# Lecture 7: Estimation procedures

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Source: Healey, Joseph F. 2015. "Statistics: A Tool for Social Research." Stamford: Cengage Learning. 10th edition. Chapter 7 (pp. 160–184).

TEXAS A&M UNIVERSITY

# Outline

- Explain the logic of estimation, role of the sample, sampling distribution, and population
- Define and explain the concepts of bias and efficiency
- Construct and interpret confidence intervals for sample means and sample proportions
- Explain relationships among confidence level, sample size, and width of the confidence interval

# Sample and population

- In estimation procedures, statistics calculated from random samples are used to estimate the value of population parameters

- Example
  - If we know that 42% of a random sample drawn from a city are Republicans, we can estimate the percentage of all city residents who are Republicans

# Terminology

- Information from samples is used to estimate information about the population

**Sample** → **Population**

- Statistics are used to estimate parameters

**Statistic** → **Parameter**

# Basic logic

- Sampling distribution is the link between sample and population

- The values of the parameters are unknown, but the characteristics of the sampling distribution are defined by two theorems (previous chapter)

# Two estimation procedures

- **A point estimate** is a sample statistic used to estimate a population value
  - 68% of a sample of randomly selected Americans support capital punishment (GSS 2010)
- **An interval estimate** consists of confidence intervals (range of values)
  - Between 65% and 71% of Americans approve of capital punishment (GSS 2010)
  - Most point estimates are actually interval estimates
  - Margin of error generates confidence intervals
  - Estimators are selected based on two criteria
    - Bias (mean) and efficiency (standard error)

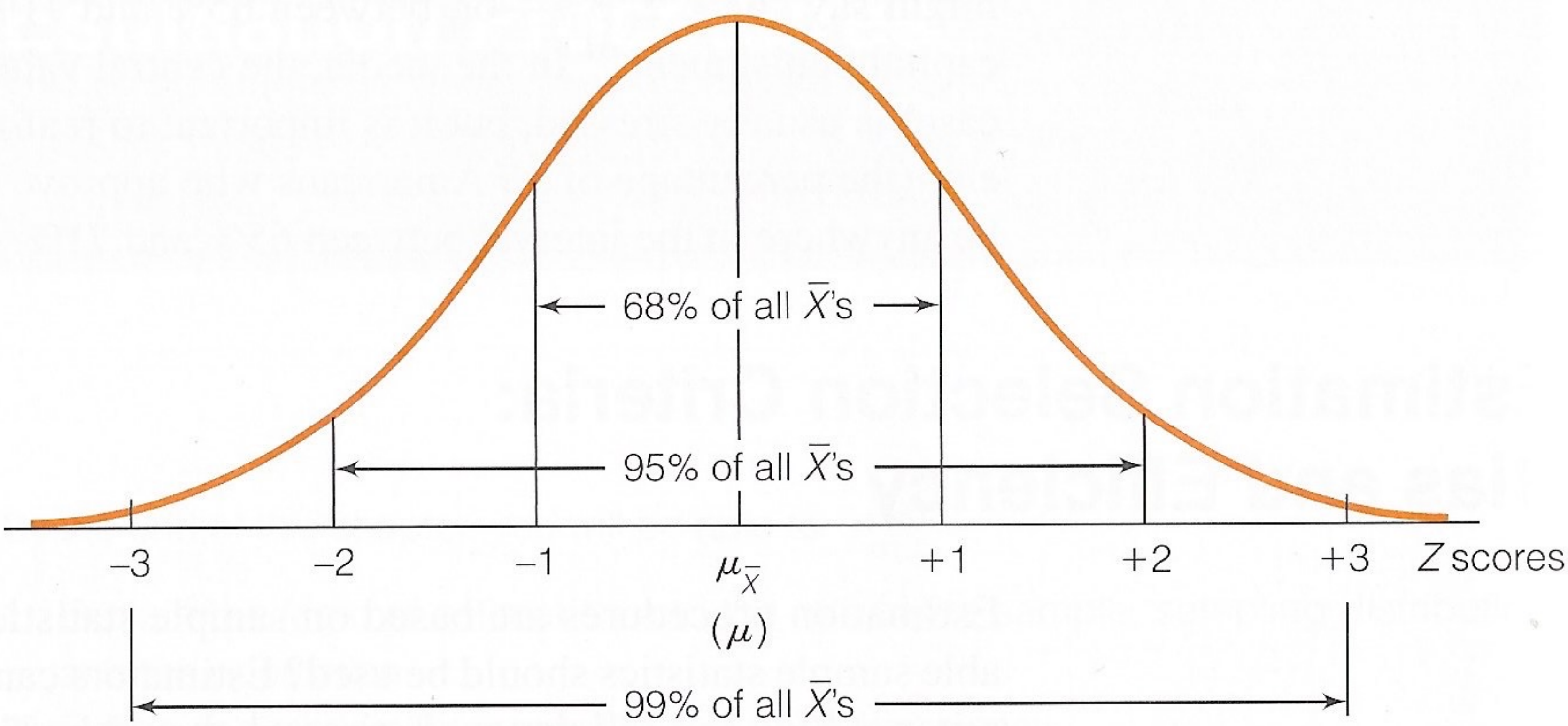
# Bias

- An estimator is unbiased if the mean of its sampling distribution is equal to the population value of interest
- The mean of the sampling distribution of sample means ($\mu_{\bar{X}}$) is the same as the population mean ($\mu$)
- Sample proportions ($P_s$) are also unbiased
  - If we calculate sample proportions from repeated random samples of size *n*...
  - Then, the sampling distribution of sample proportions will have a mean ($\mu_p$) equal to the population proportion ($P_u$)
- Sample means and proportions are unbiased estimators
  - We can determine the probability that they are within a certain distance of the population values

# Example

- Random sample to get income information
- Sample size ($n$): 500 households
- Sample mean ($\bar{X}$): $45,000
- Population mean ($\mu$): unknown parameter
- Mean of sampling distribution ($\mu_{\bar{X}} = \mu$)
  - If an estimator ($\bar{X}$) is unbiased, it is probably an accurate estimate of the population parameter ($\mu$) and sampling distribution mean ($\mu_{\bar{X}}$)
  - We use the sampling distribution (which has a normal shape) to estimate confidence intervals

# Sampling distribution




**Source: Healey 2015, p.162.**

# Efficiency

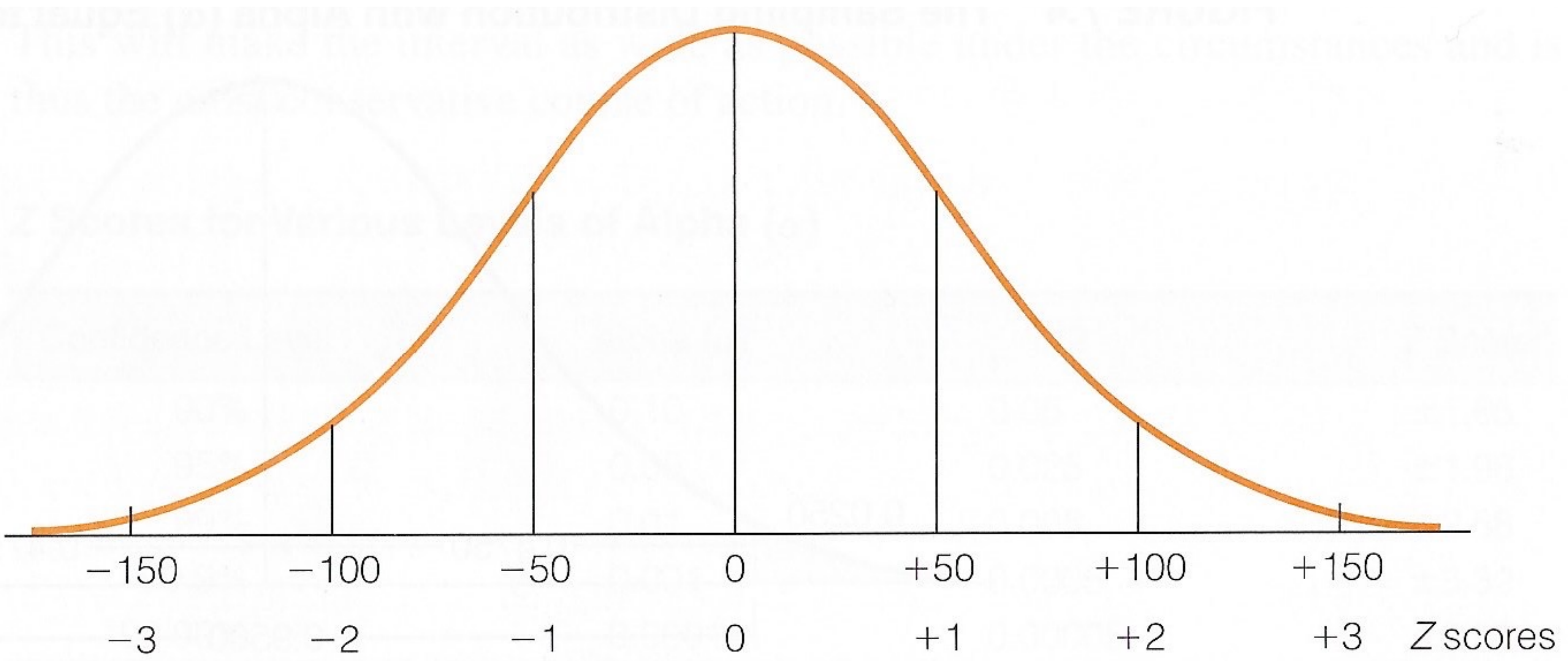
- Efficiency is the extent to which the sampling distribution is clustered around its mean
- Efficiency or clustering is a matter of dispersion
  - The smaller the standard deviation of a sampling distribution, the greater the clustering and the higher the efficiency
  - Larger samples have greater clustering and higher efficiency
  - Standard deviation of sampling distribution: $\sigma_{\bar{X}} = \sigma/\sqrt{n}$

| **Statistics** | **Sample 1** | **Sample 2** |
|---|---|---|
| Sample mean | $\bar{X}_1 = \$45{,}000$ | $\bar{X}_2 = \$45{,}000$ |
| Sample size | $n_1 = 100$ | $n_2 = 1000$ |
| Standard deviation | $\sigma_1 = \$500$ | $\sigma_2 = \$500$ |
| Standard error | $\sigma_{\bar{X}} = 500/\sqrt{100} = \$50.00$ | $\sigma_{\bar{X}} = 500/\sqrt{1000} = \$15.81$ |

Source: Healey 2015, p.163.

# Sampling distribution

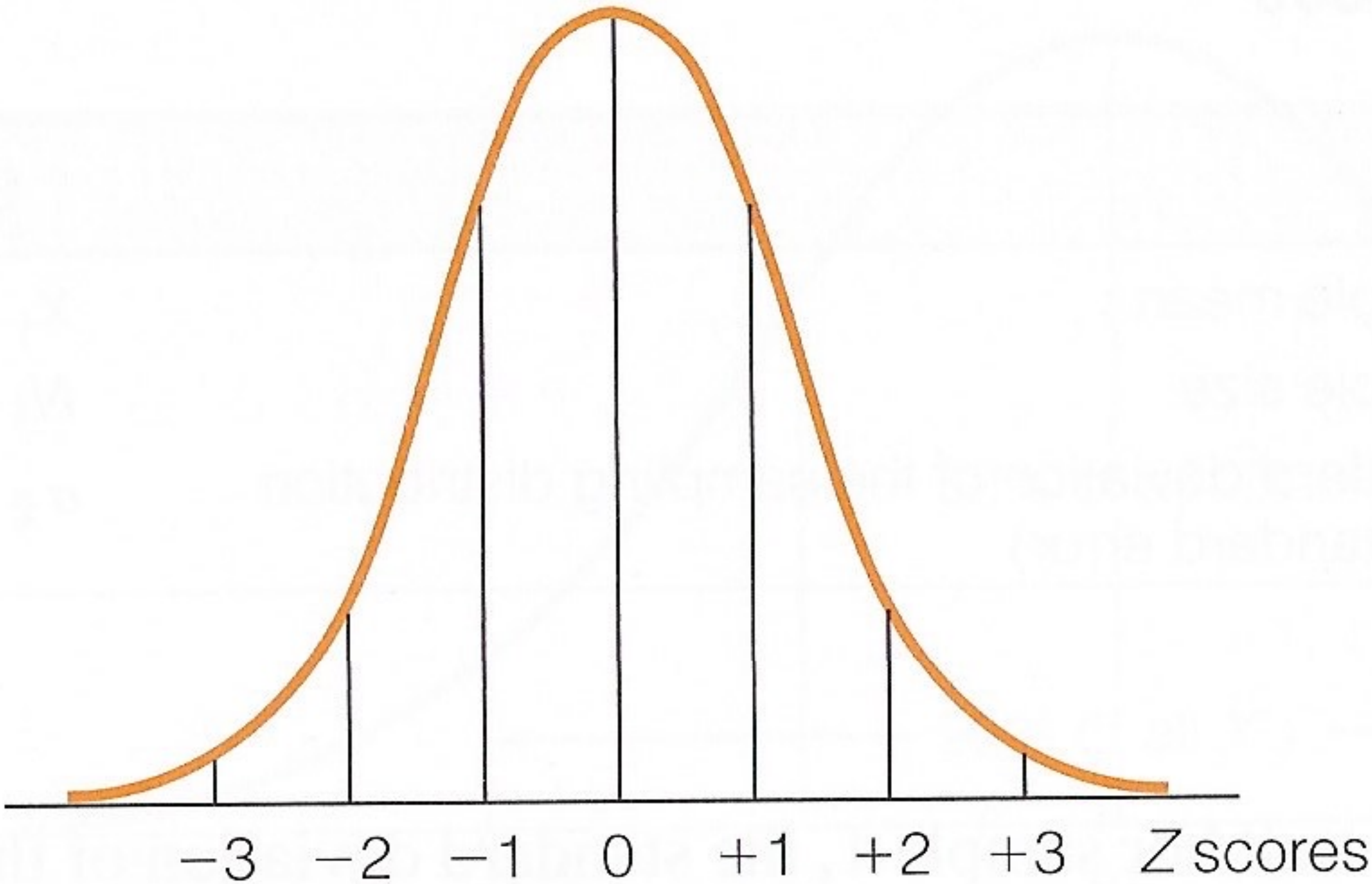
## $n$ = 100; $\sigma_{\bar{X}}$ = \$50.00




Source: Healey 2015, p.163.

# Sampling distribution

$n$ = 1000; $\sigma_{\bar{X}}$ = \$15.81




Source: Healey 2015, p.164.

# Confidence interval & level

- **Confidence interval** is a range of values used to estimate the true population parameter
  – We associate a confidence level (e.g. 0.95 or 95%) to a confidence interval
- **Confidence level** is the success rate of the procedure to estimate the confidence interval
  – Expressed as probability ($1–\alpha$) or percentage ($1–\alpha$)*100
  – $\alpha$ is the complement of the confidence level
  – Larger confidence levels generate larger confidence intervals
- Confidence level of 95% is the most common
  – Good balance between precision (width of confidence interval) and reliability (confidence level)

# Interval estimation procedures

- Set the alpha ($\alpha$)
  - Probability that the interval will be wrong
- Find the *Z* score associated with alpha
  - In column c of Appendix A of textbook
    - If the *Z* score you are seeking is between two other scores, choose the larger of the two *Z* scores
  - In Stata: **`display invnormal(α)`**
- Substitute values into appropriate equation
- Interpret the interval

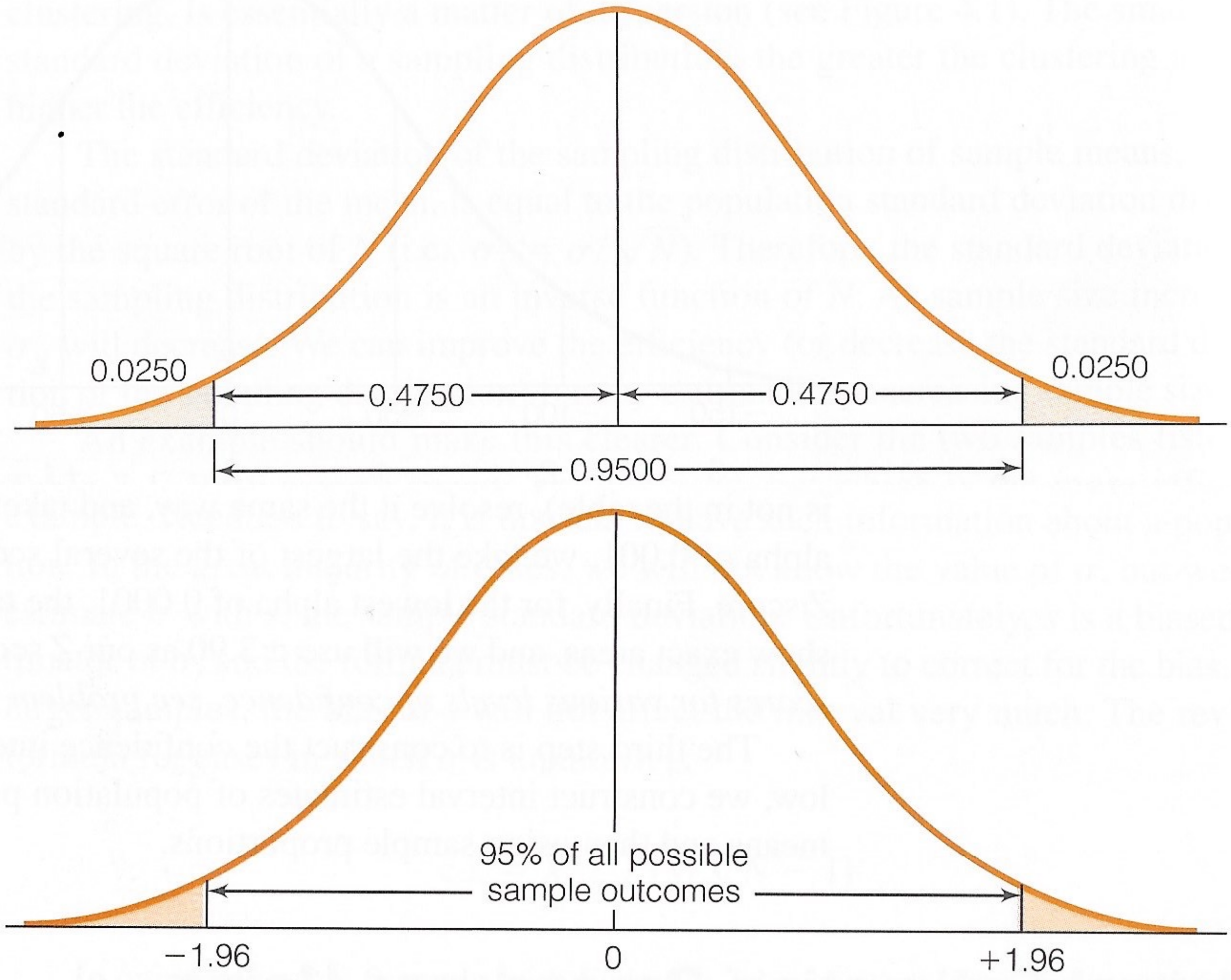
# Example to find *Z* score

- Setting alpha (*α*) equal to 0.05
  - 95% confidence level: (1–*α*)*100
  - We are willing to be wrong 5% of the time
- If alpha is equal to 0.05
  - Half of this probability is in the lower tail (*α*/2=0.025)
  - Half is in the upper tail of the distribution (*α*/2=0.025)
- Looking up this area, we find a Z = 1.96

```
di invnormal(.025)
-1.959964
```

```
di invnormal(1-.025)
di invnormal(.975)
1.959964
```

# Finding Z for sampling distribution with $\alpha$ = 0.05




Source: Healey 2015, p.164–165.

# Confidence level, $\alpha$, and Z

| Confidence level $(1 - \alpha) * 100$ | Significance level alpha ($\alpha$) | $\alpha / 2$ | $Z$ score |
|---:|---:|---:|---:|
| 90% | 0.10 | 0.05 | $\pm 1.65$ |
| **95%** | **0.05** | **0.025** | $\pm \mathbf{1.96}$ |
| 99% | 0.01 | 0.005 | $\pm 2.58$ |
| 99.9% | 0.001 | 0.0005 | $\pm 3.32$ |
| 99.99% | 0.0001 | 0.00005 | $\pm 3.90$ |

**Source: Healey 2015, p.165.**

# Confidence intervals for sample means

- For large samples ($n \geq 100$)
- Standard deviation ($\sigma$) **known** for population

$$c.i. = \bar{X} \pm Z\left(\frac{\sigma}{\sqrt{n}}\right)$$

*c.i.* = confidence interval

$\bar{X}$ = sample mean

$Z$ = score determined by the alpha level (confidence level)

$\sigma/\sqrt{n}$ = standard deviation of the sampling distribution
(standard error of the mean)

$\pm Z(\sigma/\sqrt{n})$ = margin of error

# Example for means: Large sample, $\sigma$ known

- Sample of 200 residents
- Sample mean of IQ is 105
- Population standard deviation is 15
- Calculate a confidence interval with a 95% confidence level ($\alpha$ = 0.05)
  - Same as saying: calculate a 95% confidence interval

$$c.i. = \bar{X} \pm Z\left(\frac{\sigma}{\sqrt{n}}\right) = 105 \pm 1.96\left(\frac{15}{\sqrt{200}}\right) = 105 \pm 2.08$$

  - Average IQ is somewhere between 102.92 (105–2.08) and 107.08 (105+2.08)

# Interpreting previous example

$n = 200;\ 102.92 \leq \mu \leq 107.08$

- **Correct:** We are 95% certain that the confidence interval contains the true value of $\mu$
  - If we selected several samples of size 200 and estimated their confidence intervals, 95% of them would contain the population mean ($\mu$)
  - The 95% confidence level refers to the success rate to estimate the population mean ($\mu$). It does not refer to the population mean itself
- **Wrong:** Since the value of $\mu$ is fixed, it is incorrect to say that there is a chance of 95% that the true value of $\mu$ is between the interval

**Source: Triola 2008, p.250–303.**

# Confidence intervals for sample means

- For large samples ($n \geq 100$)
- Standard deviation ($\sigma$) **unknown** for population

$$c.i. = \bar{X} \pm Z\left(\frac{s}{\sqrt{n-1}}\right)$$

*c.i.* = confidence interval

$\bar{X}$ = sample mean

$Z$ = score determined by the alpha level (confidence level)

$s/\sqrt{n-1}$ = standard deviation of the sampling distribution (standard error of the mean)

$\pm Z(s/\sqrt{n-1})$ = margin of error

# Example for means: Large sample, $\sigma$ unknown

- Sample of 500 residents
- Sample mean income is \$45,000
- Sample standard deviation is \$200
- Calculate a 95% confidence interval

$$c.i. = \bar{X} \pm Z\left(\frac{s}{\sqrt{n-1}}\right) = 45{,}000 \pm 1.96\left(\frac{200}{\sqrt{500-1}}\right)$$

$$c.i. = 45{,}000 \pm 17.54$$

  – Average income is between \$44,982.46 (45,000–17.54) and \$45,017.54 (45,000+17.54)

# Example from ACS

- We are 95% certain that the confidence interval from $49,926.89 to $50,161.07 contains the true average wage and salary income for the U.S. population in 2018

```
. ***95% confidence level
. svy, subpop(if income!=. & income!=0): mean income
(running mean on estimation sample)

Survey: Mean estimation

Number of strata =   2,351      Number of obs    =     3,214,539
Number of PSUs   = 1410976      Population size  =   327,167,439
                                Subpop. no. obs  =     1,574,313
                                Subpop. size     =   163,349,075
                                Design df        =     1,408,625

--------------------------------------------------------------
             |             Linearized
             |       Mean   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
      income |   50043.98   59.74195      49926.89    50161.07
--------------------------------------------------------------

.
. ***Standard deviation
. estat sd

-------------------------------------
             |       Mean   Std. Dev.
-------------+-----------------------
      income |   50043.98    61547.67
-------------------------------------
```

Obs.: Only individuals with some wage and salary income are included (exclude those with zero income).

Source: 2018 American Community Survey.

# Edited table

**Table 1. Summary statistics for individual average wage and salary income of the U.S. population, 2018**

| Summary statistics | Value |
|---|---:|
| Mean | 50,043.98 |
| Standard deviation | 61,547.67 |
| Standard error | 59.74 |
| 95% confidence interval | |
| Lower bound | 49,926.89 |
| Upper bound | 50,161.07 |
| Sample size | 1,574,313 |

Obs.: Only individuals with some wage and salary income are included (exclude those with zero income).
Source: 2018 American Community Survey.

# Interpreting previous example

$n$ = 1,574,313; 49,926.89 $\leq \mu \leq$ 50,161.07

- **Correct:** We are 95% certain that the confidence interval contains the true value of $\mu$
  - If we selected several samples of size 1,574,313 and estimated their confidence intervals, 95% of them would contain the population mean ($\mu$)
  - The 95% confidence level refers to the success rate to estimate the population mean ($\mu$). It does not refer to the population mean itself
- **Wrong:** Since the value of $\mu$ is fixed, it is incorrect to say that there is a chance of 95% that the true value of $\mu$ is between the interval

**Source: Triola 2008, p.250–303.**

# Example from GSS

- We are 95% certain that the confidence interval from $35,324.83 to $39,889.96 contains the true average income for the U.S. adult population in 2004

```
. svy: mean conrinc, over(year)
(running mean on estimation sample)

Survey: Mean estimation

Number of strata =     307          Number of obs   =      4,522
Number of PSUs   =     597          Population size = 4,611.7099
                                    Design df       =        290

         2004: year = 2004
         2010: year = 2010
         2016: year = 2016

--------------------------------------------------------------
             |             Linearized
        Over |       Mean   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
conrinc      |
        2004 |   37607.39   1159.734      35324.83    39889.96
        2010 |   31537.11   1216.566      29142.69    33931.53
        2016 |    34649.3   1267.614      32154.41    37144.19
--------------------------------------------------------------
Note: Variance scaled to handle strata with a single sampling
      unit.
```

Source: 2004, 2010, 2016 General Social Surveys.

# Edited table

**Table 1. Mean, standard error, 95% confidence interval, and sample size of individual average income of the U.S. adult population, 2004, 2010, and 2016**

| Year | Mean | Standard Error | 95% Confidence Interval | | Sample Size |
|---|---|---|---|---|---|
| | | | Lower Bound | Upper Bound | |
| **2004** | 37,607.39 | 1,159.73 | 35,324.83 | 39,889.96 | 1,688 |
| **2010** | 31,537.11 | 1,216.57 | 29,142.69 | 33,931.53 | 1,202 |
| **2016** | 34,649.30 | 1,267.61 | 32,154.41 | 37,144.19 | 1,632 |

Source: 2004, 2010, 2016 General Social Surveys.

# Confidence intervals for sample proportions

$$c.i. = P_s \pm Z\sqrt{\frac{P_u(1-P_u)}{n}}$$

*c.i.* = confidence interval

$P_s$ = sample proportion

$Z$ = score determined by the alpha level (confidence level)

$\sqrt{P_u(1-P_u)/n}$ = standard deviation of the sampling distribution (standard error of the proportion)

$\pm Z(\sqrt{P_u(1-P_u)/n})$ = margin of error

# Note about sample proportions

- The formula for the standard error includes the population value
  - We do not know and are trying to estimate ($P_u$)
- By convention we set $P_u$ equal to 0.50
  - The numerator [$P_u(1–P_u)$] is at its maximum value
  - $P_u(1–P_u) = (0.50)(1–0.50) = 0.25$
- The calculated confidence interval will be at its maximum width
  - This is considered the most statistically conservative technique

# Example for proportions

- Estimate the proportion of students who missed at least one day of classes last semester
  - In a random sample of 200 students, 60 students reported missing one day of class last semester
  - Thus, the sample proportion is 0.30 (60/200)
  - Calculate a 95% (alpha = 0.05) confidence interval

$$c.i. = P_s \pm Z\sqrt{\frac{P_u(1-P_u)}{n}} = 0.3 \pm 1.96\sqrt{\frac{0.5(1-0.5)}{200}}$$

$$c.i. = \ 0.3 \pm 0.08$$

# Example from ACS

- We are 95% certain that the confidence interval from 5.2% to 5.3% contains the true proportion of internal migrants in the U.S. population in 2018

```
. svy: prop migrant
(running proportion on estimation sample)

Survey: Proportion estimation

Number of strata =   2,351                Number of obs   =     3,184,099
Number of PSUs   = 1410889                Population size =   323,541,502
                                          Design df       =     1,408,538

-----------------------------------------------------------------------------
                       |             Linearized            Logit
                       |  Proportion   Std. Err.     [95% Conf. Interval]
-----------------------+-----------------------------------------------------
               migrant |
           Non-migrant |    .9418963     .000259      .9413866     .9424019
      Internal migrant |    .0524799    .0002463      .0519993     .0529647
 International migrant |    .0056239    .0000823      .0054649     .0057874
-----------------------------------------------------------------------------
```

Source: 2018 American Community Survey.

# Edited table

**Table 2. Summary statistics for migration status of the U.S. population, 2018**

| Migration status | Proportion | Standard Error | 95% Confidence Interval | |
|---|---|---|---|---|
| | | | Lower Bound | Upper Bound |
| Non-migrant | 0.9419 | 0.0003 | 0.9414 | 0.9424 |
| Internal migrant | 0.0525 | 0.0003 | 0.0520 | 0.0530 |
| International migrant | 0.0056 | 0.0001 | 0.0055 | 0.0058 |

Obs.: Sample size of 3,184,099 individuals.
Source: 2018 American Community Survey.

# Interpreting previous example

$$n = 3{,}184{,}099;\ 5.2 \leq P_u \leq 5.3$$

- **Correct:** We are 95% certain that the confidence interval contains the true value of $P_u$
  - If we selected several samples of size 3,184,099 and estimated their confidence intervals, 95% of them would contain the population proportion ($P_u$)
  - The 95% confidence level refers to the success rate to estimate the population proportion ($P_u$). It does not refer to the population proportion itself
- **Wrong:** Since the value of $P_u$ is fixed, it is incorrect to say that there is a chance of 95% that the true value of $P_u$ is between the interval

**Source: Triola 2008, p.250–303.**

# Example from GSS

- We are 95% certain that the confidence interval from 2.6% to 4.7% contains the true proportion of the U.S. adult population who thinks the number of immigrants to the country should increase a lot in 2004

```
. svy: prop letin1 if year==2004
(running proportion on estimation sample)

Survey: Proportion estimation

Number of strata =     109          Number of obs   =      1,983
Number of PSUs   =     218          Population size = 1,979.3435
                                    Design df       =        109

      _prop_1: letin1 = increased a lot
      _prop_2: letin1 = increased a little
      _prop_3: letin1 = remain the same as it is
      _prop_4: letin1 = reduced a little
      _prop_5: letin1 = reduced a lot

--------------------------------------------------------------
             |             Linearized
             |  Proportion   Std. Err.     [95% Conf. Interval]
-------------+------------------------------------------------
letin1       |
     _prop_1 |    .0348265    .005221      .0258369    .0467936
     _prop_2 |    .0653852   .0060495      .0543699     .078447
     _prop_3 |    .3517117   .0128957      .3265967    .3776749
     _prop_4 |    .2829629   .0118188      .2601357    .3069621
     _prop_5 |    .2651137   .0127052      .2407073    .2910462
--------------------------------------------------------------
```

Source: 2004 General Social Survey.

# Edited table

**Table 2. Proportion, standard error, 95% confidence interval, and sample size of opinion of the U.S. adult population about how should the number of immigrants to the country be nowadays, 2004, 2010, and 2016**

| Opinion About Number of Immigrants | Proportion | Standard Error | 95% Confidence Interval | | Sample Size |
|---|---|---|---|---|---|
| | | | Lower Bound | Upper Bound | |
| **2004** | | | | | 1,983 |
| Increase a lot | 0.0348 | 0.0052 | 0.0258 | 0.0468 | |
| Increase a little | 0.0654 | 0.0060 | 0.0544 | 0.0784 | |
| Remain the same | 0.3517 | 0.0129 | 0.3266 | 0.3777 | |
| Reduce a little | 0.2830 | 0.0118 | 0.2601 | 0.3070 | |
| Reduce a lot | 0.2651 | 0.0127 | 0.2407 | 0.2910 | |
| **2010** | | | | | 1,393 |
| Increase a lot | 0.0426 | 0.0061 | 0.0320 | 0.0564 | |
| Increase a little | 0.0944 | 0.0096 | 0.0771 | 0.1152 | |
| Remain the same | 0.3589 | 0.0166 | 0.3268 | 0.3923 | |
| Reduce a little | 0.2452 | 0.0121 | 0.2220 | 0.2700 | |
| Reduce a lot | 0.2588 | 0.0146 | 0.2310 | 0.2887 | |
| **2016** | | | | | 1,845 |
| Increase a lot | 0.0586 | 0.0069 | 0.0462 | 0.0740 | |
| Increase a little | 0.1163 | 0.0091 | 0.0993 | 0.1358 | |
| Remain the same | 0.4028 | 0.0117 | 0.3797 | 0.4264 | |
| Reduce a little | 0.2305 | 0.0097 | 0.2118 | 0.2504 | |
| Reduce a lot | 0.1918 | 0.0101 | 0.1724 | 0.2128 | |

Source: 2004, 2010, 2016 General Social Surveys.

# Width of confidence interval

- The width of confidence intervals can be controlled by manipulating the confidence level
  - The confidence level increases
  - The alpha decreases
  - The *Z* score increases
  - The confidence interval is wider

**Example: $\bar{X}$ = \$45,000; *s* = \$200; *n* = 500**

| Confidence level | Alpha ($\alpha$) | *Z* score | Confidence interval | Interval width |
|---:|---:|---:|---:|---:|
| 90% | 0.10 | $\pm$1.65 | \$45,000 $\pm$ \$14.77 | \$29.54 |
| 95% | 0.05 | $\pm$1.96 | \$45,000 $\pm$ \$17.54 | \$35.08 |
| 99% | 0.01 | $\pm$2.58 | \$45,000 $\pm$ \$23.09 | \$46.18 |
| 99.9% | 0.001 | $\pm$3.32 | \$45,000 $\pm$ \$29.71 | \$59.42 |

# Width of confidence interval

- The width of confidence intervals can be controlled by manipulating the sample size
  - The sample size increases
  - The confidence interval is narrower

**Example: $\bar{X}$ = \$45,000; *s* = \$200; *α* = 0.05**

| *n* | Confidence interval | Interval width |
|---|---|---|
| 100 | *c.i.* = \$45,000 ± 1.96(200/√99) = \$45,000 ± \$39.40 | \$78.80 |
| 500 | *c.i.* = \$45,000 ± 1.96(200/√499) = \$45,000 ± \$17.55 | \$35.10 |
| 1000 | *c.i.* = \$45,000 ± 1.96(200/√999) = \$45,000 ± \$12.40 | \$24.80 |
| 10000 | *c.i.* = \$45,000 ± 1.96(200/√9999) = \$45,000 ± \$3.92 | \$7.84 |

# Summary: Confidence intervals

- Sample means, large samples (*n*>100), population standard deviation known

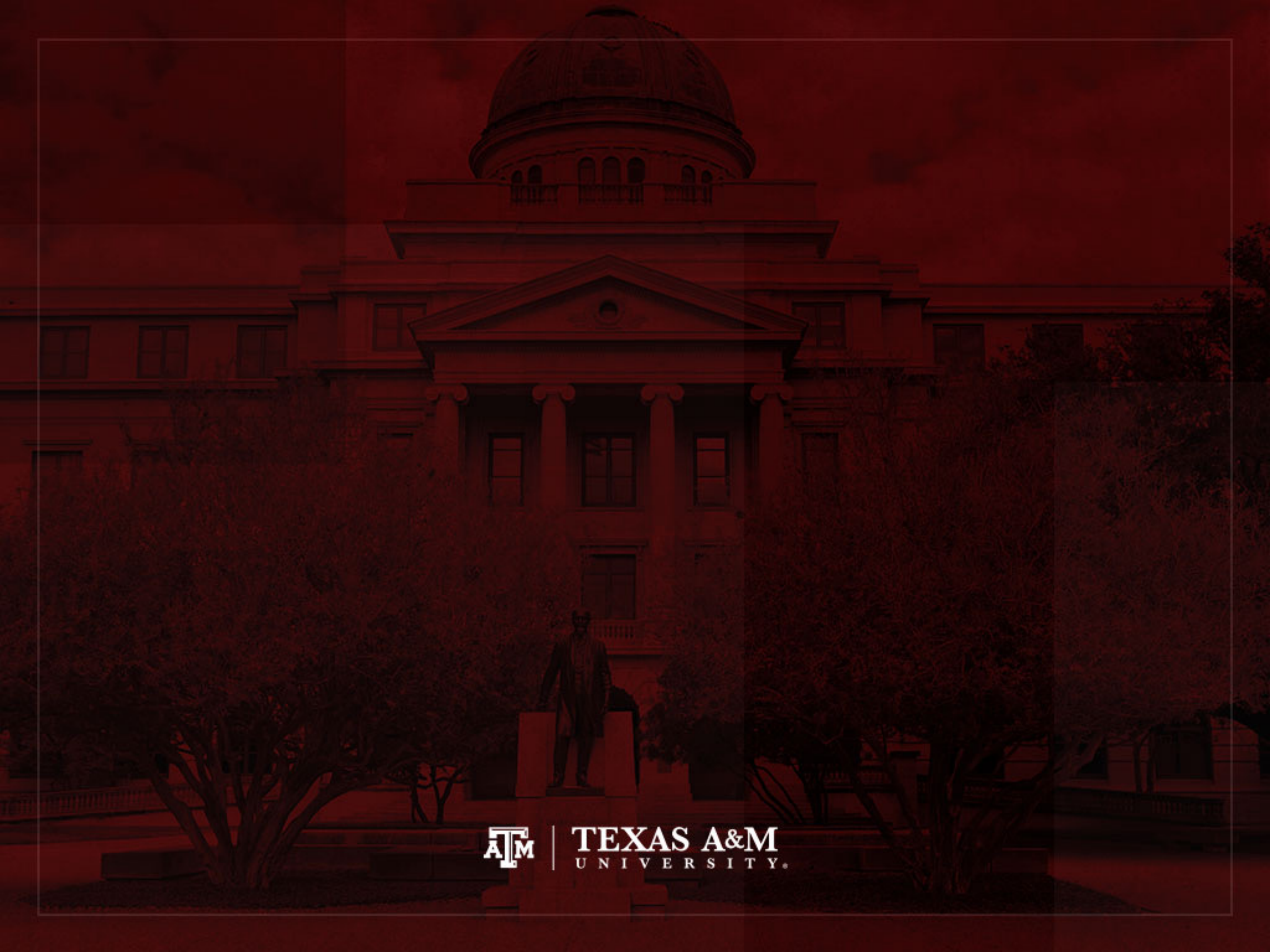
$$c.i. = \bar{X} \pm Z\left(\frac{\sigma}{\sqrt{n}}\right)$$

- Sample means, large samples (*n*>100), population standard deviation unknown

$$c.i. = \bar{X} \pm Z\left(\frac{s}{\sqrt{n-1}}\right)$$

- Sample proportions, large samples (*n*>100)

$$c.i. = P_s \pm Z\sqrt{\frac{P_u(1-P_u)}{n}}$$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