Lecture (chapter 15): Partial correlation, multiple regression, and correlation

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Source: Healey, Joseph F. 2015. "Statistics: A Tool for Social Research." Stamford: Cengage Learning. 10th edition. Chapter 15 (pp. 405–441).



## Chapter learning objectives

- Compute and interpret partial correlation coefficients
- Find and interpret the least-squares multiple regression equation with partial slopes
- Find and interpret standardized partial slopes or beta-weights (b\*)
- Calculate and interpret the coefficient of multiple determination  $(R^2)$
- Explain the limitations of partial and regression analysis



## Multiple regression

- Discuss ordinary least squares (OLS) multiple regressions
  - OLS: linear regression
  - Multiple: at least two independent variables
- Disentangle and examine the separate effects of the independent variables
- Use all of the independent variables to predict Y
- Assess the combined effects of the independent variables on Y



#### Partial correlation

• Partial correlation measures the correlation between *X* and *Y*, controlling for *Z* 

- Comparing the bivariate (zero-order) correlation to the partial (first-order) correlation
  - Allows us to determine if the relationship between X and Y is direct, spurious, or intervening
  - Interaction cannot be determined with partial correlations



#### Formula for partial correlation

• Formula for partial correlation coefficient for *X* and *Y*, controlling for *Z* 

$$r_{yx,z} = \frac{r_{yx} - (r_{yz})(r_{xz})}{\sqrt{1 - r_{yz}^2}\sqrt{1 - r_{xz}^2}}$$

• We must first calculate the zero-order coefficients between all possible pairs of variables (Y and X, Y and Z, X and Z) before solving this formula



#### Example

- Husbands' hours of housework per week (Y)
- Number of children (X)
- Husbands' years of education (Z)

Scores on Three Variables for 12 Dual–Wage-Earner Families

Family	Husband's Housework (Y)	Number of Children (X)	Husband's Years of Education ( <i>Z</i> )
A	1	1	12
В	2	1	14
С	3	1	16
D	5	p de vorte a	16
E	3	2	18
F	1	2	16
G	5	3	12
Н	0	3	12
1	6	4	10
J	3	4	12
К	7	5	10
L	4	5	16

#### **Correlation matrix**

- The bivariate (zero-order) correlation between husbands' housework and number of children is +0.50
  - This indicates a positive relationship

#### **Zero-Order Correlations**

↓	Husband's Housework (Y)	Number of Children (X)	Husband's Years of Education ( <i>Z</i> )
Husband's Housework ( <i>Y</i> )	1.00	0.50	-0.30
Number of Children ( <i>X</i> )		1.00	-0.47
Husband's Years of Education ( <i>Z</i> )		olizati sili Manufati other sa	1.00



#### **First-order correlation**

 Calculate the partial (first-order) correlation between husbands' housework (Y) and number of children (X), controlling for husbands' years of education (Z)

$$r_{yx,z} = \frac{r_{yx} - (r_{yz})(r_{xz})}{\sqrt{1 - r_{yz}^2}\sqrt{1 - r_{xz}^2}}$$

$$r_{yx,z} = \frac{(0.50) - (-0.30)(-0.47)}{\sqrt{1 - (-0.30)^2}\sqrt{1 - (-0.47)^2}}$$

$$r_{yx,z} = 0.43$$



#### Interpretation

• Comparing the bivariate correlation (+0.50) to the partial correlation (+0.43) finds little change

 The relationship between number of children and husbands' housework has not changed, controlling for husbands' education

Therefore, we have evidence of a direct relationship



## **Bivariate & multiple regressions**

Bivariate regression equation

$$Y = a + bX = \beta_0 + \beta_1 X$$

 $-a = \beta_0 = Y$  intercept

 $-b = \beta_1 = \text{slope}$ 

Multivariate regression equation

 $Y = a + b_1 X_1 + b_2 X_2 = \beta_0 + \beta_1 X_1 + \beta_2 X_2$ 

- $-b_1 = \beta_1 =$  partial slope of the linear relationship between the first independent variable and *Y*
- $-b_2 = \beta_1 =$  partial slope of the linear relationship between the second independent variable and Y



Multiple regression  $Y = a + b_1 X_1 + b_2 X_2 = \beta_0 + \beta_1 X_1 + \beta_2 X_2$ 

- $a = \beta_0$  = the Y intercept, where the regression line crosses the Y axis
- $b_1 = \beta_1 = partial slope for X_1 on Y$ 
  - $\beta_1$  indicates the change in Y for one unit change in  $X_1$ , controlling for  $X_2$
- $b_2 = \beta_2 = partial slope for X_2 on Y$ 
  - $\beta_2$  indicates the change in Y for one unit change in  $X_2$ , controlling for  $X_1$



#### Partial slopes

• The partial slopes indicate the effect of each independent variable on Y

• While controlling for the effect of the other independent variables

- This control is called ceteris paribus
  - Other things equal
  - Other things held constant
  - All other things being equal















### Interpretation of partial slopes

 The partial slopes show the effects of the X's in their original units

• These values can be used to predict scores on Y

• Partial slopes must be computed before computing the Y intercept ( $\beta_0$ )



# Formulas of partial slopes $b_1 = \beta_1 = \left(\frac{s_y}{s_1}\right) \left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)$ $b_2 = \beta_2 = \left(\frac{s_y}{s_2}\right) \left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)$

- $b_1 = \beta_1$  = partial slope of  $X_1$  on Y  $b_2 = \beta_2$  = partial slope of  $X_2$  on Y  $s_y$  = standard deviation of Y  $s_1$  = standard deviation of the first independent variable ( $X_1$ )  $s_2$  = standard deviation of the second independent variable ( $X_2$ )  $r_{y1}$  = bivariate correlation between Y and  $X_1$
- $r_{y2}$  = bivariate correlation between Y and  $X_2$
- $r_{12}$  = bivariate correlation between  $X_1$  and  $X_2$



#### Formula of constant

• Once  $b_1(\beta_1)$  and  $b_2(\beta_2)$  have been calculated, use those values to calculate the Y intercept

$$a = \overline{Y} - b_1 \overline{X}_1 - b_2 \overline{X}_2$$

$$\beta_0 = \overline{Y} - \beta_1 \overline{X}_1 - \beta_2 \overline{X}_2$$



#### Example

• Using information below, calculate the slopes

Husband's Housework	Number of Children	Husband's Education
$\overline{Y} = 3.3$	$\overline{X}_1 = 2.7$	$\overline{X}_{2} = 13.7$
<i>s<sub>y</sub></i> = 2.1	$s_1 = 1.5$	$s_2 = 2.6$
	Zero-Order Correlatio	ns
	$r_{y1} = 0.50 r_{y2} = -0.30 r_{12} = -0.47$	



#### Result and interpretation of $b_1$

$$b_1 = \beta_1 = \left(\frac{s_y}{s_1}\right) \left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)$$

$$b_1 = \beta_1 = \left(\frac{2.1}{1.5}\right) \left(\frac{0.50 - (-0.30)(-0.47)}{1 - (-0.47)^2}\right) = 0.65$$

 As the number of children in a dual-career household increases by one, the husband's hours of housework per week increases on average by 0.65 hours (about 39 minutes), controlling for husband's education



#### Result and interpretation of $b_2$

$$b_2 = \beta_2 = \left(\frac{s_y}{s_2}\right) \left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)$$

$$b_2 = \beta_2 = \left(\frac{2.1}{2.6}\right) \left(\frac{-0.30 - (0.50)(-0.47)}{1 - (-0.47)^2}\right) = -0.07$$

 As the husband's years of education increases by one year, the number of hours of housework per week decreases on average by 0.07 (about 4 minutes), controlling for the number of children



# Result and interpretation of a $a = \overline{Y} - b_1 \overline{X}_1 - b_2 \overline{X}_2$ $\beta_0 = \overline{Y} - \beta_1 \overline{X}_1 - \beta_2 \overline{X}_2$ $a = \beta_0 = 3.3 - (0.65)(2.7) - (-0.07)13.7$

$$a = \beta_0 = 2.5$$

 With zero children in the family and a husband with zero years of education, that husband is predicted to complete 2.5 hours of housework per week on average



#### Final regression equation

• In this example, this is the final regression equation

$$Y = a + b_1 X_1 + b_2 X_2$$

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

$$Y = 2.5 + (0.65)X_1 + (-0.07)X_2$$

 $Y = 2.5 + 0.65X_1 - 0.07X_2$ 



#### Prediction

 Use the regression equation to predict a husband's hours of housework per week when he has 11 years of schooling and the family has 4 children

$$Y' = 2.5 + 0.65X_1 - 0.07X_2$$
$$Y' = 2.5 + (0.65)(4) + (-0.07)(11)$$
$$Y' = 4.3$$

• Under these conditions, we would predict 4.3 hours of housework per week

## Standardized coefficients (b\*)

- Partial slopes  $(b_1 = \beta_1; b_2 = \beta_2)$  are in the original units of the independent variables
  - This makes assessing relative effects of independent variables difficult when they have different units
  - It is easier to compare if we standardize to a common unit by converting to Z scores
- Compute beta-weights (b\*) to compare relative effects of the independent variables
  - Amount of change in the standardized scores of Y for a one-unit change in the standardized scores of each independent variable
    - While controlling for the effects of all other independent variables
  - They show the amount of change in standard deviations in Y for a change of one standard deviation in each X



#### Formulas

• Formulas for standardized coefficients

$$b_1^* = b_1\left(\frac{s_1}{s_y}\right) = \beta_1^* = \beta_1\left(\frac{s_1}{s_y}\right)$$

$$b_2^* = b_2 \left(\frac{s_2}{s_y}\right) = \beta_2^* = \beta_2 \left(\frac{s_2}{s_y}\right)$$



#### Example

 Which independent variable, number of children (X<sub>1</sub>) or husband's education (X<sub>2</sub>), has the stronger effect on husband's housework in dual-career families?

$$b_1^* = b_1 \left(\frac{s_1}{s_y}\right) = (0.65) \left(\frac{1.5}{2.1}\right) = 0.46$$
$$b_2^* = b_2 \left(\frac{s_2}{s_y}\right) = (-0.07) \left(\frac{2.6}{2.1}\right) = -0.09$$

- The standardized coefficient for number of children (0.46) is greater in absolute value than the standardized coefficient for husband's education (–0.09)
- Therefore, number of children has a stronger effect on husband's housework

#### Standardized coefficients

• Standardized regression equation

$$Z_y = a_z + b_1^* Z_1 + b_2^* Z_2$$

- where Z indicates that all scores have been standardized to the normal curve
- The Y intercept will always equal zero once the equation is standardized

$$Z_y = b_1^* Z_1 + b_2^* Z_2$$

• For the previous example

$$Z_y = (0.46)Z_1 + (-0.09)Z_2$$



## Multiple correlation

- The coefficient of multiple determination (*R*<sup>2</sup>) measures how much of *Y* is explained by all of the *X*'s combined
- *R*<sup>2</sup> measures the percentage of the variation in Y that is explained by all of the independent variables combined
- The coefficient of multiple determination is an indicator of the strength of the entire regression equation

$$R^{2} = r_{y1}^{2} + r_{y2.1}^{2} \left(1 - r_{y1}^{2}\right)$$

- $R^2$  = coefficient of multiple determination
- $-r_{y1}^2$  = zero-order correlation between Y and  $X_1$
- $r_{y2.1}^2$  = partial correlation of Y and  $X_2$ , while controlling for  $X_1$



### Partial correlation of Y and $X_2$

• Before estimating  $R^2$ , we need to estimate the partial  $r_y$ correlation of Y and  $X_2(r_{y2.1})$ 

$$_{y2.1} = \frac{r_{y2} - (r_{y1})(r_{12})}{\sqrt{1 - r_{y1}^2}\sqrt{1 - r_{12}^2}}$$

• We need three correlations

$$r_{y2.1} = \frac{(-0.30) - (0.50)(-0.47)}{\sqrt{1 - (0.50)^2}\sqrt{1 - (-0.47)^2}}$$

- Between  $X_1$  and Y: 0.50
- Between  $X_2$  and Y: -0.30

Between 
$$X_1$$
 and  $X_2$ : -0.47

$$r_{y2.1} = -0.08$$



#### Result and interpretation

 For this example, R<sup>2</sup> will tell us how much of husband's housework is explained by the combined effects of the number of children (X<sub>1</sub>) and husband's education (X<sub>2</sub>)

$$R^{2} = r_{y1}^{2} + r_{y2.1}^{2} \left(1 - r_{y1}^{2}\right)$$
$$R^{2} = (0.50)^{2} + (-0.08)^{2} (1 - 0.50^{2})$$
$$R^{2} = 0.255$$

 Number of children and husband's education explain 25.5% of the variation in husband's housework

## Normality assumption

 OLS regressions require normal distribution for its interval-ratio-level variables

• We can analyze histograms to determine if variables have a normal distribution



#### Histogram of income



#### Income = F(age, education)

. \*\*\*Repondent's income by age and years of schooling

. svy: reg conrinc age educ if year==2016

(running regress on estimation sample)

Survey: Linear regression

Number	of	strata	=	65	Num	ıber	of ol	os	=	1,626
Number	of	PSUs	=	130	Рор	oulat	ion s	size	=	1,688.1407
					Des	sign	df		=	65
					F (	2,		64)	=	74.94
					Pro	ob >	F		=	0.0000
					R-s	quar	red		=	0.1434

conrinc	Coef.	Linearized Std. Err.	t	P> t	[95% Conf.	Interval]
age	435.2034	46.70745	9.32	0.000	341.9222	528.4846
educ	4209.814	449.8191	9.36	0.000	3311.464	5108.165
_cons	-43706	6198.866	-7.05	0.000	-56085.99	-31326.01



#### Standardized coefficients

- . **\*\*\*Standardized regression coefficients**
- . \*\*\*(i.e., standardized partial slopes, beta-weights)
- . \*\*\*It does not allow the use of GSS complex survey design
- . reg conrinc age educ if year==2016, beta

Source	SS	df	MS	Number of obs	=	1,626
				F(2, 1623)	=	109.52
Model	2.5113e+11	2	1.2557e+11	Prob > F	=	0.0000
Residual	1.8608e+12	1,623	1.1465e+09	R-squared	=	0.1189
			· · · · · · · · · · · · · · · · · · ·	Adj R-squared	=	0.1178
Total	2.1120e+12	1,625	1.2997e+09	Root MSE	=	33861

Beta	P> t	t	Std. Err.	Coef.	conrinc
.1376138	0.000	5.90	59.38046	350.4332	age
.3107139	0.000	13.32	292.0824	3891.937	educ
	0.000	-7.36	4885.588	-35944.9	_cons



#### **Power transformation**

• Lawrence Hamilton ("Regression with Graphics", 1992, p.18–19)

 $\begin{array}{rcl} Y^3 & \longrightarrow & q = 3 \\ Y^2 & \longrightarrow & q = 2 \\ Y^1 & \longrightarrow & q = 1 \\ Y^{0.5} & \longrightarrow & q = 0.5 \\ \log(Y) & \longrightarrow & q = 0 \\ -(Y^{-0.5}) & \longrightarrow & q = -0.5 \\ -(Y^{-1}) & \longrightarrow & q = -1 \end{array}$ 

- q>1: reduce concentration on the right (reduce negative skew)
- q=1: original data
- q<1: reduce concentration on the left (reduce positive skew)
- log(x+1) may be applied when x=0. If distribution of log(x+1) is normal, it is called lognormal distribution



### Histogram of log of income



Source: 2016 General Social Survey.

## Interpretation of coefficients

(with continuous independent variables)

- With the logarithm of the dependent variable
  - Coefficients are interpreted as percentage changes
- If coefficient of  $X_1$  equals 0.12
  - $-\exp(\beta_1)$  times
    - X<sub>1</sub> increases by one unit, Y increases on average <u>1.13 times</u>, controlling for other independent variables
  - $100*[exp(\beta_1)-1]$  percent
    - X<sub>1</sub> increases by one unit, Y increases on average by <u>13%</u>, controlling for other independent variables
- If coefficient has a small magnitude:  $-0.3 < \beta < 0.3$ 
  - $-100^*\beta$  percent
    - X<sub>1</sub> increases by one unit, Y increases on average <u>approximately by 12%</u>, controlling for other independents

## log income = F(age, education)

. \*\*\*Log of respondent's income by age and years of schooling
. svy: reg lnconrinc age educ if year==2016
(running regress on estimation sample)

Survey: Linear regression

Number of	f strata	=	65	Number of obs	=	1,626
Number of	f PSUs	=	130	Population size	=	1,688.1407
				Design df	=	65
				F( <b>2,64</b> )	=	99.50
				Prob > F	=	0.0000
				R-squared	=	0.1327

lnconrinc	Coef.	Linearized Std. Err.	t	P> t	[95% Conf.	Interval]
age	.0157926	.0021115	7.48	0.000	.0115756	.0200096
educ	.1229175	.0109476	11.23	0.000	.1010536	.1447814
_cons	7.506136	.1702918	44.08	0.000	7.16604	7.846233



#### Interpretation of example

(with continuous independent variables)

- Coefficient for <u>age</u> equals 0.016
  - $-\exp(\beta_1)$  times
    - When age increases by one unit, income increases on average by <u>1.0161 times</u>, controlling for education
  - $100*[exp(\beta_1)-1]$  percent
    - When age increases by one unit, income increases on average by <u>1.61%</u>, controlling for education
  - $-100^*\beta_1$  percent
    - When age increases by one unit, income increases on average approximately by 1.6%, controlling for education

#### Standardized coefficients

- . \*\*\*Standardized regression coefficients
- . \*\*\*(i.e., standardized partial slopes, beta-weights)
- . \*\*\*It does not allow the use of GSS complex survey design
- . reg lnconrinc age educ if year==2016, beta

Source	SS	df	MS	Number of obs	=	1,626
				F(2, 1623)	=	101.26
Model	240.448673	2	120.224336	Prob > F	=	0.0000
Residual	1926.88361	1,623	1.18723574	R-squared	=	0.1109
				Adj R-squared	=	0.1098
Total	2167.33229	1,625	1.33374294	Root MSE	=	1.0896

Beta	P> t	t	Std. Err.	Coef.	lnconrinc
. 1454577	0.000	6.21	.0019108	.0118659	age
. 2938647	0.000	12.55	.009399	.1179159	educ
	0.000	49.31	.1572141	7.752344	_cons



### **Dummy variables**

- Many variables that are important in social life are nominal-level variables
  - They cannot be included in a regression equation or correlational analysis (e.g., sex, race/ethnicity)
- We can create dummy variables
  - Two categories, one coded as 0 and the other as 1

0

0

	Sex	Male	Female	Race/	White	Black	Hispanic
	1	1	0	ethnicity			
ŀ	2	0	1	1	1	0	0
L				2	0	1	0
				3	0	0	1

Other

0

0

0

1

 $\mathbf{0}$ 

#### Age in interval-ratio level

Age does not have a normal distribution



Generate age group variable (categorical)
 18–24; 25–34; 35–49; 50–64; 65+



Source: 2016 General Social Survey.

#### Age in ordinal level

#### Age has five categories

. egen agegr = cut(age), at(18,25,35,50,65,90)
(22 missing values generated)

. table agegr, contents(min age max age count age)

agegr	min(age)	max(age)	N(age)
18	18	24	671
25	25	34	1,452
35	35	49	2,116
50	50	64	2,022
65	65	89	1,440

• Generate dummy variables for age...



#### Dummies for age

Generate dummy variables for age group

Age group	Age 18–24	Age 25–35	Age 35–49	Age 50–64	Age 65–89
18–24	1	0	0	0	0
25–34	0	1	0	0	0
35–49	0	0	1	0	0
50–64	0	0	0	1	0
65–89	0	0	0	0	1

. tab agegr

agegr Freq. Use the category with Percent Cum. 8.71 8.71 18 671 25 1,452 18.85 27.57 largest sample size 35 2,116 27.48 55.04 50 2,022 26.26 81.30 65 1,440 18.70 100.00 as the reference (35–49) 7,701 Total 100.00

#### Education in interval-ratio level

• Education does not have a normal distribution



- Use the categorical education variable (degree)
  - Less than high school; high school; junior college; bachelor; graduate



Source: 2016 General Social Survey.

#### Education in ordinal level

- Education has five categories
  - . tab degree

rs highest degree	Freq.	Percent	Cum.
lt high school	997	12.92	12.92
high school	3,897	50.52	63.44
junior college	585	7.58	71.03
bachelor	1,418	18.38	89.41
graduate	817	10.59	100.00
Total	7,714	100.00	

Generate dummy variables for education...



#### **Dummies for education**

Generate dummy variables for education group

Education group	<hs< th=""><th>HS</th><th>JC</th><th>BA</th><th>GR</th></hs<>	HS	JC	BA	GR
Less than high school	1	0	0	0	0
High school	0	1	0	0	0
Junior college	0	0	1	0	0
Bachelor	0	0	0	1	0
Graduate	0	0	0	0	1

. tab degree

 Use the category with largest sample size as the reference (HS)

rs highest degree	Freq.	Percent	Cum.
lt high school	997	12.92	12.92
high school	3,897	50.52	63.44
junior college	585	7.58	71.03
bachelor	1,418	18.38	89.41
graduate	817	10.59	100.00
Total	7,714	100.00	

## log income = F(age, education)

. svy: reg lnconrinc agegr1 agegr2 agegr4 agegr5 educgr1 educgr3 educgr4 educgr5 if year==2016
(running regress on estimation sample)

Survey: Linear regression

= 1,62 $= 1,688.140$ $= 6$ $= 53.4$ $= 0.000$ $= 0.198$	= 2e = : 3) = = =	of obs tion size df , 58) F red	Number Popula Design F( <b>8</b> Prob > R-squa		65 130	ata = 5 =	Number of stra Number of PSUs
nf. Interval	₅ Conf.	[95%	P> t	t	Linearized Std. Err.	Coef.	lnconrinc
5 –.923120 9 –.187549	L0805 L5379	-1.410 4815	0.000	-9.56 -4.55	.1220959 .0736023	-1.166963 3345438	agegr1 agegr2

agegr2	3345438	.0/36023	-4.55	0.000	48153/9	18/5498
agegr4	0050007	.0638917	-0.08	0.938	1326013	.1225999
agegr5	4155278	.096474	-4.31	0.000	6081997	2228559
educgr1	4276264	.1163403	-3.68	0.000	6599739	1952789
educgr3	.2367316	.0940649	2.52	0.014	.0488711	.4245921
educgr4	.4559903	.0843136	5.41	0.000	.2876045	.6243761
educgr5	.8516728	.0920326	9.25	0.000	.667871	1.035475
_cons	9.949482	.0471336	211.09	0.000	9.855349	10.04361

### Interpretation of example

(with dummies as independent variables)

- High school is reference category for <u>education</u>
- Coefficient for junior college equals 0.237
  - $\exp(\beta_1)$  times
    - People with junior college degree have on average earnings <u>1.27</u>
       <u>times higher</u> than earnings of high school graduates, controlling for the other independent variables
  - $100*[exp(\beta_1)-1]$  percent
    - People with junior college degree have on average earnings <u>27%</u> <u>higher</u> than earnings of high school graduates, controlling for the other independent variables
  - $100^*\beta_1$  percent
    - People with junior college degree have on average earnings <u>approximately 23.7% higher</u> than earnings of high school graduates, controlling for the other independent variables

#### Interpretation of example

(with dummies as independent variables)

- 35–49 age group is reference category for <u>age</u>
- Coefficient for 18–24 age group equals –1.167
  - $\exp(\beta_1)$  times
    - People between 18 and 24 years of age have on average earnings
       <u>0.31 times</u> the earnings of people between 35 and 49 years of age, controlling for the other independent variables
  - $100*[exp(\beta_1)-1]$  percent
    - People between 18 and 24 years of age have on average earnings
       <u>69% lower</u> than earnings of people between 35 and 49 years of age, controlling for the other independent variables
  - $100^*\beta_1$  percent: result is not good because the magnitude is high
    - People between 18 and 24 years of age have on average earnings <u>approximately 117% lower</u> than high school graduates, controlling for the other independent variables

#### Standardized coefficients

. reg lnconrinc agegr1 agegr2 agegr4 agegr5 educgr1 educgr3 educgr4 educgr5 if year==2016, beta

Source	SS	df	MS	Number of obs	=	1,626
			·····	F(8, 1617)	=	45.13
Model	395.582999	8	49.4478749	Prob > F	=	0.0000
Residual	1771.74929	1,617	1.09570148	R-squared	=	0.1825
			· · · · · · · · · · · · · · · · · · ·	Adj R-squared	=	0.1785
Total	2167.33229	1,625	1.33374294	Root MSE	=	1.0468

lnconrinc	Coef.	Std. Err.	t	P> t	Beta
agegr1 agegr2 agegr4 agegr5 educgr1 educgr3 educgr4 educgr5	-1.133145 3035091 .0151364 4761732 4255014 .2160742 .5121465 .7994411	.1077317 .072592 .0656439 .103716 .1019697 .0977414 .0675632 .0810907	-10.52 -4.18 0.23 -4.59 -4.17 2.21 7.58 9.86	0.000 0.000 0.818 0.000 0.000 0.027 0.000 0.000 0.000	2566052 1091483 .0061022 1110716 0970916 .0516399 .1828828 .2359353
_cons	3.32714J	.0341100	102.42	0.000	·



#### **Edited table**

Table 1. Coefficients and standard errors estimated with ordinary least squares models for the logarithm of respondent's income as the dependent variable, U.S. adult population, 2004, 2010, and 2016

Indonandant	2004		20	)10	2016	
variables	Coefficients	Standardized coefficients	Coefficients	Standardized coefficients	Coefficients	Standardized coefficients
Constant	10.030***		9.919***		9.949***	
	(0.063)		(0.090)		(0.047)	
Age groups						
18–24	-1.114***	-0.269	-1.438***	-0.327	-1.167***	-0.257
	(0.104)		(0.188)		(0.122)	
25–34	-0.306***	-0.118	-0.406***	-0.140	-0.335***	-0.109
	(0.074)		(0.102)		(0.074)	
35–49	ref.	ref.	ref.	ref.	l ref.	ref.
50–64	0.132*	0.041	0.043	0.015	-0.005	0.006
	(0.068)		(0.092)		(0.064)	
65+	-0.596***	-0.120	-0.720***	-0.168	-0.416***	-0.111
	(0.165)		(0.175)		(0.097)	
Education groups						
Less than high school	-0.410***	-0.101	-0.477***	-0.139	-0.428***	-0.097
	(0.117)		(0.125)		(0.116)	
High school	ref.	ref.	ref.	ref.	ref.	ref.
Junior college	0.276***	0.071	0.142	0.018	0.237**	0.052
eenner eenege	(0.097)		(0.122)		(0.094)	0.002
Bachelor	0.620***	0.219	0.579***	0.197	0.456***	0.183
	(0.062)		(0.099)		(0.084)	
Graduate	0.785***	0.233	0.983***	0.251	0.852***	0.236
	(0.097)		(0.088)		(0.092)	
R <sup>2</sup>	0.242	0.222	0.288	0.272	0.198	0.183
Number of observations	1,685	1,685	1,201	1,201	1,626	1,626

Note: Coefficients and standard errors were generated with the complex survey design of the General Social Survey. The standardized coefficients were generated without the complex survey design. Standard errors are reported in parentheses. \*Significant at p<0.10; \*\*Significant at p<0.05; \*\*\*Significant at p<0.01. Source: 2004, 2010, 2016 General Social Surveys.

#### Limitations

- Multiple regression and correlation are among the most powerful techniques available to researchers
  - But powerful techniques have high demands
- These techniques require
  - Every variable is measured at the interval-ratio level
  - Each independent variable has a linear relationship with the dependent variable
  - Independent variables do not interact with each other
  - Independent variables are uncorrelated with each other
  - When these requirements are violated (as they often are), these techniques will produce biased and/or inefficient estimates
  - There are more advanced techniques available to researchers that can correct for violations of these requirements

