Lecture (chapter 10): Hypothesis testing III: The analysis of variance

Ernesto F. L. Amaral

March 19–21, 2018
Advanced Methods of Social Research (SOCI 420)

Source: Healey, Joseph F. 2015. "Statistics: A Tool for Social Research." Stamford: Cengage Learning. 10th edition. Chapter 10 (pp. 247–275).

TEXAS A&M UNIVERSITY

Chapter learning objectives

- Identify and cite examples of situations in which analysis of variance (ANOVA) is appropriate
- Explain the logic of hypothesis testing as applied to ANOVA
- Perform the ANOVA test, using the five-step model as a guide, and correctly interpret the results
- Define and explain the concepts of population variance, total sum of squares, sum of squares between, sum of squares within, mean square estimates
- Explain the difference between the statistical significance and the importance (magnitude) of relationships between variables

ANOVA application

- ANOVA can be used in situations where the researcher is interested in the differences in sample means across three or more categories
  - How do Protestants, Catholics, and Jews vary in terms of number of children?
  - How do Republicans, Democrats, and Independents vary in terms of income?
  - How do older, middle-aged, and younger people vary in terms of frequency of church attendance?

Extension of *t*-test

- We can think of ANOVA as an extension of *t*-test for more than two groups
  - Are the differences between the samples large enough to reject the null hypothesis and justify the conclusion that the populations represented by the samples are different?

- Null hypothesis, H_0
  - H_0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_k$
  - All population means are similar to each other

- Alternative hypothesis, H_1
  - At least one of the populations means is different

Logic of ANOVA

- Could there be a relationship between age and support for capital punishment?
  - No difference between groups

Support for Capital Punishment by Age Group (fictitious data)

	18–29	30–45	46–64	65+
Mean	10.3	11.0	10.1	9.9
Standard deviation	2.4	1.9	2.2	1.7

  - Difference between groups

Support for Capital Punishment by Age Group (fictitious data)

	18–29	30–45	46–64	65+
Mean	10.0	13.0	16.0	22.0
Standard deviation	2.4	1.9	2.2	1.7

Source: Healey 2015, p.249.

Between and within differences

- If the H_0 is true, the sample means should be about the same value
  - If the H_0 is true, there will be little difference between sample means
- If the H_0 is false
  - There should be substantial differences **between** sample means (between categories)
  - There should be relatively little difference **within** categories
    - The sample standard deviations should be small within groups

Likelihood of rejecting H_0

- The greater the difference **between** categories (as measured by the means)
  - Relative to the differences **within** categories (as measured by the standard deviations)
  - The more likely the H_0 can be rejected

- When we reject H_0
  - We are saying there are differences between the **populations** represented by the sample

Computation of ANOVA

1. Find total sum of squares (SST)

$$SST = \sum X_i^2 - N\bar{X}^2$$

2. Find sum of squares between (SSB)

$$SSB = \sum N_k(\bar{X}_k - \bar{X})^2$$

   - SSB = sum of squares between categories
   - N_k= number of cases in a category
   - $\bar{X}_k$= mean of a category

3. Find sum of squares within (SSW)

SSW = SST – SSB

4. Degrees of freedom

$$\text{dfw} = N - k$$

- dfw = degrees of freedom within
- N = total number of cases
- k = number of categories

$$\text{dfb} = k - 1$$

- dfb = degrees of freedom between
- *k* = number of categories

Final estimations

5. Find mean square estimates

$$Mean\ square\ within = \frac{SSW}{dfw}$$

$$Mean\ square\ between = \frac{SSB}{dfb}$$

6. Find the *F* ratio

$$F(obtained) = \frac{Mean\ square\ between}{Mean\ square\ within}$$

Example

- Support for capital punishment
- Sample of 16 people who are equally divided across four age groups

Support for Capital Punishment by Age Group (fictitious data)

18–29		30–45		46–64		65+	
X_i	X_i^2	X_i	X_i^2	X_i	X_i^2	X_i	X_i^2
7	49	10	100	12	144	17	289
8	64	12	144	15	225	20	400
10	100	13	169	17	289	24	576
15	225	17	289	20	400	27	729
40	438	52	702	64	1058	88	1994
$\bar{X}_k = 10.0$		$\bar{X}_k = 13.0$		$\bar{X}_k = 16.0$		$\bar{X}_k = 22.0$	

$\bar{X} = 15.25$

Source: Healey 2015, p.252.

Step 1: Assumptions,requirements

- Independent random samples
- Interval-ratio level of measurement
- Normally distributed populations
- Equal population variances

Step 2: Null hypothesis

- Null hypothesis, H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$
  - The null hypothesis asserts there is no difference between the populations

- Alternative hypothesis, H_1
  - At least one of the populations means is different

Step 3: Distribution, critical region

- Sampling distribution
  - *F* distribution
- Significance level
  - Alpha (α) = 0.05
- Degrees of freedom
  - dfw = $N - k = 16 - 4 = 12$
  - dfb = $k - 1 = 4 - 1 = 3$
- Critical *F*
  - F(critical) = 3.49

Step 4: Test statistic

1. Total sum of squares (SST)

$$SST = \sum X_i^2 - N\bar{X}^2$$

$$SST = (438 + 702 + 1058 + 1994) - (16)(15.25)^2$$
$$SST = 471.04$$

2. Sum of squares between (SSB)

$$SSB = \sum N_k(\bar{X}_k - \bar{X})^2$$

$$\mathrm{SSB} = 4(10 - 15.25)^2 + 4(13 - 15.25)^2$$
$$+ 4(16 - 15.25)^2 + 4(22 - 15.25)^2 = 314.96$$

3. Sum of squares within (SSW)

SSW = SST – SSB = 471.04 – 314.96 = 156.08

4. Degrees of freedom

dfw = *N* – *k* = 16 – 4 = 12

dfb = *k* – 1 = 4 – 1 = 3

5. Mean square estimates

$$Mean\ square\ within = \frac{SSW}{dfw} = \frac{156.08}{12} = 13.00$$

$$Mean\ square\ between = \frac{SSB}{dfb} = \frac{314.96}{3} = 104.99$$

6. *F* ratio

$$F(obtained) = \frac{Mean\ square\ between}{Mean\ square\ within} = \frac{104.99}{13.00} = 8.08$$

Step 5: Decision, interpret

- *F*(obtained) = 8.08
- This is beyond *F*(critical) = 3.49
- The obtained test statistic falls in the critical region, so we ***reject*** the H_0
- Support for capital punishment does differ across age groups

Example from 2016 GSS

- We know the average income by race/ethnicity

```
. table raceeth [aweight=wtssall], c(mean conrinc sd conrinc n conrinc)
```

Race/Ethnicity	mean(conrinc)	sd(conrinc)	N(conrinc)
White	38845.61946	39157.17	1,072
Black	23243.0413	19671.53	273
Hispanic	23128.91777	21406.31	215
Other	50156.34855	59219.9	72

- Does at least one category of the race/ethnicity variable have average income different than the others?
  - This is not a perfect example for ANOVA, because the race/ethnicity variable does not have equal numbers of cases across its categories

Example from GSS: Result

- The probability of not rejecting H_0 is small (p<0.01)
  - At least one category of the race/ethnicity variable has average income different than the others with a 99% confidence level
  - However, ANOVA does not inform which category has an average income significantly different than the others in 2016

```
. oneway conrinc raceeth [aweight=wtssall]

                        Analysis of Variance
    Source              SS         df      MS            F     Prob > F
------------------------------------------------------------------------
Between groups      1.0142e+11      3   3.3806e+10     26.23     0.0000
 Within groups      2.0980e+12   1628   1.2887e+09
------------------------------------------------------------------------
    Total           2.1994e+12   1631   1.3485e+09
```

Source: 2016 General Social Survey.

Edited table

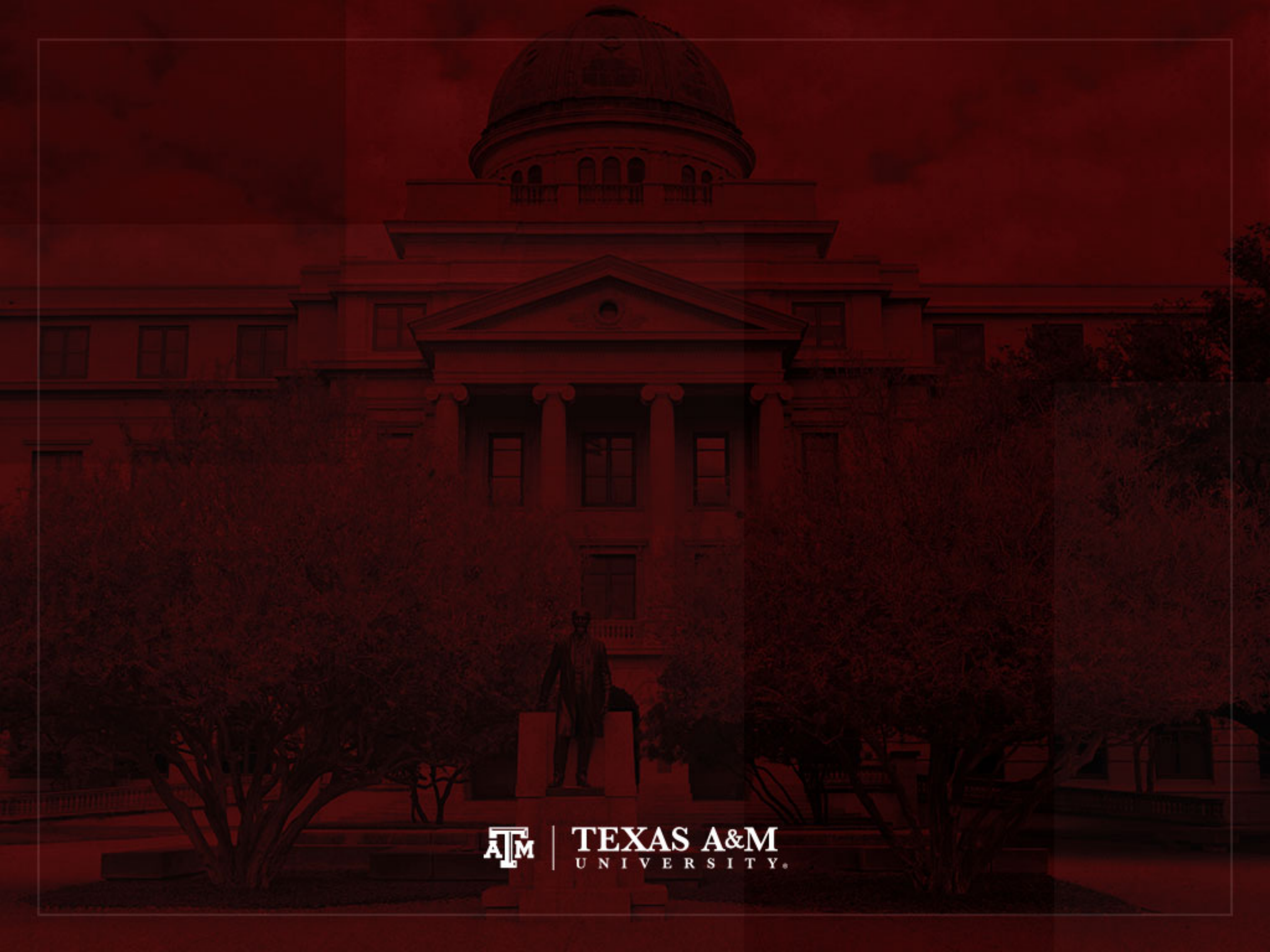
Table 1. One-way analysis of variance for individual average income of the U.S. adult population by race/ethnicity, 2004, 2010, and 2016

Source	Sum of Squares	Degrees of Freedom	Mean of Squares	F-test	Prob > F
2004					
Between groups	5.92e+10	3	1.97e+10	16.36	0.0000
Within groups	2.03e+12	1,682	1.21e+09		
Total	2.09e+12	1,685	1.24e+09		
2010					
Between groups	6.02e+10	3	2.01e+10	24.50	0.0000
Within groups	9.79e+11	1,195	819,590,864		
Total	1.04e+12	1,198	867,818,893		
2016					
Between groups	1.01e+11	3	3.38e+10	26.23	0.0000
Within groups	2.10e+12	1,628	1.29e+09		
Total	2.20e+12	1,631	1.35e+09		

Source: 2004, 2010, 2016 General Social Surveys.

Limitations of ANOVA

- Requires interval-ratio level measurement of the dependent variable
- Requires roughly equal numbers of cases in the categories of the independent variable
- Statistically significant differences are not necessarily important (small magnitude)
- The alternative (research) hypothesis is not specific
  - It only asserts that at least one of the population means differs from the others


ATM
TEXAS A&M
UNIVERSITY