

# Lecture 6: Introduction to inferential statistics

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# Outline

- Explain the purpose of inferential statistics in terms of generalizing from a sample to a population
- Define and explain the basic techniques of random sampling
- Explain and define these key terms: population, sample, parameter, statistic, representative, EPSEM sampling techniques
- Differentiate between the sampling distribution, the sample, and the population
- Explain two theorems



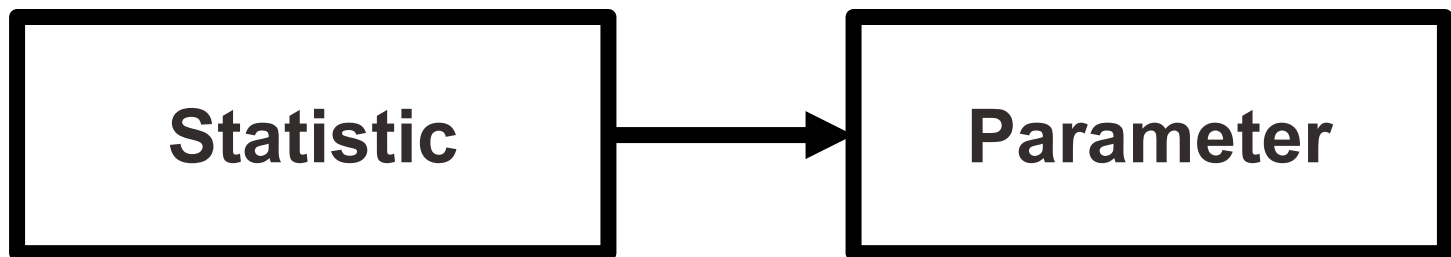
# Basic logic and terminology

- **Problem**
- The populations we wish to study are almost always so large that we are unable to gather information from every case
  
- **Solution**
- We choose a sample – a carefully chosen subset of the population – and use information gathered from the cases in the sample to generalize to the population



# Basic logic and terminology

- **Statistics** are mathematical characteristics of samples
- **Parameters** are mathematical characteristics of populations
- **Statistics** are used to estimate **parameters**



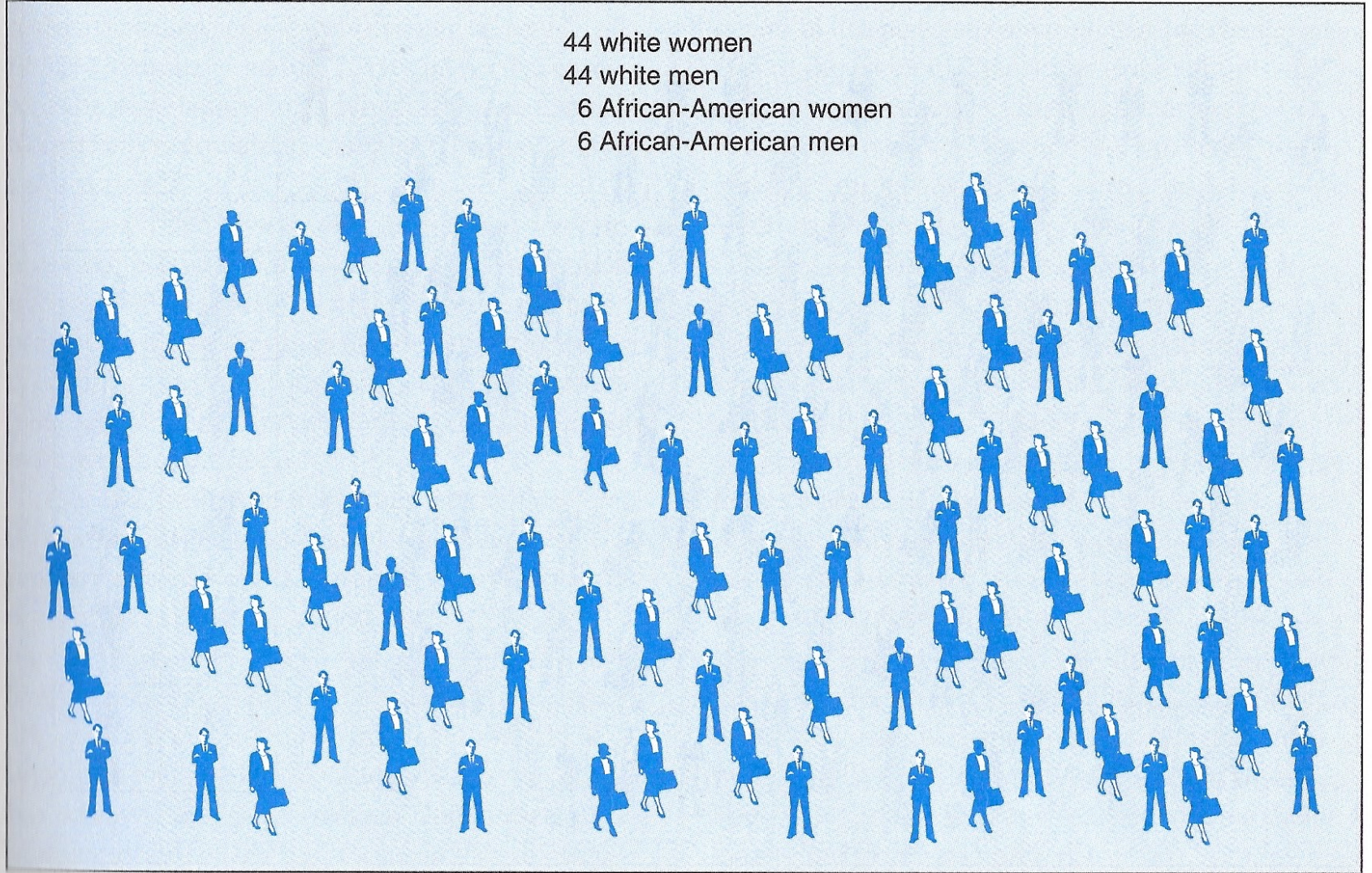
# Samples

- Must be representative of the population
  - Representative: The sample has the same characteristics as the population
- How can we ensure samples are representative?
  - Samples drawn according to the rule of **EPSEM** (equal probability of selection method)
  - If every case in the population has the same chance of being selected, the sample is likely to be representative

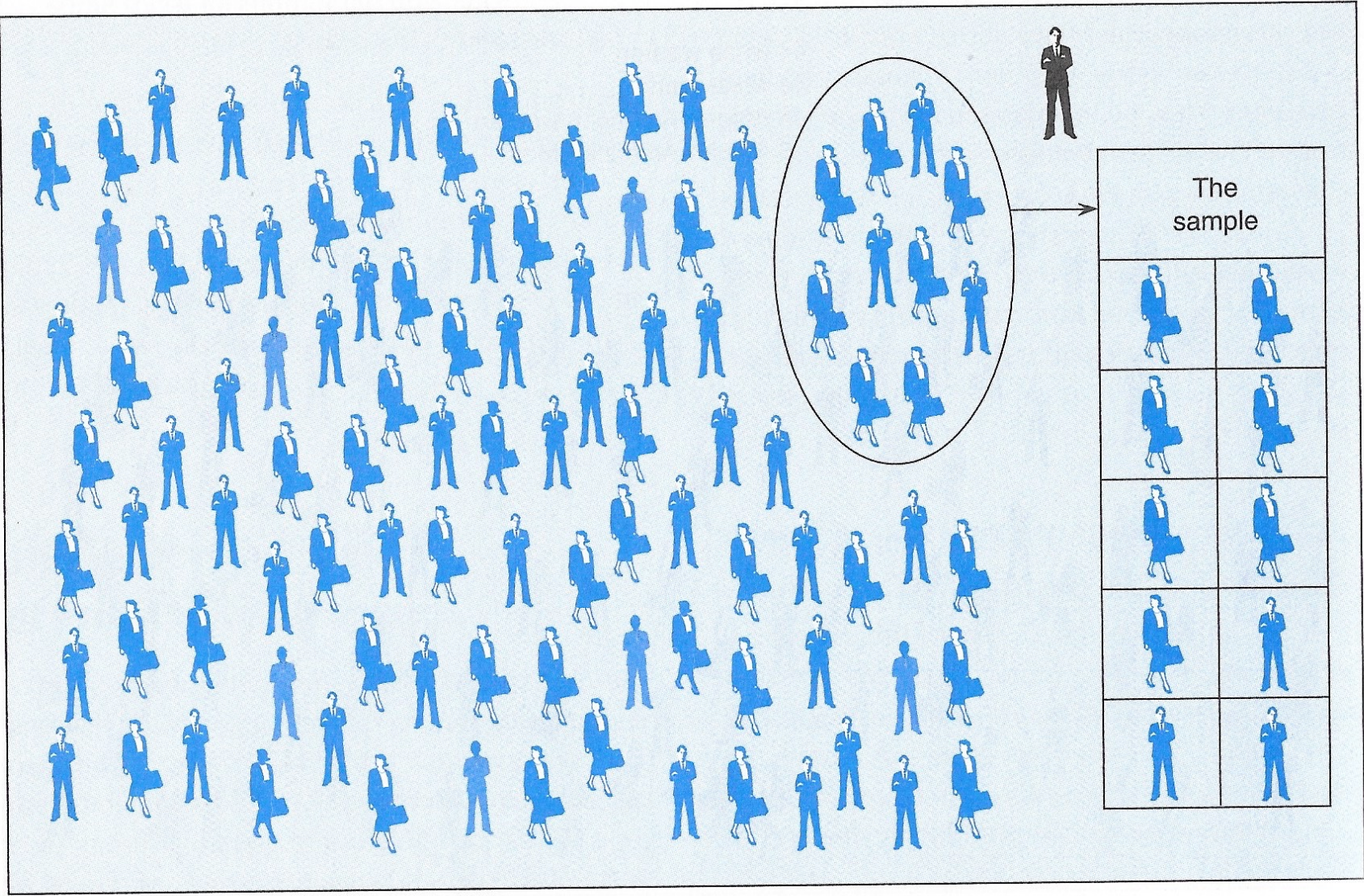


# A population of 100 people

44 white women  
44 white men  
6 African-American women  
6 African-American men



# Nonprobability sampling



# EPSEM sampling techniques

1. Simple random sampling
2. Systematic sampling
3. Stratified sampling
4. Cluster sampling





# 1. Simple random sampling

- To begin, we need
  - A list of the population
- Then, we need a method for selecting cases from the population, so each case has the same probability of being selected
  - The principle of EPSEM
  - A sample selected this way is very likely to be representative of the population
  - Variable in population should have a normal distribution or  $n > 30$



# Example

- You want to know what percent of students at a large university work during the semester
- Draw a sample size ( $n$ ) of 500 from a list of all students ( $N=20,000$ )
- Assume the list is available from the Registrar
- How can you draw names, so every student has the same chance of being selected?



# Example

- Each student has a unique, 6 digit ID number that ranges from 000001 to 999999
- Use a table of random numbers or a computer program to select 500 ID numbers with 6 digits each
- Each time a randomly selected 6 digit number matches the ID of a student, that student is selected for the sample
- Continue until 500 names are selected



# Example

- **Stata**

```
set obs 500
```

```
generate student = runiformint(1,999999)
```

```
sum student
```

Variable	Obs	Mean	Std. Dev.	Min	Max
-----+-----					
student	500	482562.6	283480.9	3652	997200

- **Excel**

- **RANDBETWEEN** (minimum,maximum)

- Returns a random number between those you specify
- Drag the function to 500 cells

=RANDBETWEEN(1,999999)

- **RANDARRAY** (rows,columns,minimum,maximum)

=RANDARRAY(500,1,1,999999)



# Example

- Disregard duplicate numbers
- Ignore cases in which no student ID matches the randomly selected number
- After questioning each of these 500 students, you find that 368 (74%) work during the semester

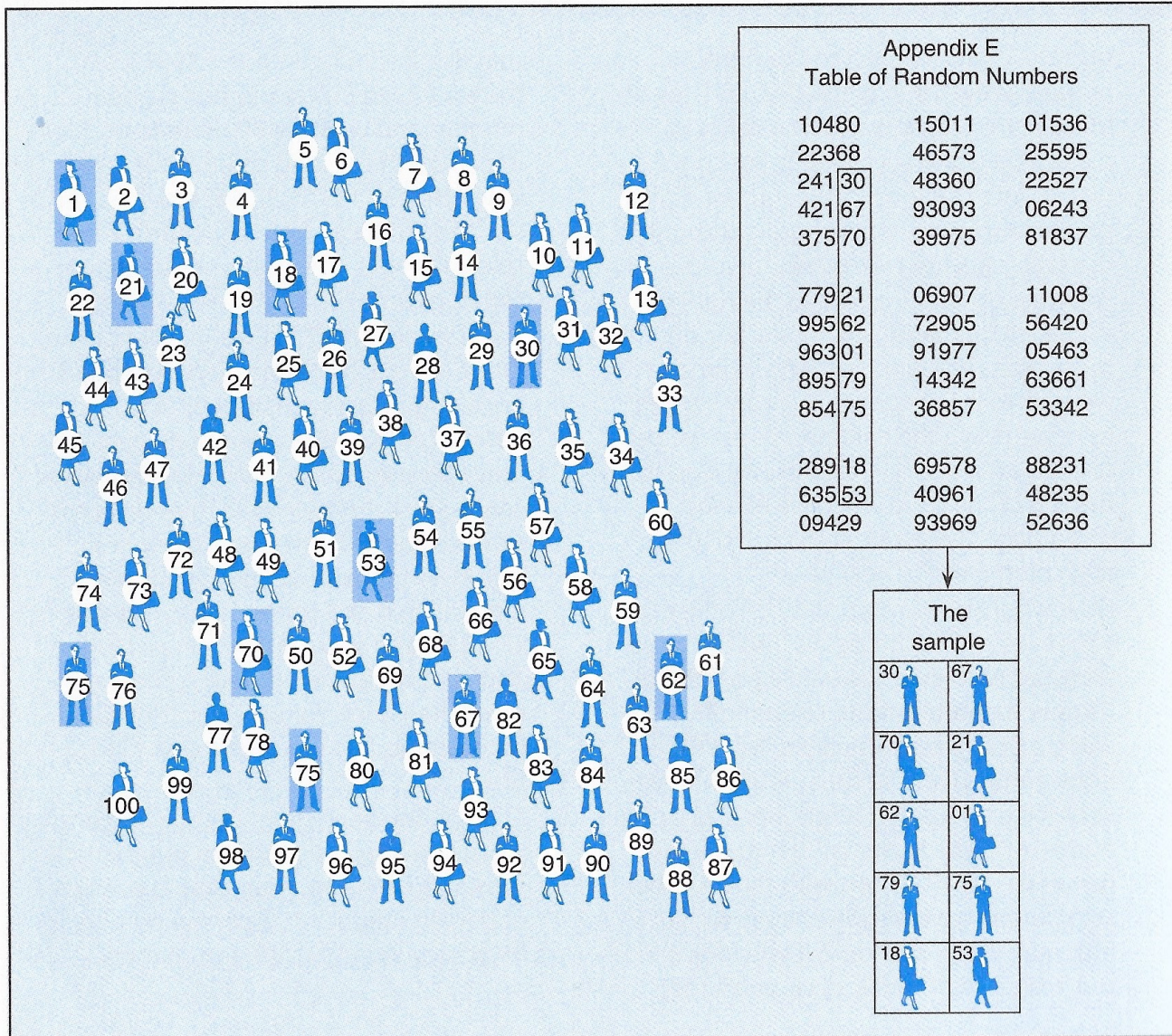


# Applying logic and terminology

- In the previous example:
- **Population:** All 20,000 students
- **Sample:** 500 students selected and interviewed
- **Statistic:** 74% (percentage of sample that held a job during the semester)
- **Parameter:** Percentage of all students in the population who held a job



# Simple random sample



## 2. Systematic sampling

- Useful for large populations
- Randomly select the first case then select every  $k^{\text{th}}$  case
- **Sampling interval**
  - Distance between elements selected in the sample
  - Population size ( $N$ ) divided by sample size ( $n$ )
- **Sampling ratio**
  - Proportion of selected elements in the population
  - Sample size ( $n$ ) divided by population size ( $N$ )
- Can be problematic if the list of cases is not truly random or demonstrates some patterning





# Example

- If a list contained 10,000 elements and we want a sample of 1,000
- Sampling interval
  - Population size / sample size =  $10,000 / 1,000 = 10$
  - We would select every 10th element for our sample
- Sampling ratio
  - Sample size / population size =  $1,000 / 10,000 = 1/10$
  - Proportion of selected elements in population
- Select the first element at random



# 3. Stratified sampling

- It guarantees the sample will be representative on the selected (stratifying) variables
  - Stratification variables relate to research interests
- First, divide the population list into subsets, according to some relevant variable
  - **Homogeneity within subsets**
    - E.g., only women in a subset; only men in another subset
  - **Heterogeneity between subsets**
    - E.g., subset of women is different than subset of men
- Second, sample from the subsets
  - Select the number of cases from each subset proportional to the population

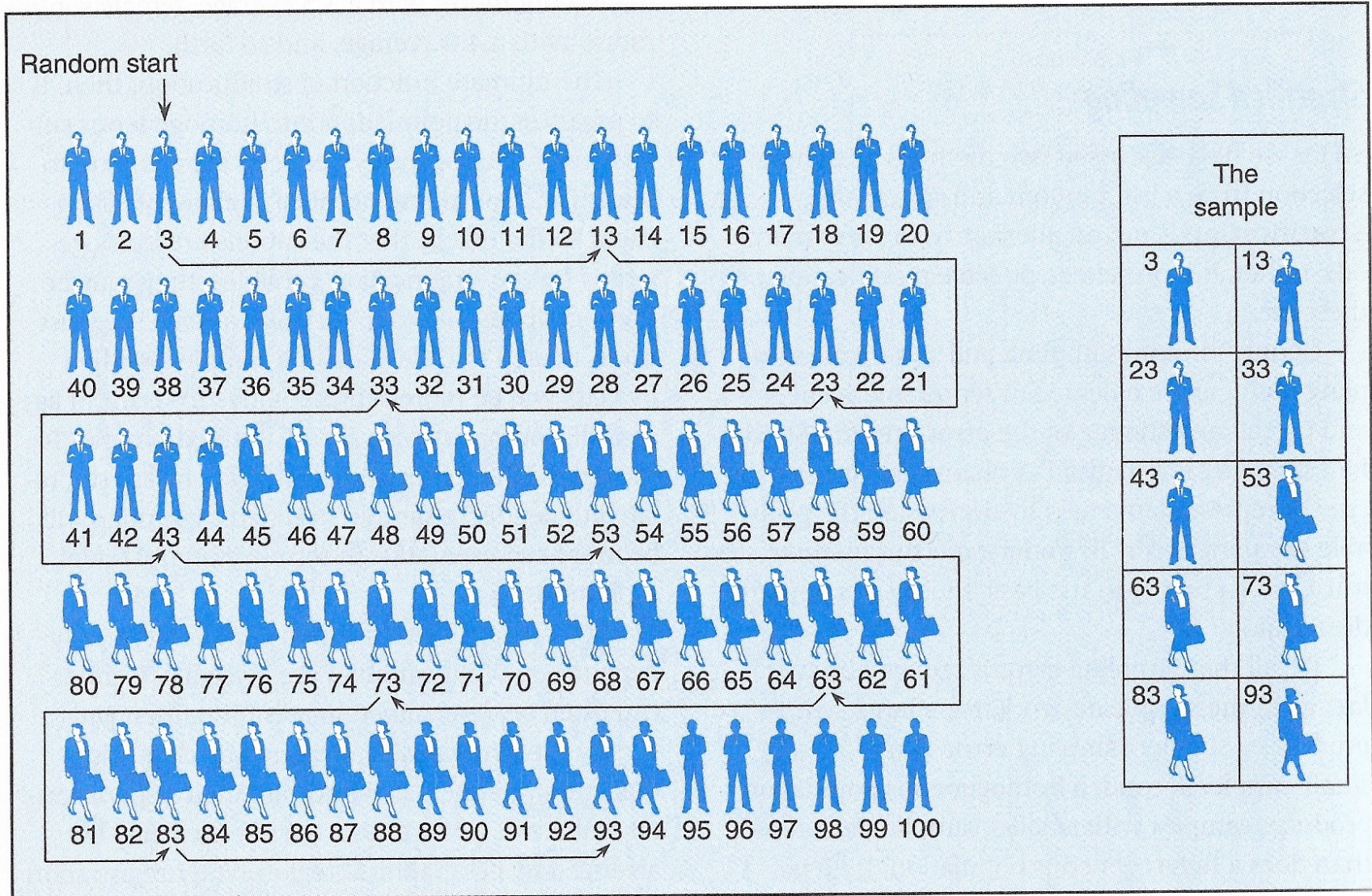


# Example

- If you want a sample of 1,000 students
  - That would be representative to the population of students by sex and GPA
- You need to know the population composition
  - E.g., women with a 4.0 average compose 15 percent of the student population
- Your sample should follow that composition
  - In a sample of 1,000 students, you would select 150 women with a 4.0 average



# Stratified, systematic sample



# 4. Cluster sampling

- Select groups (or clusters) of cases rather than single cases
  - **Heterogeneity within subsets**
    - E.g., each subset has both women and men, following same proportional distribution as population
  - **Homogeneity between subsets**
    - E.g., all subsets with both women and men should be similar
- Clusters are often geographically based
  - For example, cities or voting districts
- Sampling often proceeds in stages
  - Multi-stage cluster sampling
  - Less representative than simple random sampling



# Stratified vs. cluster sampling

- **Stratified**

- Homogeneity within subsets
- Heterogeneity between subsets
- Select cases from each subset

Subset of  
women

Subset of  
men

- **Cluster**

- Heterogeneity within subsets (groups, clusters, areas)
- Homogeneity between subsets
- Select groups (e.g., area 1) rather than single cases

Area 1:  
women & men

Area 2:  
women & men



# Sampling distribution

- Sampling distribution is the probabilistic distribution of a statistic for all possible samples of a given size ( $n$ )
  - It is the distribution of a statistic (e.g., proportion, mean) for all possible outcomes of a certain size
- Central tendency and dispersion
  - Mean is the same as the population mean
  - Standard deviation is referred as standard error
    - It is the population standard deviation divided by the square root of  $n$
    - We have to take into account the complex survey design to estimate the standard error (`svyset` command in Stata)



# Linking sample and population

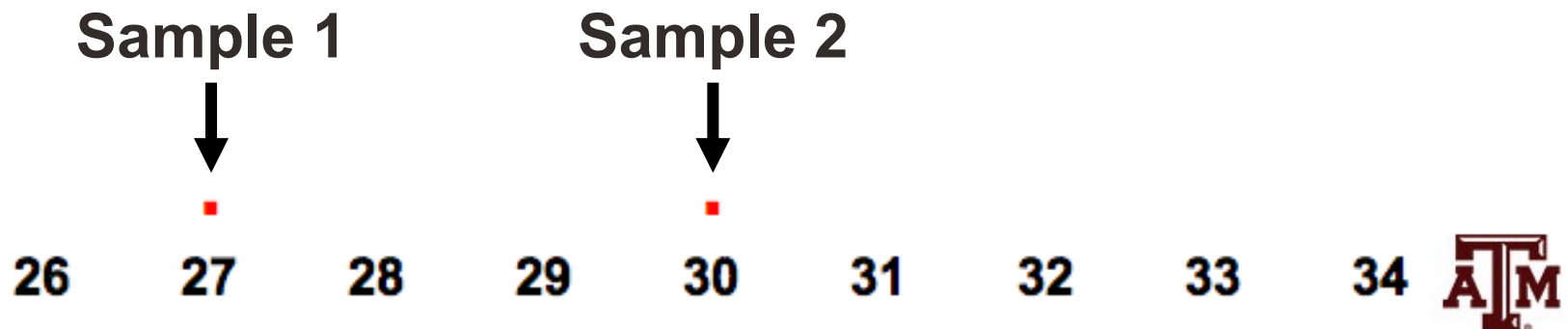
- Every application of inferential statistics involves three different distributions
  - Population: empirical; unknown
  - Sampling distribution: theoretical; known
  - Sample: empirical; known
- In inferential statistics, the sample distribution links the sample with the population





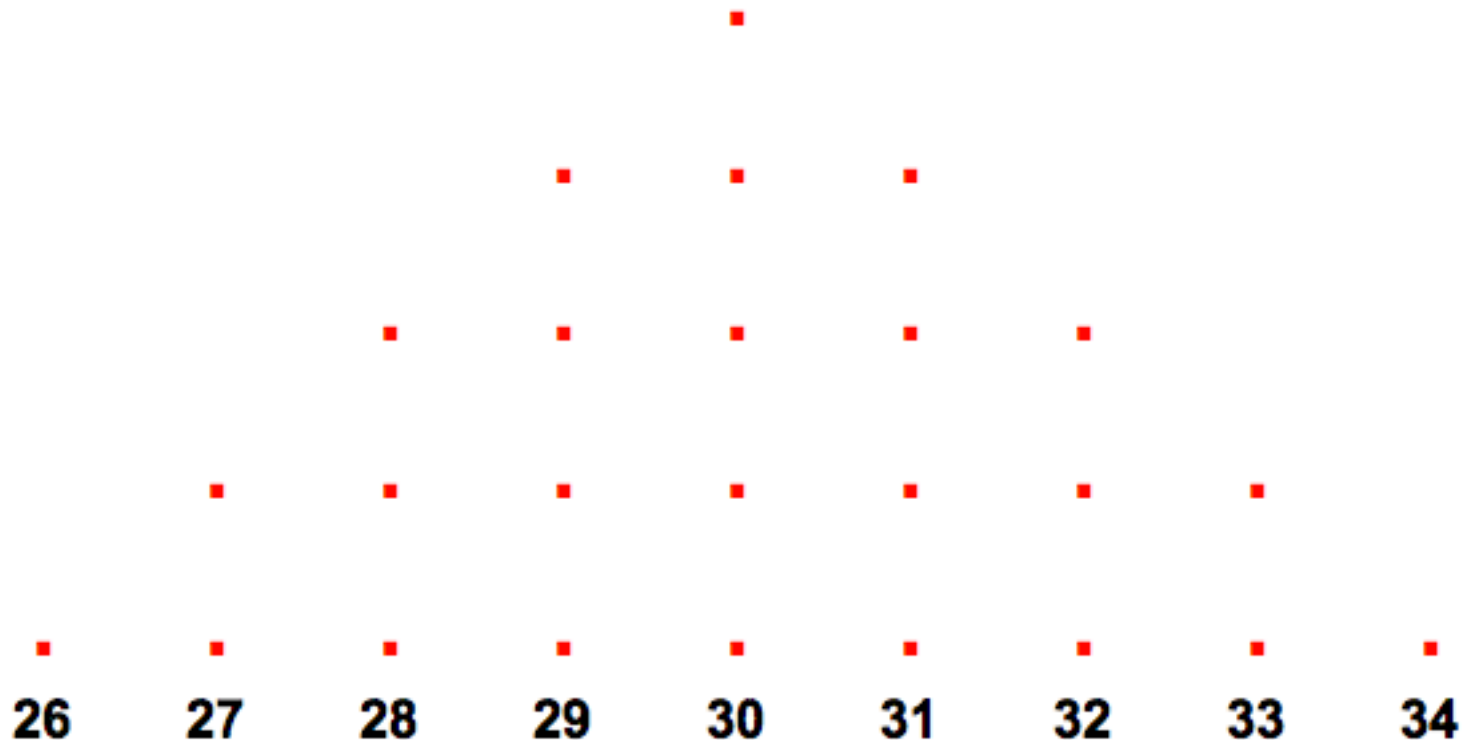
# Example

- Suppose we want to gather information on the age of a community of 10,000 individuals
  - Sample 1:  $n=100$  people, plot sample's mean of 27
  - Replace people in the sample back to the population
  - Sample 2:  $n=100$  people, plot sample's mean of 30
  - Replace people in the sample back to the population

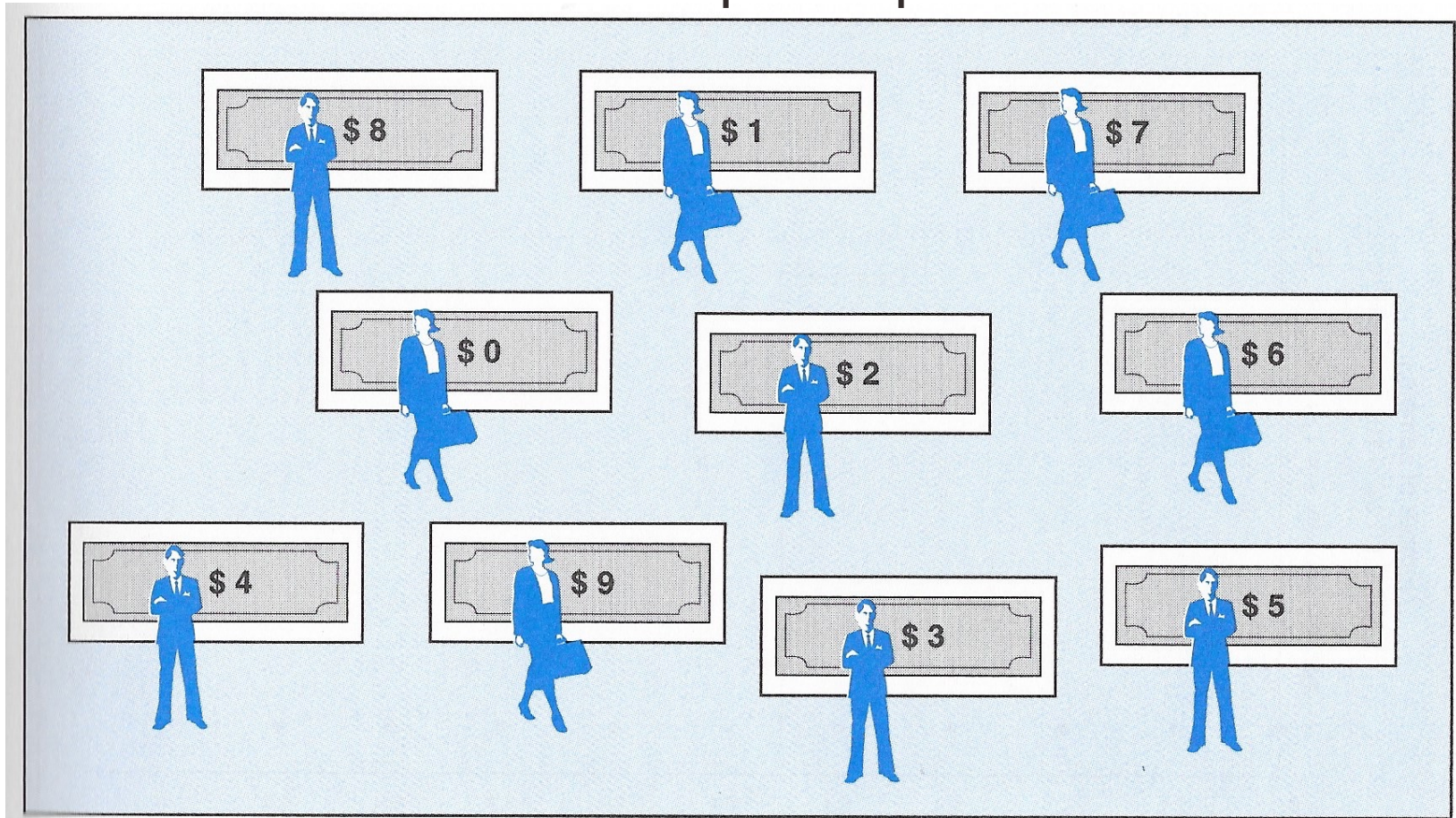


# Example

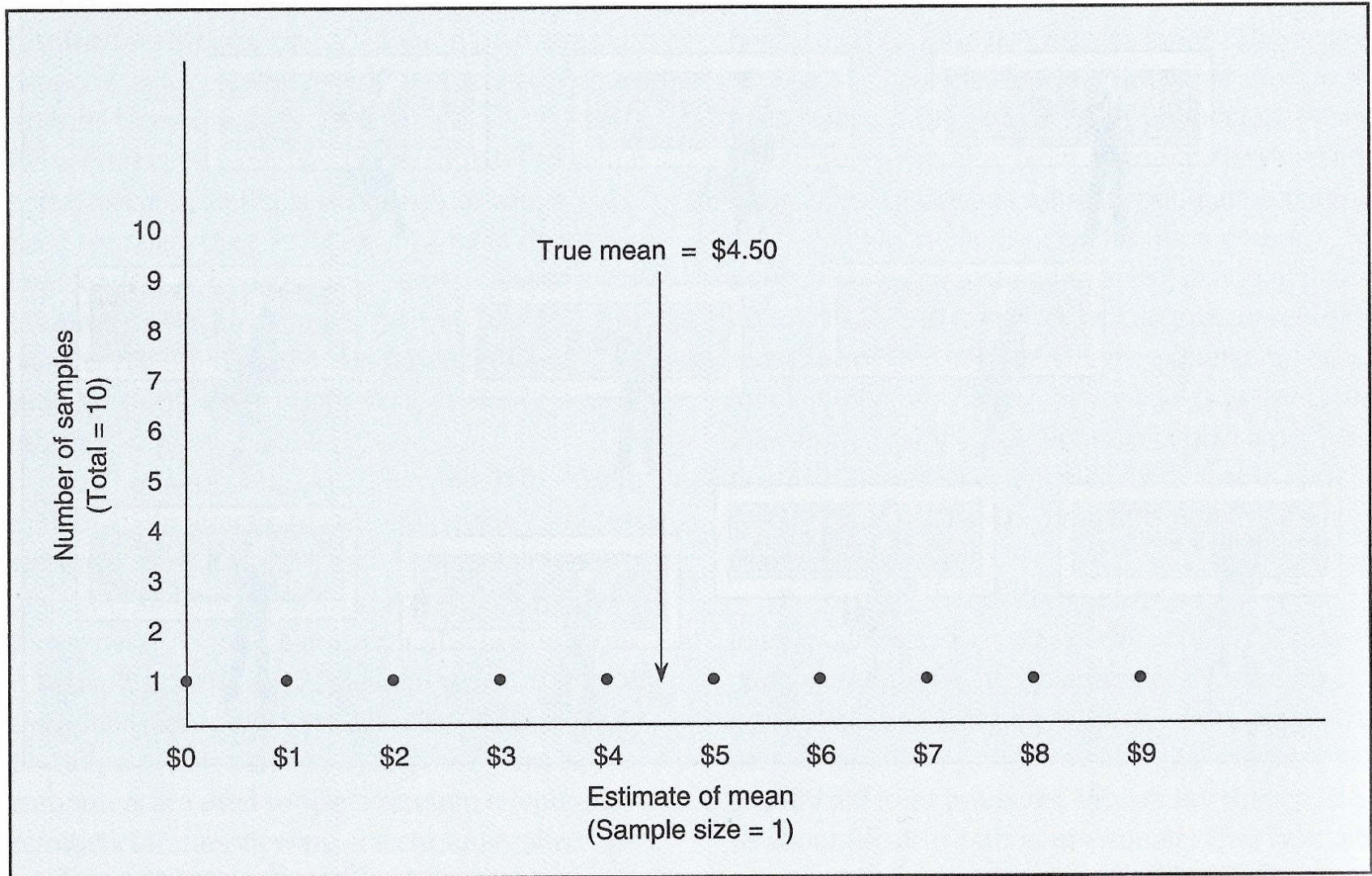
- We repeat this procedure: sampling, replacing
  - Until we have exhausted every possible combination of 100 people from the population of 10,000
  - Sampling distribution has a normal shape



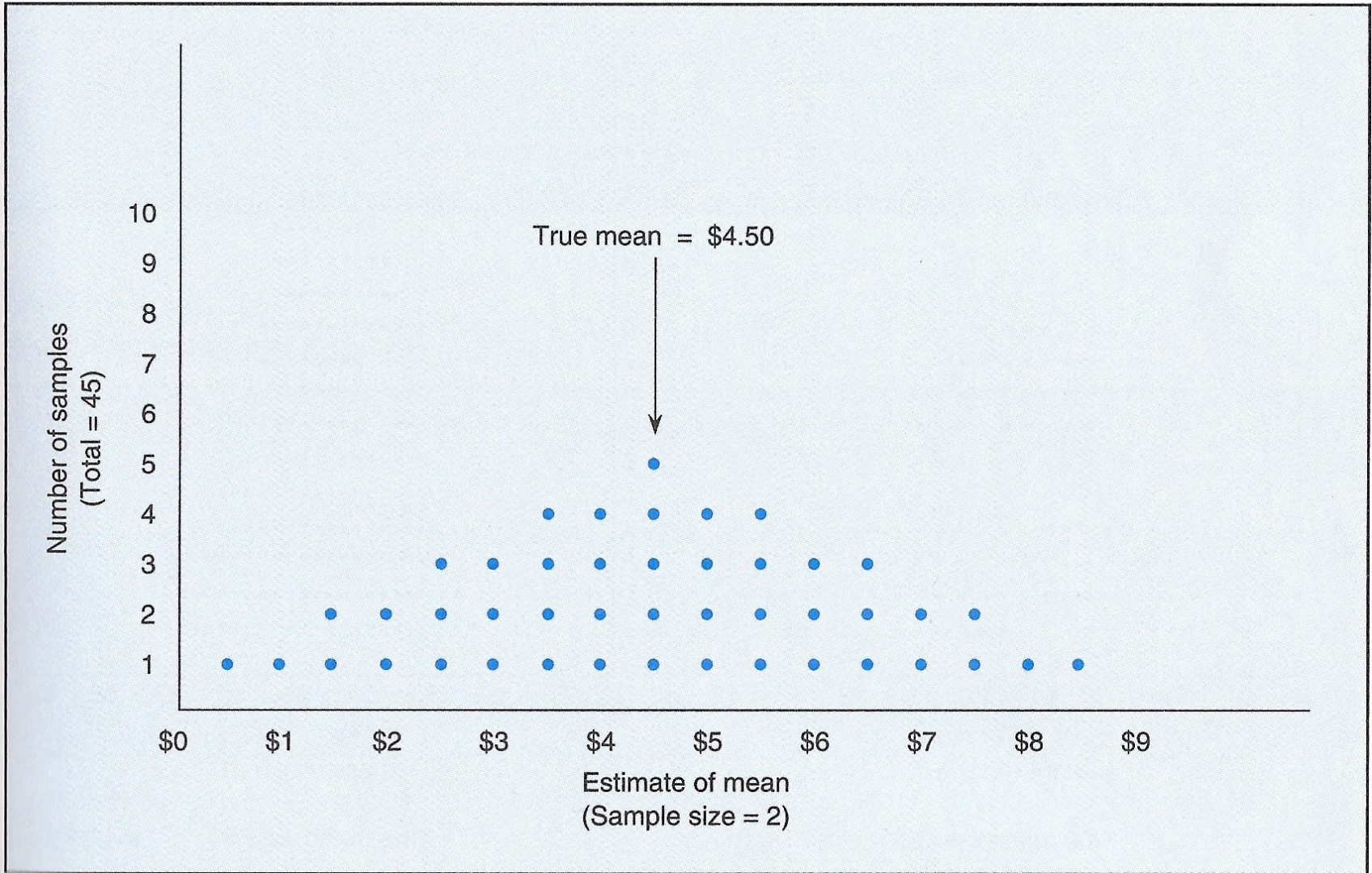
# Another example: A population of 10 people with \$0–\$9



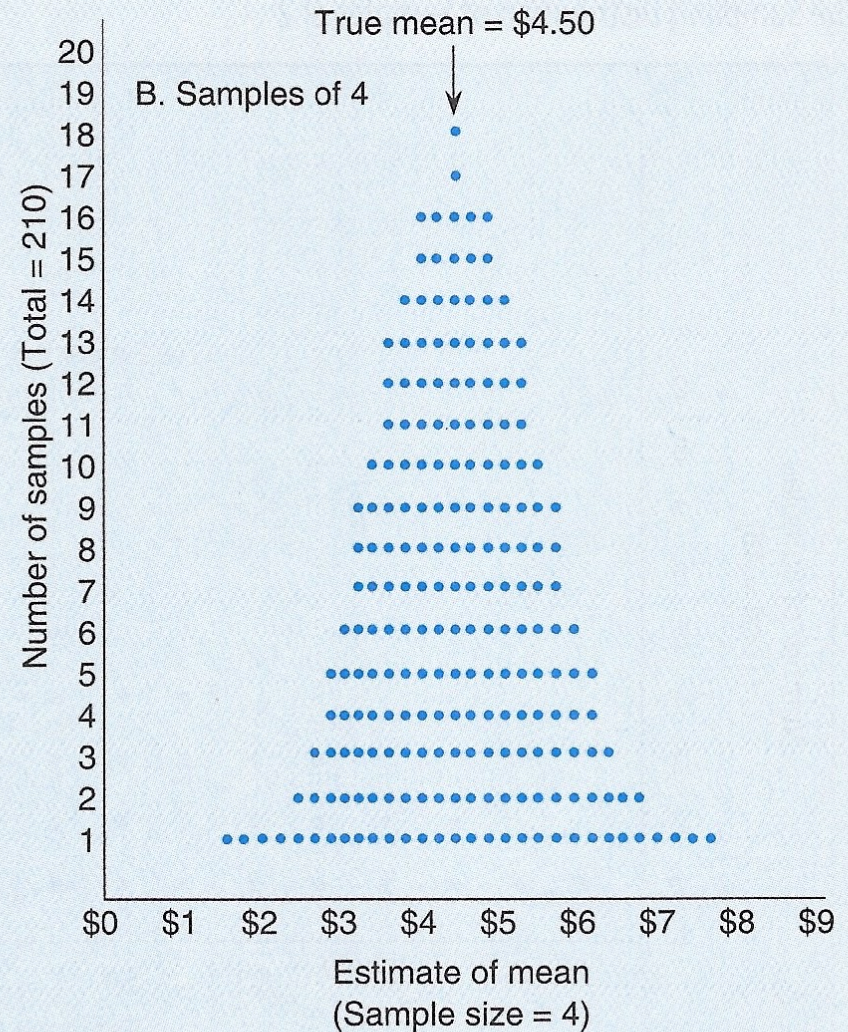
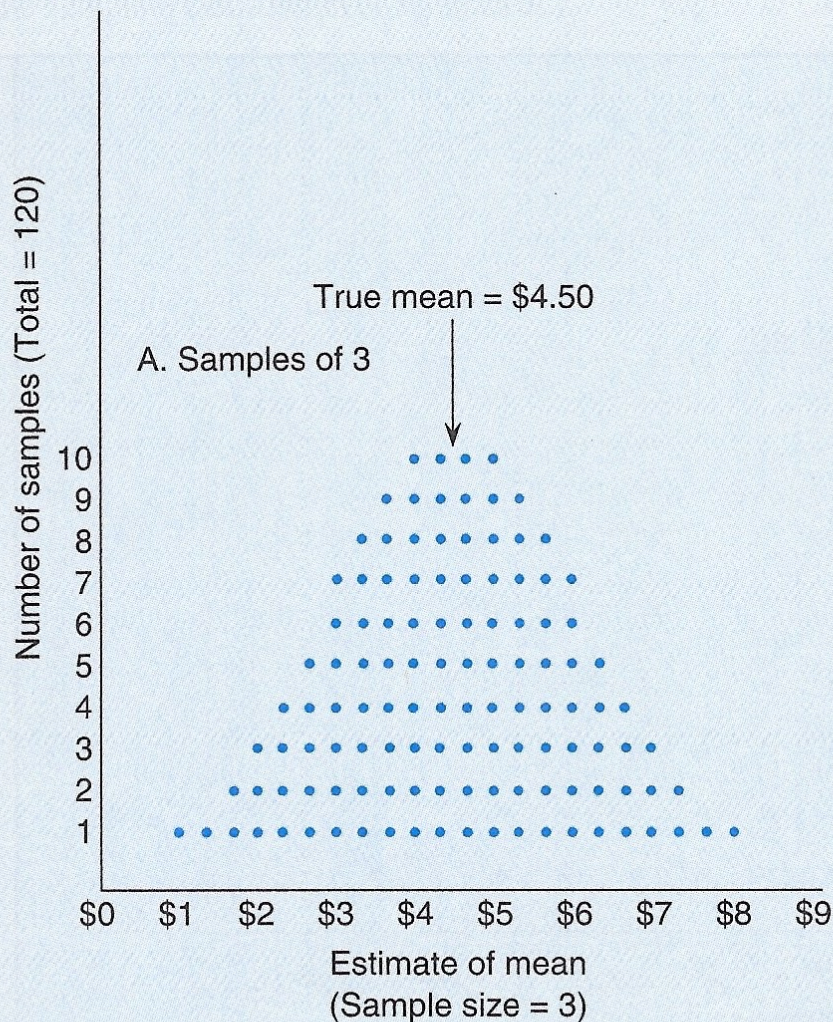
# The sampling distribution ( $n=1$ )



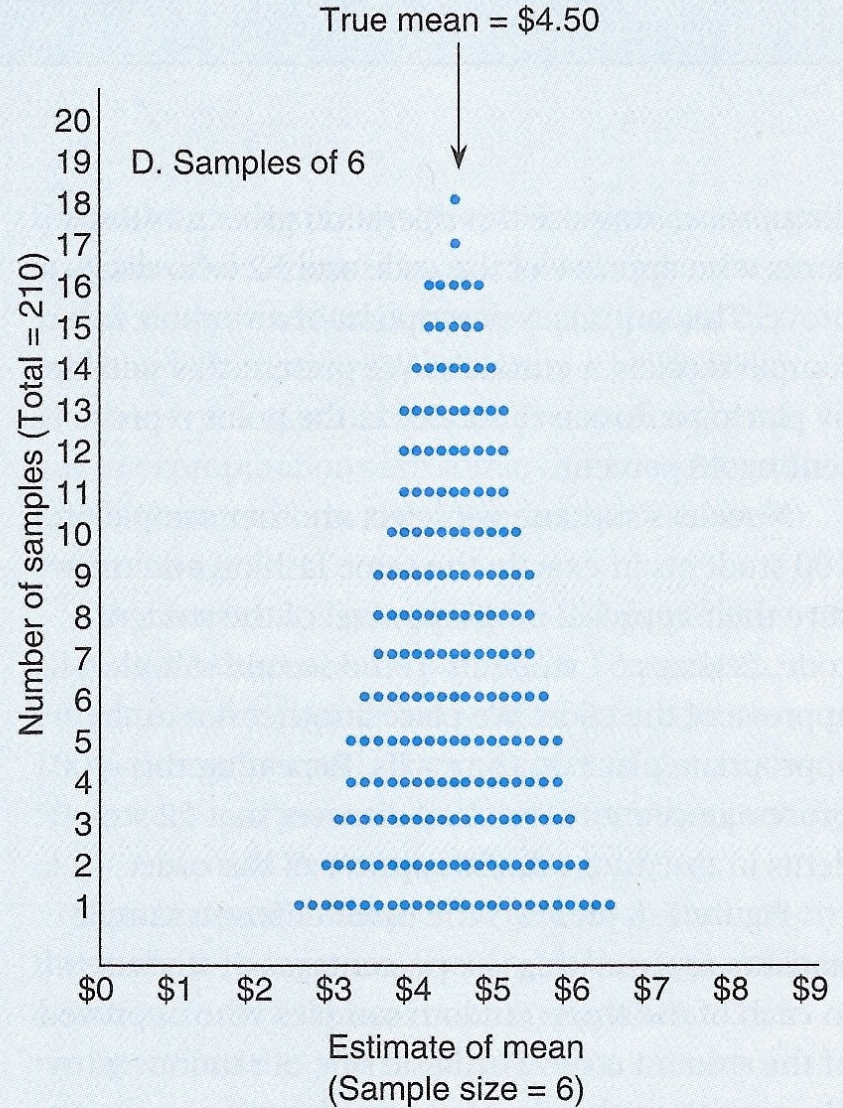
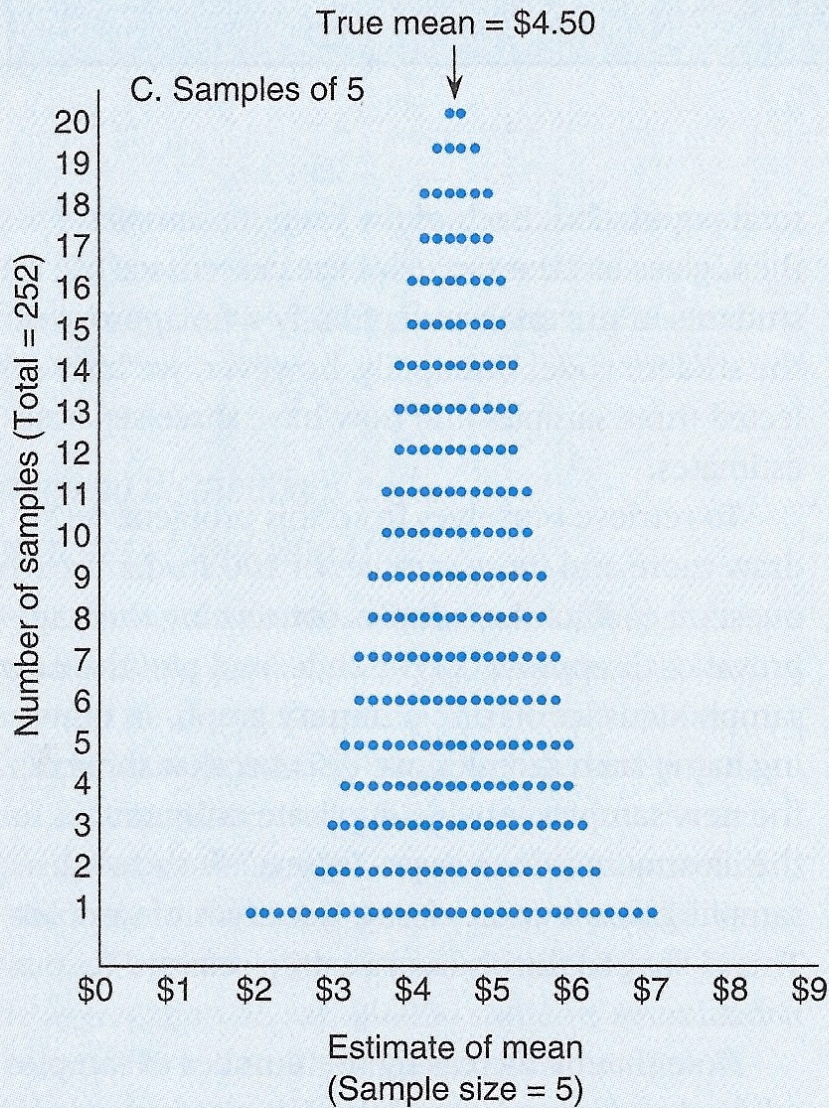
# The sampling distribution ( $n=2$ )



# The sampling distribution



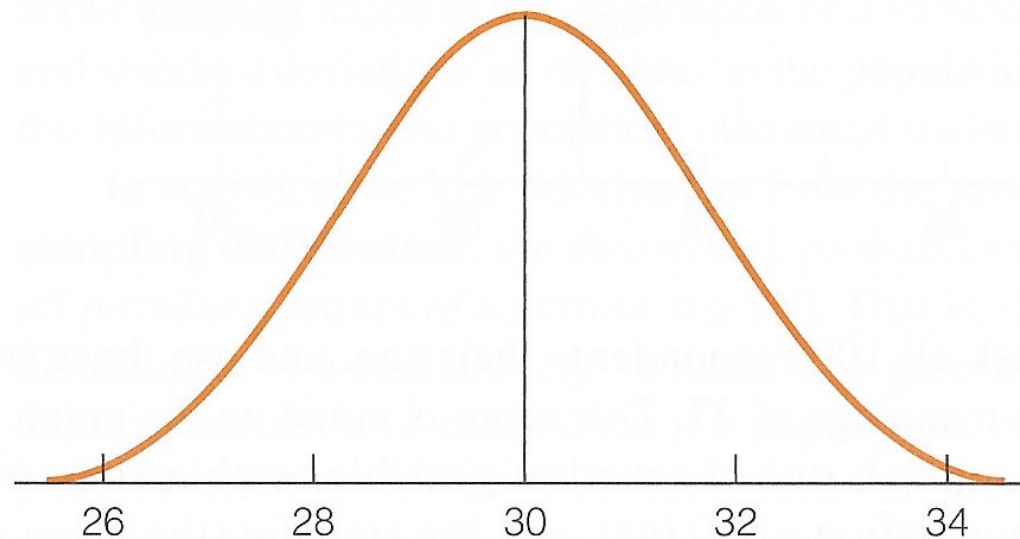
# The sampling distribution



# Properties of sampling distribution

- It has a mean ( $\mu_{\bar{X}}$ ) equal to the population mean ( $\mu$ )
- It has a standard deviation (standard error,  $\sigma_{\bar{X}}$ ) equal to the population standard deviation ( $\sigma$ ) divided by the square root of  $n$
- It has a normal distribution

A Sampling Distribution of Sample Means





# First theorem

- Tells us the shape of the sampling distribution and defines its mean and standard deviation
- If repeated random samples of size  $n$  are drawn from a **normal population** with mean  $\mu$  and standard deviation  $\sigma$ 
  - Then, the sampling distribution of sample means will **have a normal distribution** with...
  - A mean:  $\mu_{\bar{X}} = \mu$
  - A standard error of the mean:  $\sigma_{\bar{X}} = \sigma/\sqrt{n}$



# First theorem

- Begin with a characteristic that is normally distributed across a population (IQ, height)
- Take an infinite number of equally sized random samples from that population
- The sampling distribution of sample means will be normal

# Central limit theorem

- If repeated random samples of size  $n$  are drawn from **any population** with mean  $\mu$  and standard deviation  $\sigma$ 
  - Then, as  $n$  becomes large, the sampling distribution of sample means will **approach normality** with...
    - A mean:  $\mu_{\bar{X}} = \mu$
    - A standard error of the mean:  $\sigma_{\bar{X}} = \sigma/\sqrt{n}$
- This is true for any variable, even those that are not normally distributed in the population
  - As sample size grows larger, the sampling distribution of sample means will become normal in shape



# Central limit theorem

- The importance of the central limit theorem is that it removes the constraint of normality in the population
  - Applies to large samples ( $n \geq 100$ )
- If the sample is small ( $n < 100$ )
  - We must have information on the normality of the population before we can assume the sampling distribution is normal

# Additional considerations

- The sampling distribution is normal
  - We can estimate areas under the curve (Appendix A)
  - Or in Stata: `display normal(z)`
- We do not know the value of the population mean ( $\mu$ )
  - But the mean of the sampling distribution ( $\mu_{\bar{x}}$ ) is the same value as  $\mu$
- We do not know the value of the population standard deviation ( $\sigma$ )
  - But the standard deviation of the sampling distribution ( $\sigma_{\bar{x}}$ ) is equal to  $\sigma$  divided by the square root of  $n$



# Symbols

Distribution	Shape	Mean	Standard deviation	Proportion
Samples	Varies	$\bar{X}$	$s$	$P_s$
Populations	Varies	$\mu$	$\sigma$	$P_u$
Sampling distributions	Normal	$\mu_{\bar{X}}$		
of means		$\mu_{\bar{X}}$	$\sigma_{\bar{X}} = \sigma/\sqrt{n}$	
of proportions		$\mu_p$	$\sigma_p$	





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