sampling distribution, and population
- Define and explain the concepts of bias and efficiency
- Construct and interpret confidence intervals for sample means and sample proportions
- Explain relationships among confidence level, sample size, and width of the confidence interval

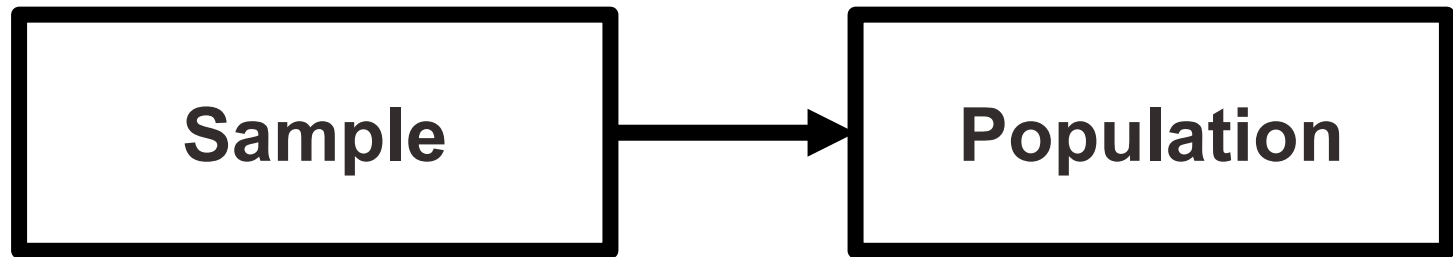


# Sample and population

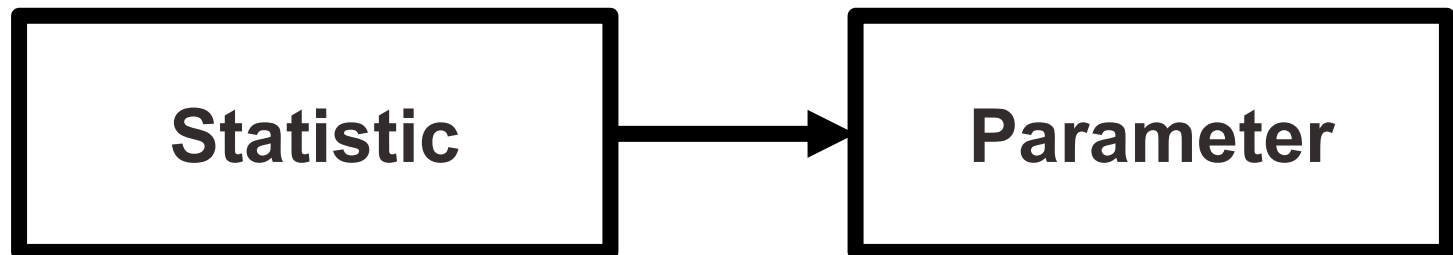
- In estimation procedures, statistics calculated from random samples are used to estimate the value of population parameters
- Example
  - If we know that 42% of a random sample drawn from a city are Republicans, we can estimate the percentage of all city residents who are Republicans

# Terminology

- Information from samples is used to estimate information about the population



- Statistics are used to estimate parameters



# Basic logic

- Sampling distribution is the link between sample and population
- The values of the parameters are unknown, but the characteristics of the sampling distribution are defined by two theorems (previous chapter)



# Two estimation procedures

- **A point estimate** is a sample statistic used to estimate a population value
  - 68% of a sample of randomly selected Americans support capital punishment (GSS 2010)
- **An interval estimate** consists of confidence intervals (range of values)
  - Between 65% and 71% of Americans approve of capital punishment (GSS 2010)
  - Most point estimates are actually interval estimates
  - Margin of error generates confidence intervals
  - Estimators are selected based on two criteria...



# Bias

- An estimator is unbiased if the mean of its sampling distribution is equal to the population value of interest
- The mean of the sampling distribution of sample means ( $\mu_{\bar{X}}$ ) is the same as the population mean ( $\mu$ )
- Sample proportions ( $P_s$ ) are also unbiased
  - If we calculate sample proportions from repeated random samples of size  $N$ ...
  - Then, the sampling distribution of sample proportions will have a mean ( $\mu_p$ ) equal to the population proportion ( $P_u$ )
- Sample means and proportions are unbiased estimators
  - We can determine the probability that they are within a certain distance of the population values

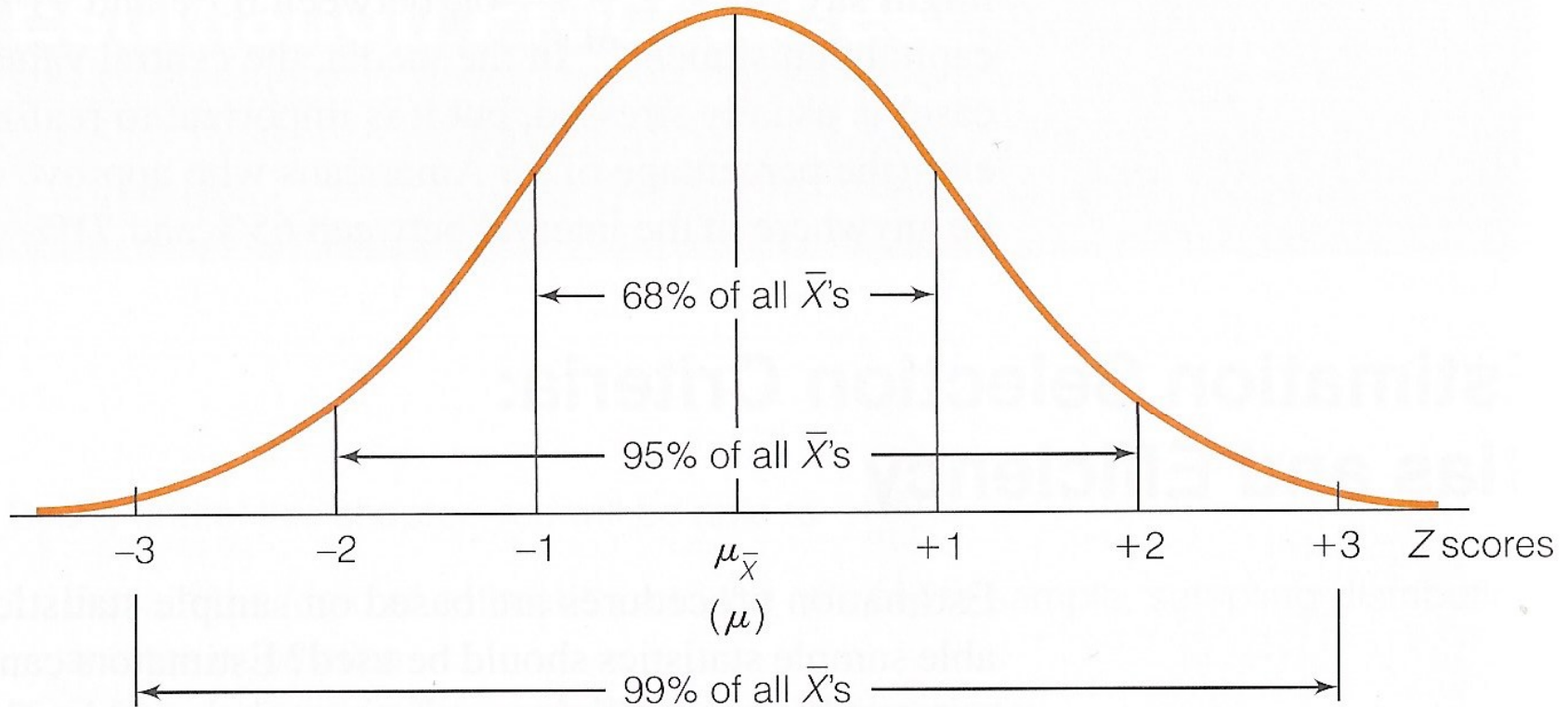
# Example

- Random sample to get income information
- Sample size ( $N$ ): 500 households
- Sample mean ( $\bar{X}$ ): \$45,000
- Population mean ( $\mu$ ): unknown parameter
- Mean of sampling distribution ( $\mu_{\bar{X}} = \mu$ )
  - If an estimator ( $\bar{X}$ ) is unbiased, it is probably an accurate estimate of the population parameter ( $\mu$ ) and sampling distribution mean ( $\mu_{\bar{X}}$ )
  - We use the sampling distribution (which has a normal shape) to estimate confidence intervals





# Sampling distribution



# Efficiency

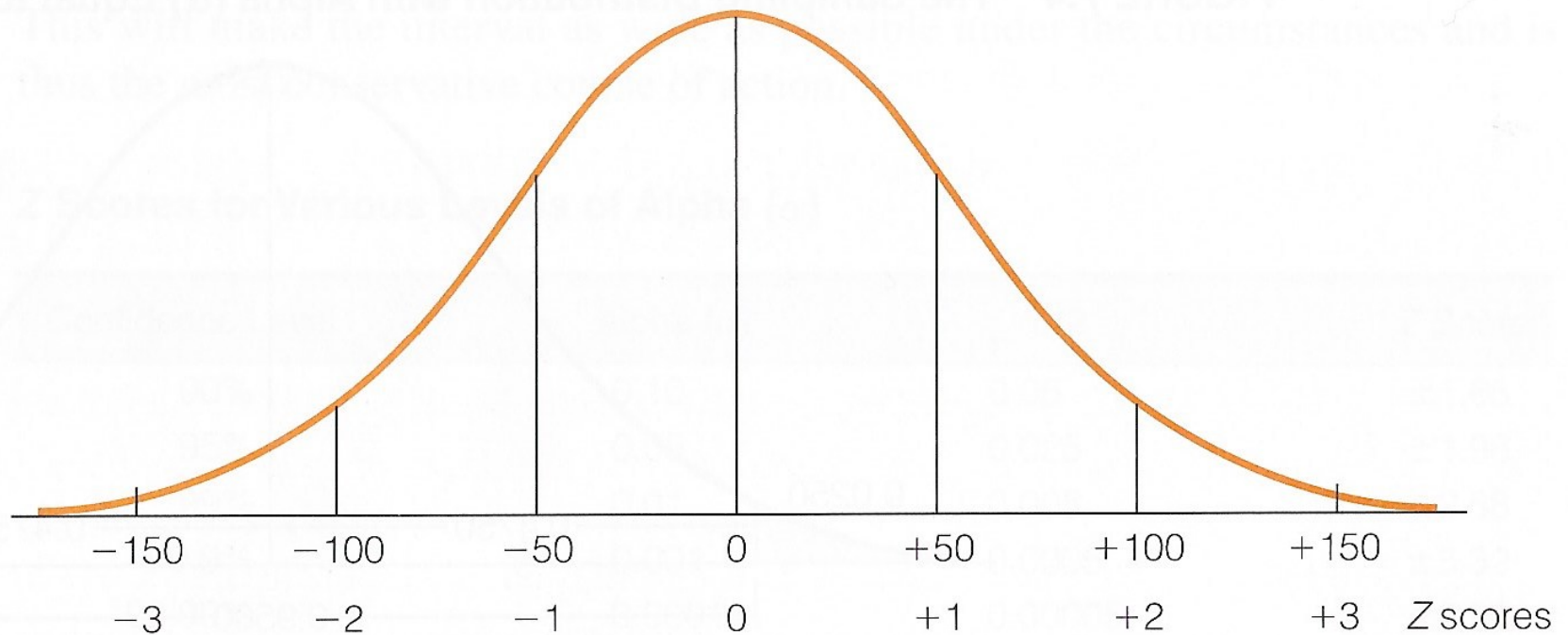
- Efficiency is the extent to which the sampling distribution is clustered around its mean
- Efficiency or clustering is a matter of dispersion
  - The smaller the standard deviation of a sampling distribution, the greater the clustering and the higher the efficiency
  - Larger samples have greater clustering and higher efficiency
  - Standard deviation of sampling distribution:  $\sigma_{\bar{X}} = \sigma/\sqrt{N}$

Statistics	Sample 1	Sample 2
Sample mean	$\bar{X}_1 = \$45,000$	$\bar{X}_2 = \$45,000$
Sample size	$N_1 = 100$	$N_2 = 1000$
Standard deviation	$\sigma_1 = \$500$	$\sigma_2 = \$500$
Standard error	$\sigma_{\bar{X}} = 500/\sqrt{100} = \$50.00$	$\sigma_{\bar{X}} = 500/\sqrt{1000} = \$15.81$



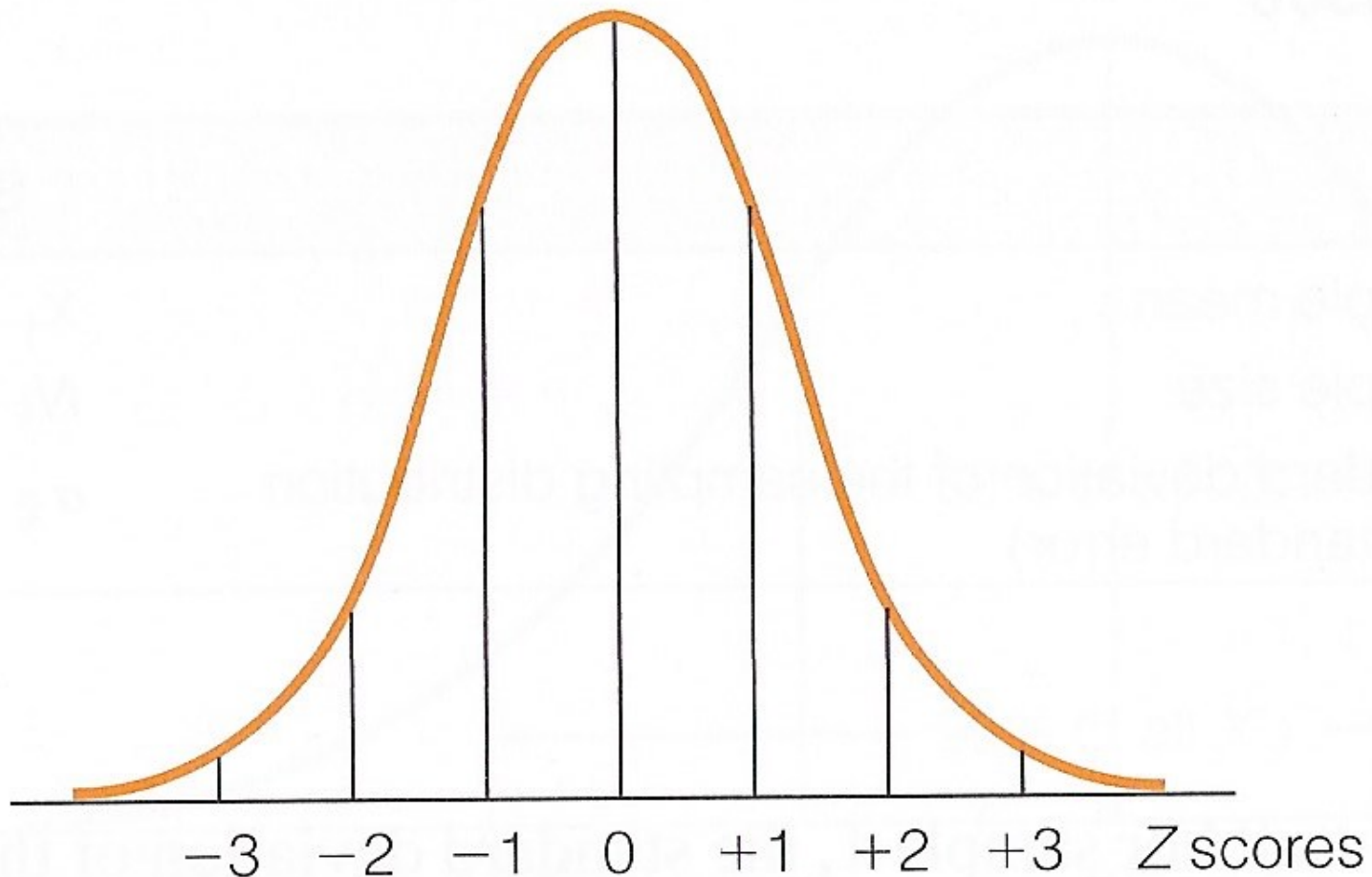
# Sampling distribution

$$N = 100; \sigma_{\bar{X}} = \$50.00$$



# Sampling distribution

$$N = 1000; \sigma_{\bar{X}} = \$15.81$$



# Confidence interval & level

- **Confidence interval** is a range of values used to estimate the true population parameter
  - We associate a confidence level (e.g. 0.95 or 95%) to a confidence interval
- **Confidence level** is the success rate of the procedure to estimate the confidence interval
  - Expressed as probability  $(1-\alpha)$  or percentage  $(1-\alpha)*100$
  - $\alpha$  is the complement of the confidence level
  - Larger confidence levels generate larger confidence intervals
- Confidence level of 95% is the most common
  - Good balance between precision (width of confidence interval) and reliability (confidence level)



# Interval estimation procedures

- Set the alpha ( $\alpha$ )
  - Probability that the interval will be wrong
- Find the Z score associated with alpha
  - In column c of Appendix A of textbook
    - If the Z score you are seeking is between two other scores, choose the larger of the two Z scores
  - In Stata: **display invnormal( $\alpha$ )**
- Substitute values into appropriate equation
- Interpret the interval



# Example to find Z score

- Setting alpha ( $\alpha$ ) equal to 0.05
  - 95% confidence level:  $(1-\alpha)*100$
  - We are willing to be wrong 5% of the time
- If alpha is equal to 0.05
  - Half of this probability is in the lower tail ( $\alpha/2=0.025$ )
  - Half is in the upper tail of the distribution ( $\alpha/2=0.025$ )
- Looking up this area, we find a  $Z = 1.96$

```
di invnormal(.025)
```

```
-1.959964
```

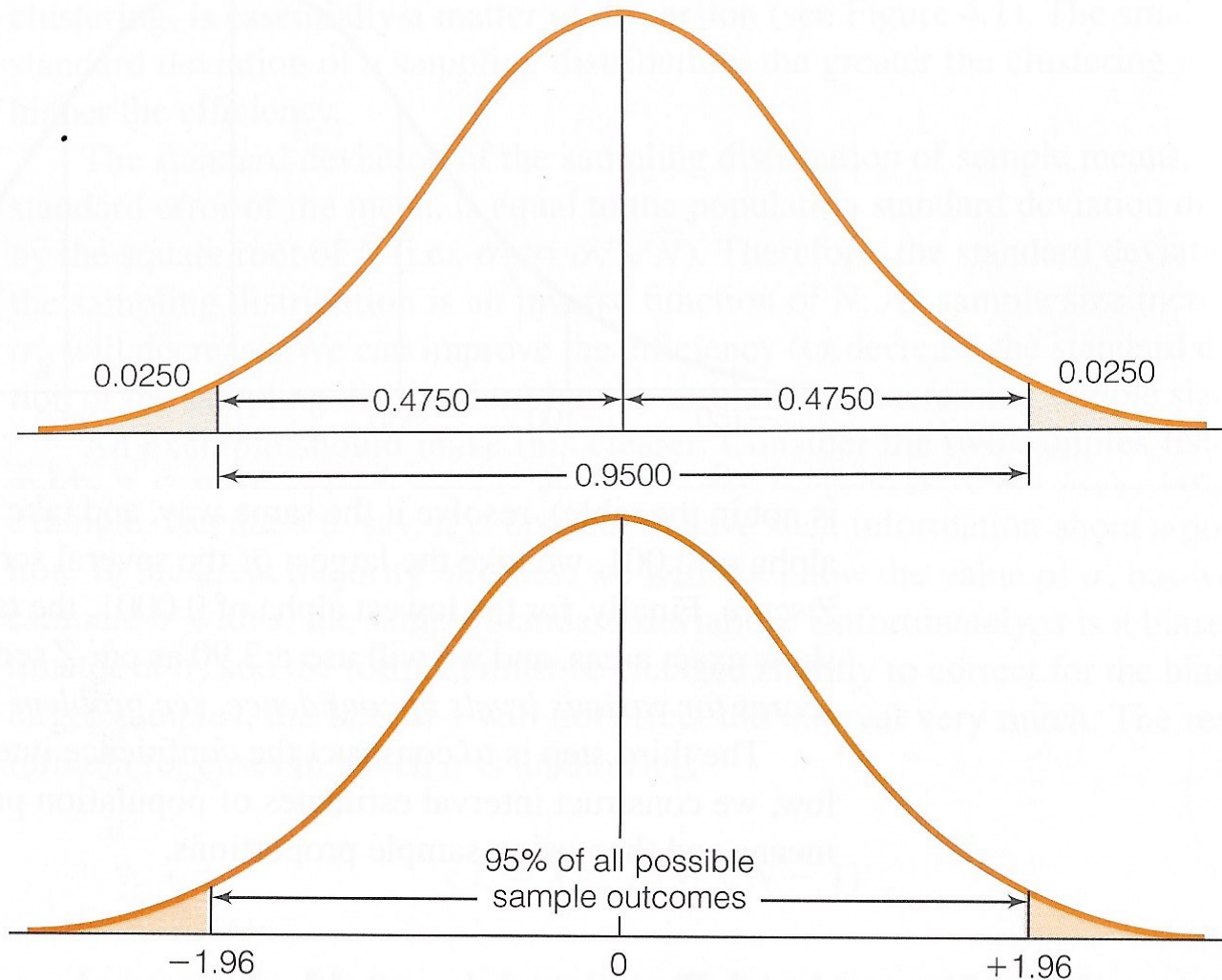
```
di invnormal(1-.025)
```

```
di invnormal(.975)
```

```
1.959964
```



# Finding Z for sampling distribution with $\alpha = 0.05$





# Z scores for various levels of $\alpha$

---

<b>Confidence level (1 - <math>\alpha</math>) * 100</b>	<b>Alpha (<math>\alpha</math>)</b>	<b><math>\alpha / 2</math></b>	<b>Z score</b>
90%	0.10	0.05	$\pm 1.65$
95%	0.05	0.025	$\pm 1.96$
99%	0.01	0.005	$\pm 2.58$
99.9%	0.001	0.0005	$\pm 3.32$
99.99%	0.0001	0.00005	$\pm 3.90$

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# Confidence intervals for sample means

- For large samples ( $N \geq 100$ )
- Standard deviation ( $\sigma$ ) **known** for population

$$c.i. = \bar{X} \pm Z \left( \frac{\sigma}{\sqrt{N}} \right)$$

$c.i.$  = confidence interval

$\bar{X}$  = the sample mean

$Z$  = the  $Z$  score as determined by the alpha level

$\sigma/\sqrt{N}$  = the sample deviation of the sampling distribution  
or the standard error of the mean



# Example for means:

## Large sample, $\sigma$ known

- Sample of 200 residents
- Sample mean of IQ is 105
- Population standard deviation is 15
- Calculate a confidence interval with a 95% confidence level ( $\alpha = 0.05$ )

– Same as saying: calculate a 95% confidence interval

$$c.i. = \bar{X} \pm Z \left( \frac{\sigma}{\sqrt{N}} \right) = 105 \pm 1.96 \left( \frac{15}{\sqrt{200}} \right) = 105 \pm 2.08$$

– Average IQ is somewhere between 102.92 (105–2.08) and 107.08 (105+2.08)



# Interpreting previous example

$$N = 200; 102.92 \leq \mu \leq 107.08$$

- **Correct:** We are 95% certain that the confidence interval contains the true value of  $\mu$ 
  - If we selected several samples of size 200 and estimated their confidence intervals, 95% of them would contain the population mean ( $\mu$ )
  - The 95% confidence level refers to the success rate to estimate the population mean ( $\mu$ ). It does not refer to the population mean itself
- **Wrong:** Since the value of  $\mu$  is fixed, it is incorrect to say that there is a chance of 95% that the true value of  $\mu$  is between the interval



# Confidence intervals for sample means

- For large samples ( $N \geq 100$ )
- Standard deviation ( $\sigma$ ) unknown for population

$$c.i. = \bar{X} \pm Z \left( \frac{s}{\sqrt{N - 1}} \right)$$

$c.i.$  = confidence interval

$\bar{X}$  = the sample mean

$Z$  = the  $Z$  score as determined by the alpha level

$s/\sqrt{N - 1}$  = the sample deviation of the sampling distribution or the standard error of the mean

For small samples, use Student's  $t$  distribution: chapter 8 

# Example for means:

## Large sample, $\sigma$ unknown

- Sample of 500 residents
- Sample mean income is \$45,000
- Sample standard deviation is \$200
- Calculate a 95% confidence interval

$$c.i. = \bar{X} \pm Z \left( \frac{s}{\sqrt{N-1}} \right) = 45,000 \pm 1.96 \left( \frac{200}{\sqrt{500-1}} \right)$$
$$c.i. = 45,000 \pm 17.54$$

- Average income is between \$44,982.46 (45,000–17.54) and \$45,017.54 (45,000+17.54)



# Example from GSS

- We are 95% certain that the confidence interval from \$35,324.83 to \$39,889.96 contains the true average income for the U.S. adult population in 2004

```
. svy: mean conrinc, over(year)
(running mean on estimation sample)
```

```
Survey: Mean estimation
```

```
Number of strata =      307      Number of obs   =      4,522
Number of PSUs   =      597      Population size = 4,611.7099
Design df        =              Design df      =      290
```

```
2004: year = 2004
2010: year = 2010
2016: year = 2016
```

Over	Mean	Linearized Std. Err.	[95% Conf. Interval]	
<b>conrinc</b>				
2004	<b>37607.39</b>	<b>1159.734</b>	<b>35324.83</b>	<b>39889.96</b>
2010	<b>31537.11</b>	<b>1216.566</b>	<b>29142.69</b>	<b>33931.53</b>
2016	<b>34649.3</b>	<b>1267.614</b>	<b>32154.41</b>	<b>37144.19</b>

Source: 2004, 2010, 2016 General Social Surveys.

Note: Variance scaled to handle strata with a single sampling unit.

# Edited table

**Table 1. Mean, standard error, 95% confidence interval, and sample size of individual average income of the U.S. adult population, 2004, 2010, and 2016**

Year	Mean	Standard Error	95% Confidence Interval		Sample Size
			Lower Bound	Upper Bound	
<b>2004</b>	37,607.39	1,159.73	35,324.83	39,889.96	1,688
<b>2010</b>	31,537.11	1,216.57	29,142.69	33,931.53	1,202
<b>2016</b>	34,649.30	1,267.61	32,154.41	37,144.19	1,632

Source: 2004, 2010, 2016 General Social Surveys.





# Confidence intervals for sample proportions

$$c.i. = P_s \pm Z \sqrt{\frac{P_u(1 - P_u)}{N}}$$

$c.i.$  = confidence interval

$P_s$  = the sample proportion

$Z$  = the  $Z$  score as determined by the alpha level

$\sqrt{P_u(1 - P_u)/N}$  = the sample deviation of the sampling distribution or the standard error of the proportion



# Note about sample proportions

- The formula for the standard error includes the population value
  - We do not know and are trying to estimate ( $P_u$ )
- By convention we set  $P_u$  equal to 0.50
  - The numerator [ $P_u(1-P_u)$ ] is at its maximum value
  - $P_u(1-P_u) = (0.50)(1-0.50) = 0.25$
- The calculated confidence interval will be at its maximum width
  - This is considered the most statistically conservative technique



# Example for proportions

- Estimate the proportion of students who missed at least one day of classes last semester
  - In a random sample of 200 students, 60 students reported missing one day of class last semester
  - Thus, the sample proportion is 0.30 (60/200)
  - Calculate a 95% (alpha = 0.05) confidence interval

$$c.i. = P_s \pm Z \sqrt{\frac{P_u(1 - P_u)}{N}} = 0.3 \pm 1.96 \sqrt{\frac{0.5(1 - 0.5)}{200}}$$

$$c.i. = 0.3 \pm 0.08$$



# Example from GSS

- We are 95% certain that the confidence interval from 2.6% to 4.7% contains the true proportion of the U.S. adult population who thinks the number of immigrants to the country should increase a lot in 2004

```
. svy: prop letin1 if year==2004
(running proportion on estimation sample)
```

Survey: Proportion estimation

```
Number of strata =      109      Number of obs   =      1,983
Number of PSUs   =      218      Population size = 1,979.3435
Design df        =                      =      109
```

```
_prop_1: letin1 = increased a lot
_prop_2: letin1 = increased a little
_prop_3: letin1 = remain the same as it is
_prop_4: letin1 = reduced a little
_prop_5: letin1 = reduced a lot
```

	Proportion	Linearized Std. Err.	[95% Conf. Interval]	
<b>letin1</b>				
_prop_1	.0348265	.005221	.0258369	.0467936
_prop_2	.0653852	.0060495	.0543699	.078447
_prop_3	.3517117	.0128957	.3265967	.3776749
_prop_4	.2829629	.0118188	.2601357	.3069621
_prop_5	.2651137	.0127052	.2407073	.2910462

Source: 2004 General Social Survey.

# Edited table

**Table 2. Proportion, standard error, 95% confidence interval, and sample size of opinion of the U.S. adult population about how should the number of immigrants to the country be nowadays, 2004, 2010, and 2016**

Opinion About Number of Immigrants	Proportion	Standard Error	95% Confidence Interval		Sample Size
			Lower Bound	Upper Bound	
<b>2004</b>					1,983
Increase a lot	0.0348	0.0052	0.0258	0.0468	
Increase a little	0.0654	0.0060	0.0544	0.0784	
Remain the same	0.3517	0.0129	0.3266	0.3777	
Reduce a little	0.2830	0.0118	0.2601	0.3070	
Reduce a lot	0.2651	0.0127	0.2407	0.2910	
<b>2010</b>					1,393
Increase a lot	0.0426	0.0061	0.0320	0.0564	
Increase a little	0.0944	0.0096	0.0771	0.1152	
Remain the same	0.3589	0.0166	0.3268	0.3923	
Reduce a little	0.2452	0.0121	0.2220	0.2700	
Reduce a lot	0.2588	0.0146	0.2310	0.2887	
<b>2016</b>					1,845
Increase a lot	0.0586	0.0069	0.0462	0.0740	
Increase a little	0.1163	0.0091	0.0993	0.1358	
Remain the same	0.4028	0.0117	0.3797	0.4264	
Reduce a little	0.2305	0.0097	0.2118	0.2504	
Reduce a lot	0.1918	0.0101	0.1724	0.2128	

Source: 2004, 2010, 2016 General Social Surveys.

# Width of confidence interval

- The width of confidence intervals can be controlled by manipulating the confidence level
  - The confidence level increases
  - The alpha decreases
  - The Z score increases
  - The confidence interval is wider

**Example:  $\bar{X} = \$45,000$ ;  $s = \$200$ ;  $N = 500$**

<b>Confidence level</b>	<b>Alpha (<math>\alpha</math>)</b>	<b>Z score</b>	<b>Confidence interval</b>	<b>Interval width</b>
90%	0.10	$\pm 1.65$	$\$45,000 \pm \$14.77$	\$29.54
95%	0.05	$\pm 1.96$	$\$45,000 \pm \$17.54$	\$35.08
99%	0.01	$\pm 2.58$	$\$45,000 \pm \$23.09$	\$46.18
99.9%	0.001	$\pm 3.32$	$\$45,000 \pm \$29.71$	\$59.42



# Width of confidence interval

- The width of confidence intervals can be controlled by manipulating the sample size
  - The sample size increases
  - The confidence interval is narrower

**Example:  $\bar{X} = \$45,000$ ;  $s = \$200$ ;  $\alpha = 0.05$**

<b><i>N</i></b>	<b>Confidence interval</b>	<b>Interval width</b>
100	$c.i. = \$45,000 \pm 1.96(200/\sqrt{99}) = \$45,000 \pm \$39.40$	\$78.80
500	$c.i. = \$45,000 \pm 1.96(200/\sqrt{499}) = \$45,000 \pm \$17.55$	\$35.10
1000	$c.i. = \$45,000 \pm 1.96(200/\sqrt{999}) = \$45,000 \pm \$12.40$	\$24.80
10000	$c.i. = \$45,000 \pm 1.96(200/\sqrt{9999}) = \$45,000 \pm \$3.92$	\$7.84



# Summary: Confidence intervals

- Sample means, large samples ( $N > 100$ ), population standard deviation known

$$c.i. = \bar{X} \pm Z \left( \frac{\sigma}{\sqrt{N}} \right)$$

- Sample means, large samples ( $N > 100$ ), population standard deviation unknown

$$c.i. = \bar{X} \pm Z \left( \frac{s}{\sqrt{N - 1}} \right)$$

- Sample proportions, large samples ( $N > 100$ )

$$c.i. = P_s \pm Z \sqrt{\frac{P_u(1 - P_u)}{N}}$$







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