Lecture 5a: Inferential statistics

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Introduction to Sociological Data Analysis (SOCI 600)

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Source: Healey, Joseph F. 2015. "Statistics: A Tool for Social Research." Stamford: Cengage Learning. 10th edition. Chapter 6 (pp. 144–159).

TEXAS A&M UNIVERSITY

Outline

- Explain the purpose of inferential statistics in terms of generalizing from a sample to a population
- Define and explain the basic techniques of random sampling
- Explain and define these key terms: population, sample, parameter, statistic, representative, EPSEM sampling techniques
- Differentiate between the sampling distribution, the sample, and the population
- Explain two theorems

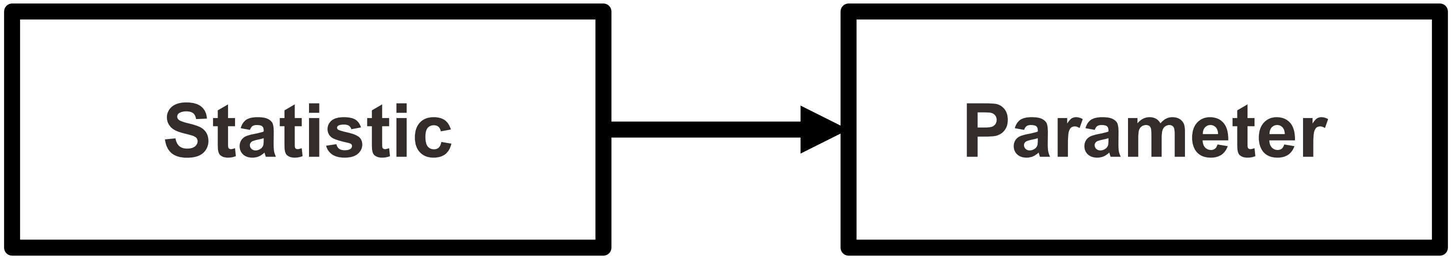
Basic logic and terminology

- **Problem**
- The populations we wish to study are almost always so large that we are unable to gather information from every case

- **Solution**
- We choose a sample – a carefully chosen subset of the population – and use information gathered from the cases in the sample to generalize to the population

Basic logic and terminology

- **Statistics** are mathematical characteristics of samples
- **Parameters** are mathematical characteristics of populations
- **Statistics** are used to estimate **parameters**

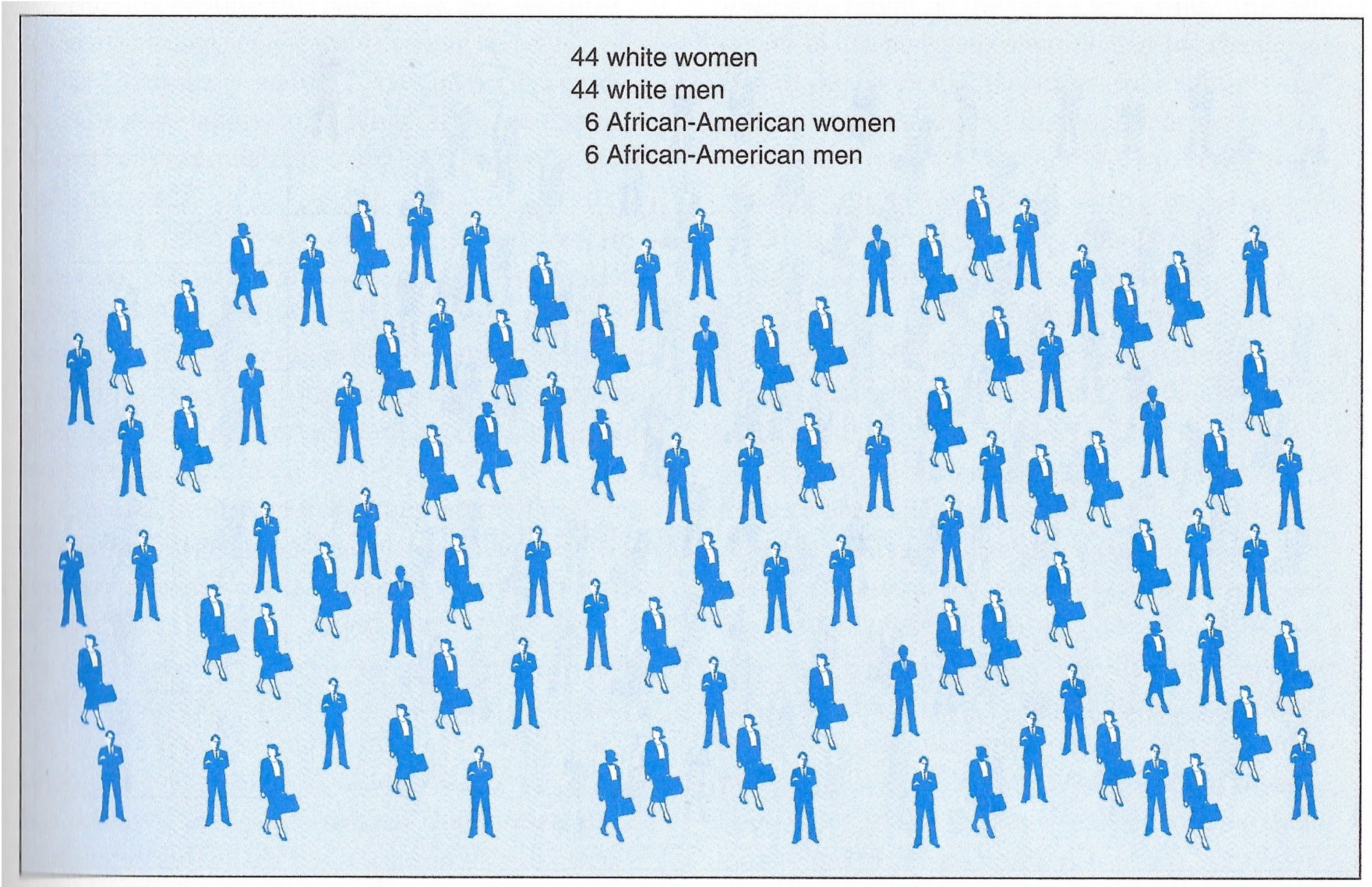


Samples

- Must be representative of the population
  - Representative: The sample has the same characteristics as the population

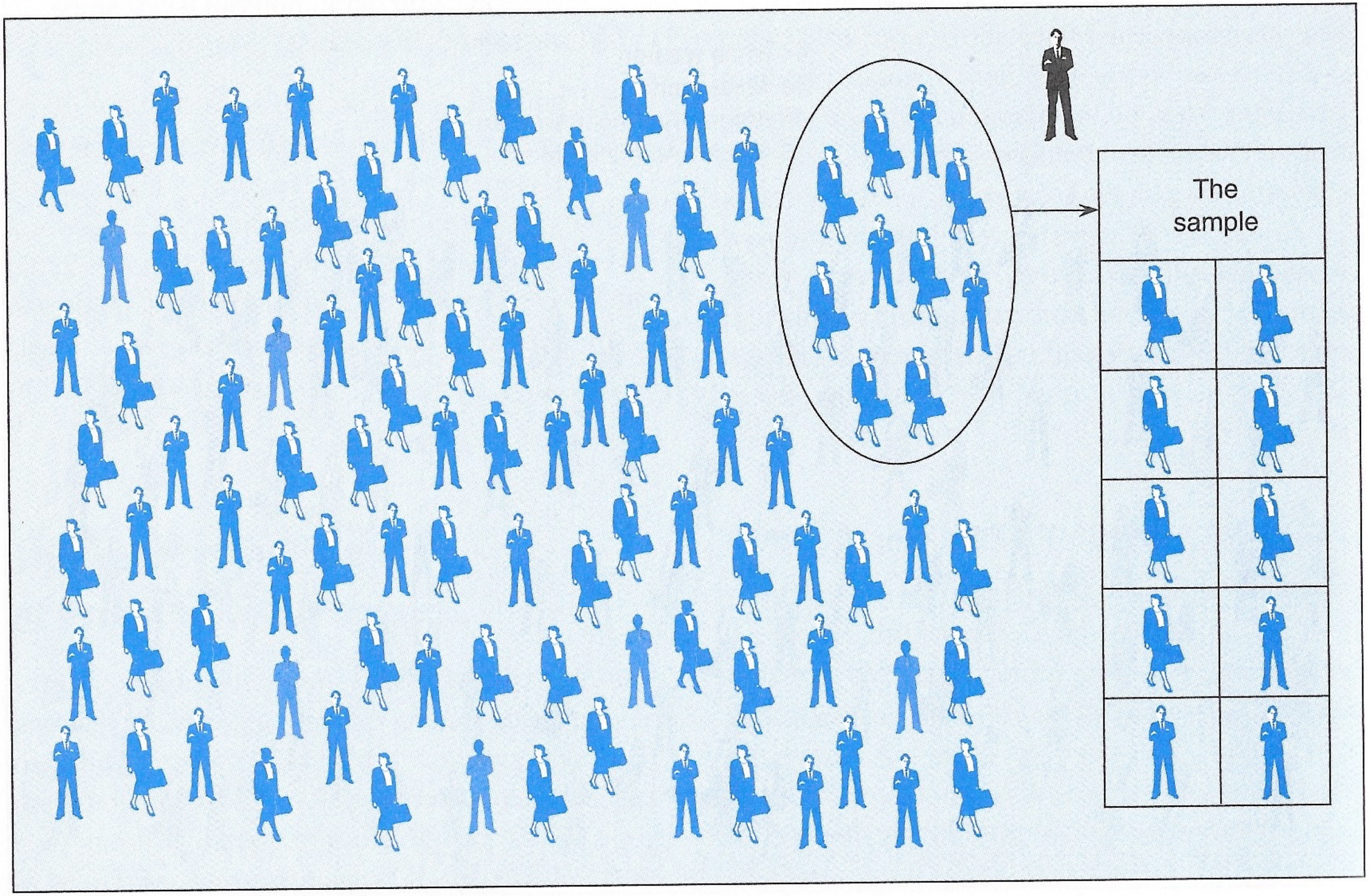
- How can we ensure samples are representative?
  - Samples drawn according to the rule of **EPSEM** (**e**qual **p**robability of **s**election **m**ethod)
  - If every case in the population has the same chance of being selected, the sample is likely to be representative

A population of 100 people



Source: Babbie 2001, p.183.

Nonprobability sampling



Source: Babbie 2001, p.184.

EPSEM sampling techniques

1. Simple random sampling

2. Systematic sampling

3. Stratified sampling

4. Cluster sampling

1. Simple random sampling

- To begin, we need
  - A list of the population
- Then, we need a method for selecting cases from the population, so each case has the same probability of being selected
  - The principle of EPSEM
  - A sample selected this way is very likely to be representative of the population
  - Variable in population should have a normal distribution or n>30

Example

- You want to know what percent of students at a large university work during the semester

- Draw a sample size (*n*) of 500 from a list of all students (*N*=20,000)

- Assume the list is available from the Registrar

- How can you draw names, so every student has the same chance of being selected?

Example

- Each student has a unique, 6 digit ID number that ranges from 000001 to 999999
- Use a table of random numbers or a computer program to select 500 ID numbers with 6 digits each
- Each time a randomly selected 6 digit number matches the ID of a student, that student is selected for the sample
- Continue until 500 names are selected

Example

- **Stata**

```
set obs 500
generate student = runiformint(1,999999)
sum student
    Variable |        Obs        Mean    Std. Dev.       Min        Max
-------------+---------------------------------------------------------
     student |        500    482562.6    283480.9       3652     997200
```

- **Excel**
  - RANDBETWEEN (minimum,maximum)
    - Returns a random number between those you specify
    - Drag the function to 500 cells

=RANDBETWEEN(1,999999)

  - RANDARRAY (rows,columns,minimum,maximum)

=RANDARRAY(500,1,1,999999)

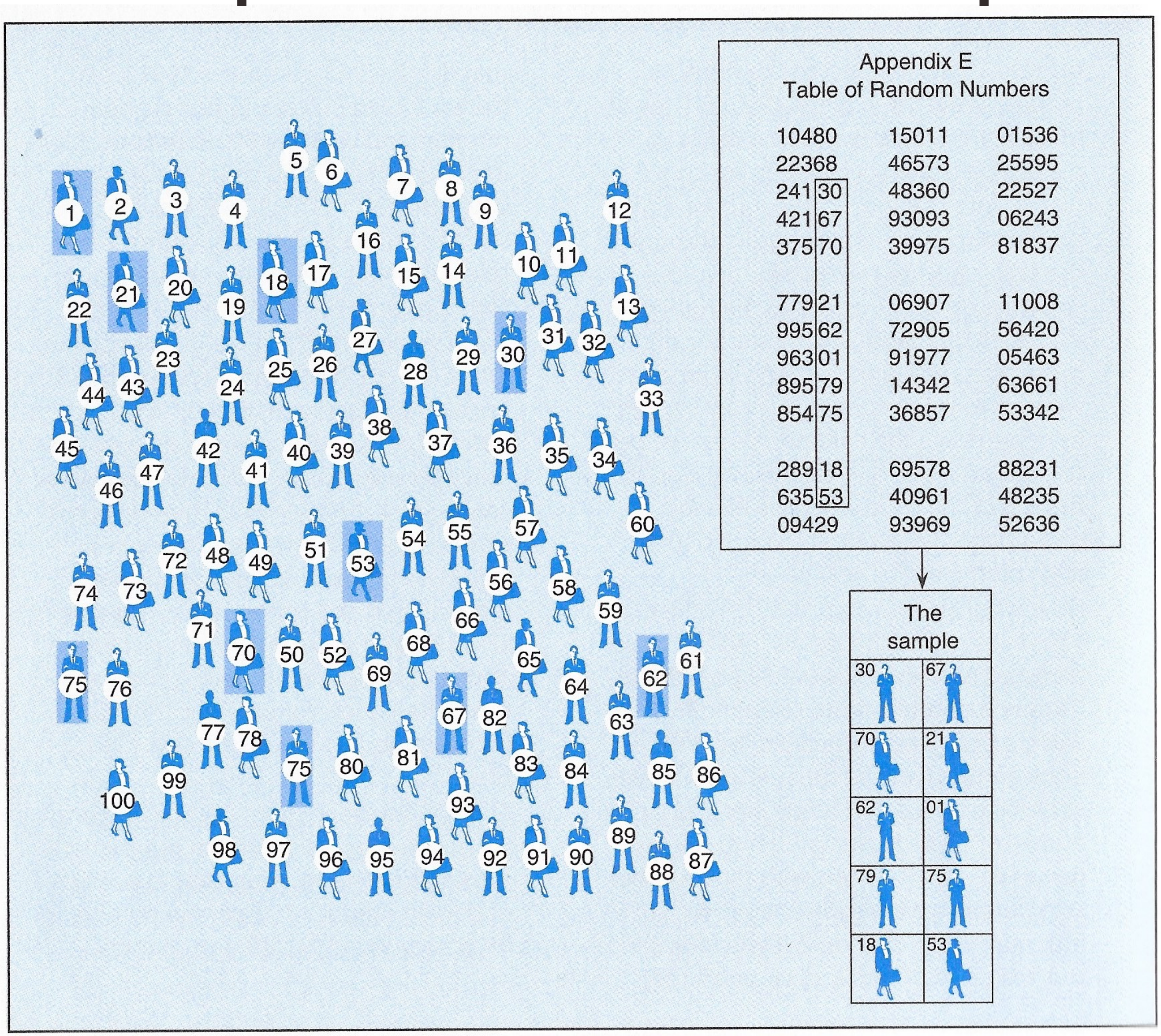
Example

- Disregard duplicate numbers
- Ignore cases in which no student ID matches the randomly selected number
- After questioning each of these 500 students, you find that 368 (74%) work during the semester

Applying logic and terminology

- In the previous example:
- **Population:** All 20,000 students
- **Sample:** 500 students selected and interviewed
- **Statistic:** 74% (percentage of sample that held a job during the semester)
- **Parameter:** Percentage of all students in the population who held a job

Simple random sample



Source: Babbie 2001, p.200.

2. Systematic sampling

- Useful for large populations
- Randomly select the first case then select every k^{th} case
- **Sampling interval**
  - Distance between elements selected in the sample
  - Population size (*N*) divided by sample size (*n*)
- **Sampling ratio**
  - Proportion of selected elements in the population
  - Sample size (*n*) divided by population size (*N*)
- Can be problematic if the list of cases is not truly random or demonstrates some patterning

Source: Babbie 2001, p.197–198.

Example

- If a list contained 10,000 elements and we want a sample of 1,000
- Sampling interval
  - Population size / sample size = 10,000 / 1,000 = 10
  - We would select every 10th element for our sample
- Sampling ratio
  - Sample size / population size = 1,000 / 10,000 = 1/10
  - Proportion of selected elements in population
- Select the first element at random

Source: Babbie 2001, p.197–198.

3. Stratified sampling

- It guarantees the sample will be representative on the selected (stratifying) variables
  - Stratification variables relate to research interests
- First, divide the population list into subsets, according to some relevant variable
  - **Homogeneity within subsets**
    - E.g., only women in a subset; only men in another subset
  - **Heterogeneity between subsets**
    - E.g., subset of women is different than subset of men
- Second, sample from the subsets
  - Select the number of cases from each subset proportional to the population

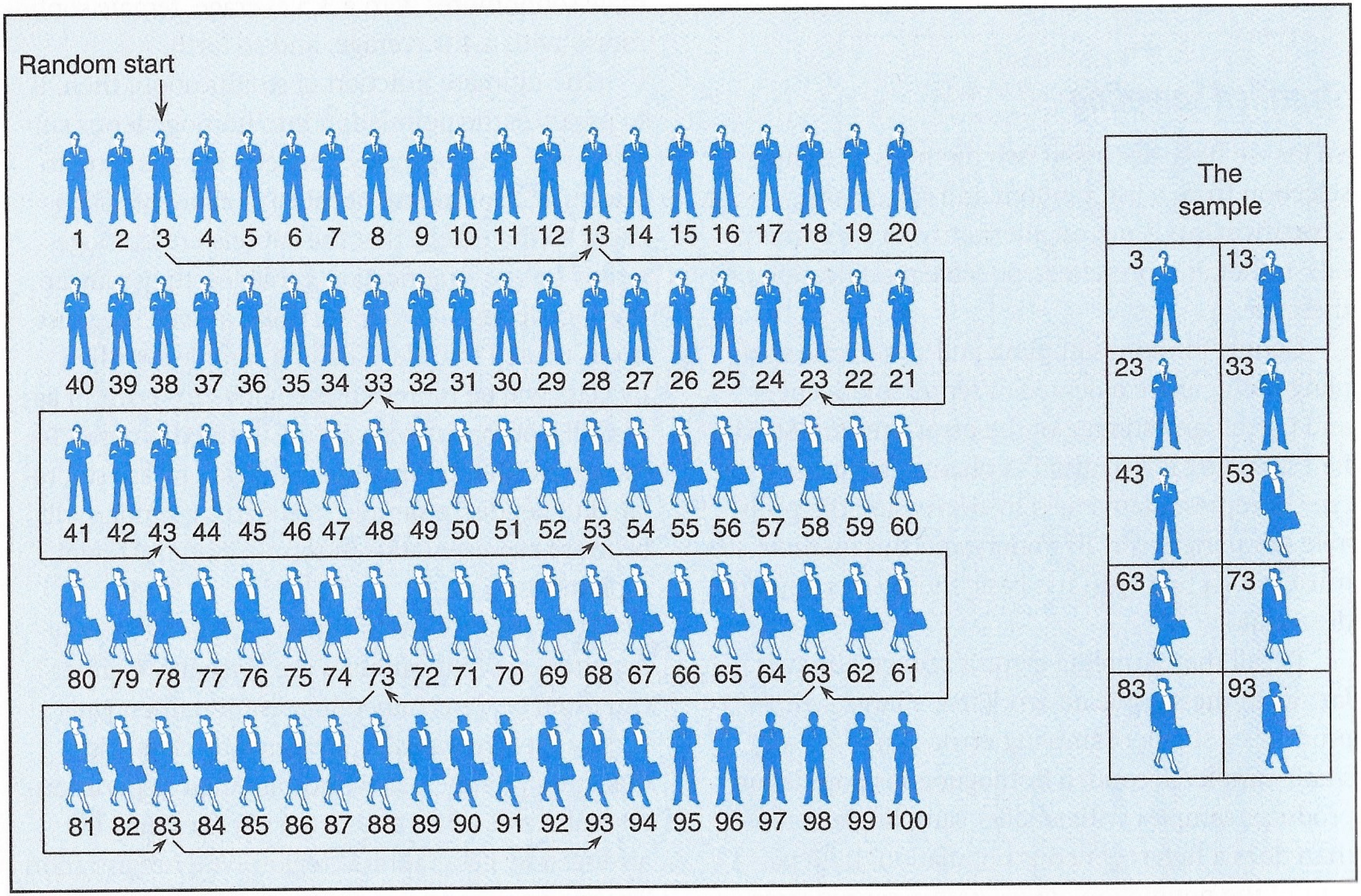
Example

- If you want a sample of 1,000 students
  - That would be representative to the population of students by sex and GPA

- You need to know the population composition
  - E.g., women with a 4.0 average compose 15 percent of the student population

- Your sample should follow that composition
  - In a sample of 1,000 students, you would select 150 women with a 4.0 average

Stratified, systematic sample



Source: Babbie 2001, p.202.

4. Cluster sampling

- Select groups (or clusters) of cases rather than single cases
  - **Heterogeneity within subsets**
    - E.g., each subset has both women and men, following same proportional distribution as population
  - **Homogeneity between subsets**
    - E.g., all subsets with both women and men should be similar
- Clusters are often geographically based
  - For example, cities or voting districts
- Sampling often proceeds in stages
  - Multi-stage cluster sampling
  - Less representative than simple random sampling

Stratified vs. cluster sampling

- **Stratified**
  - Homogeneity within subsets
  - Heterogeneity between subsets
  - Select cases from each subset

Subset of women	Subset of men

- **Cluster**
  - Heterogeneity within subsets (groups, clusters, areas)
  - Homogeneity between subsets
  - Select groups (e.g., area 1) rather than single cases

Area 1: women & men	Area 2: women & men

Sampling distribution

- Sampling distribution is the probabilistic distribution of a statistic for all possible samples of a given size (*n*)
  - It is the distribution of a statistic (e.g., proportion, mean) for all possible outcomes of a certain size
- Central tendency and dispersion
  - Mean is the same as the population mean
  - Standard deviation is referred as standard error
    - It is the population standard deviation divided by the square root of *n*
    - We have to take into account the complex survey design to estimate the standard error (**`svyset`** command in Stata)

Linking sample and population

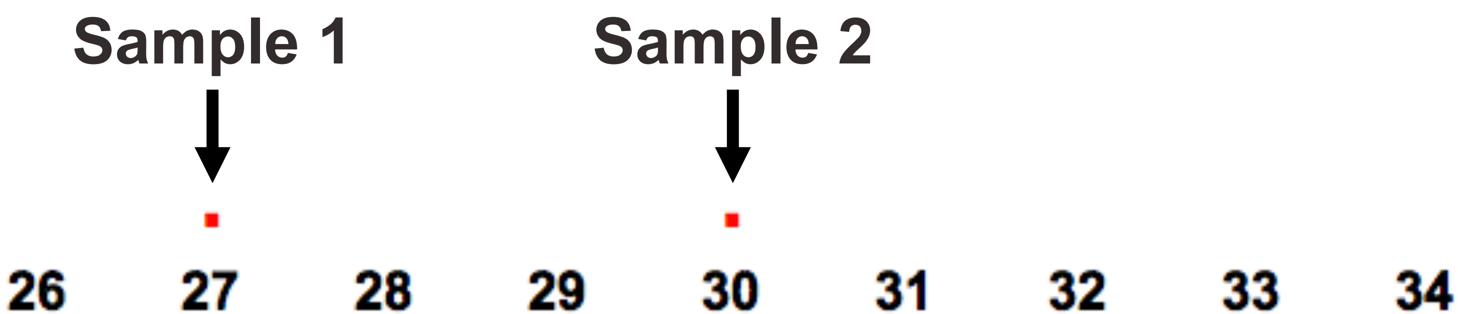
- Every application of inferential statistics involves three different distributions
  - Population: empirical; unknown
  - Sampling distribution: theoretical; known
  - Sample: empirical; known

- In inferential statistics, the sample distribution links the sample with the population

Example

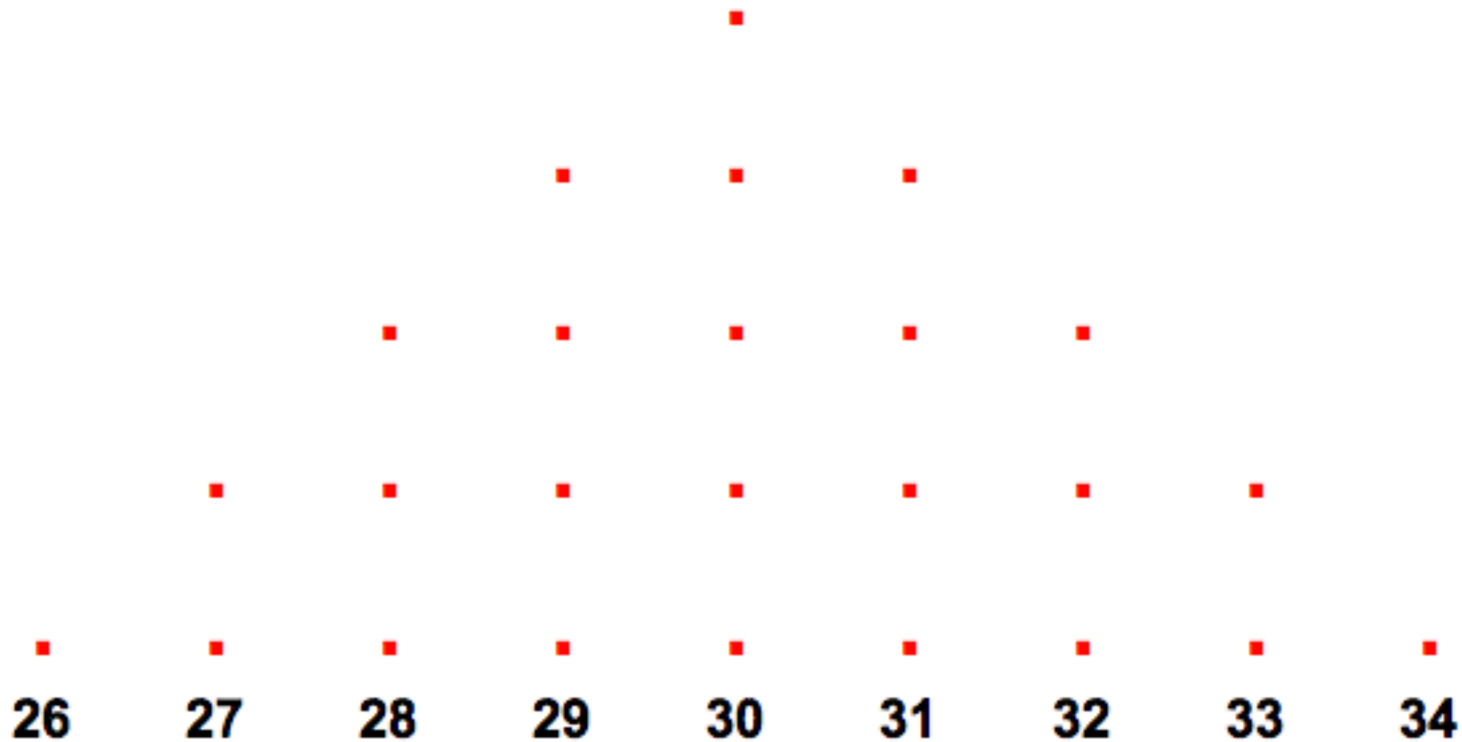
- Suppose we want to gather information on the age of a community of 10,000 individuals
  - Sample 1: *n*=100 people, plot sample's mean of 27
  - Replace people in the sample back to the population

  - Sample 2: *n*=100 people, plot sample's mean of 30
  - Replace people in the sample back to the population

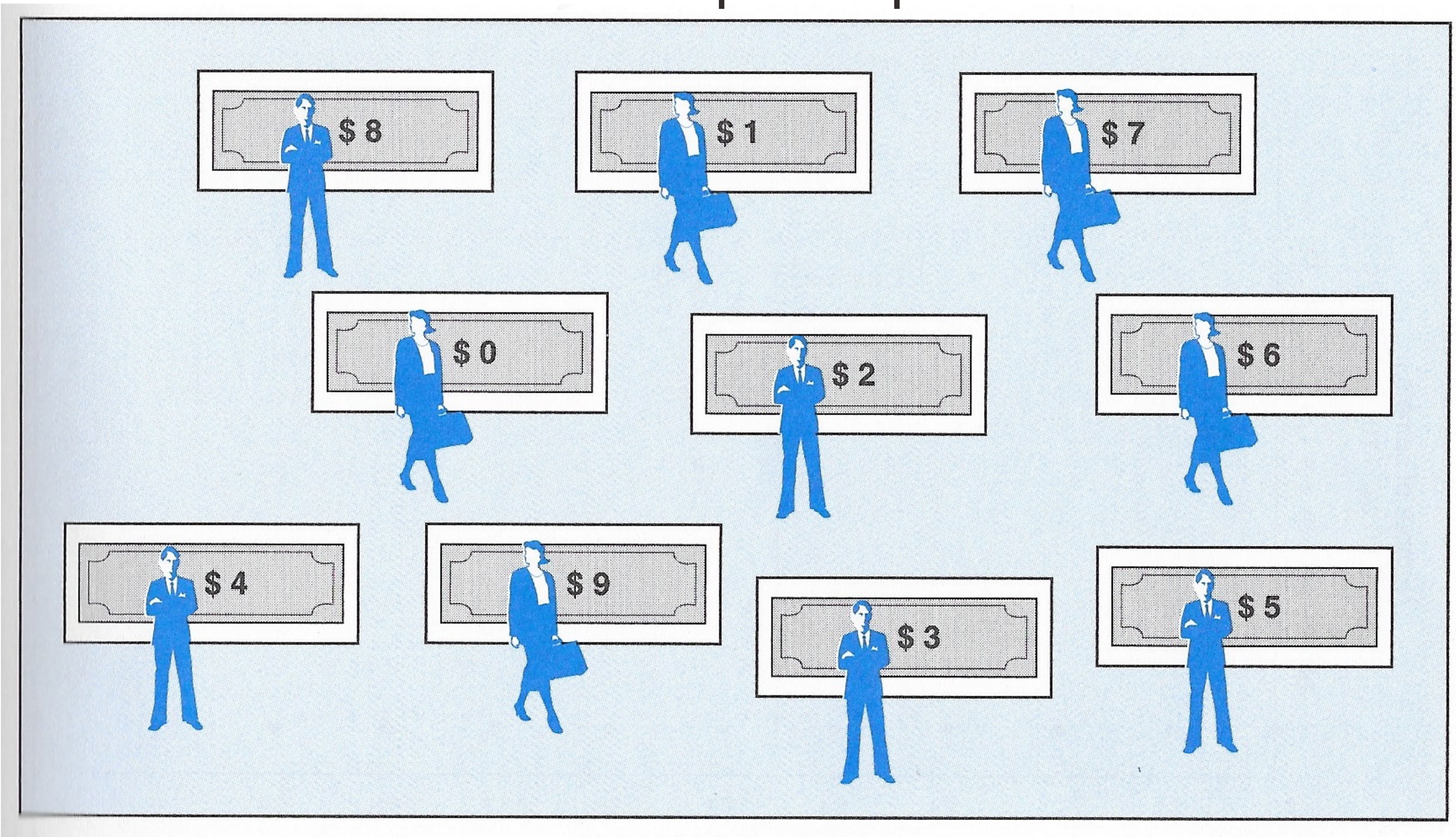


Example

- We repeat this procedure: sampling, replacing
  - Until we have exhausted every possible combination of 100 people from the population of 10,000
  - Sampling distribution has a normal shape

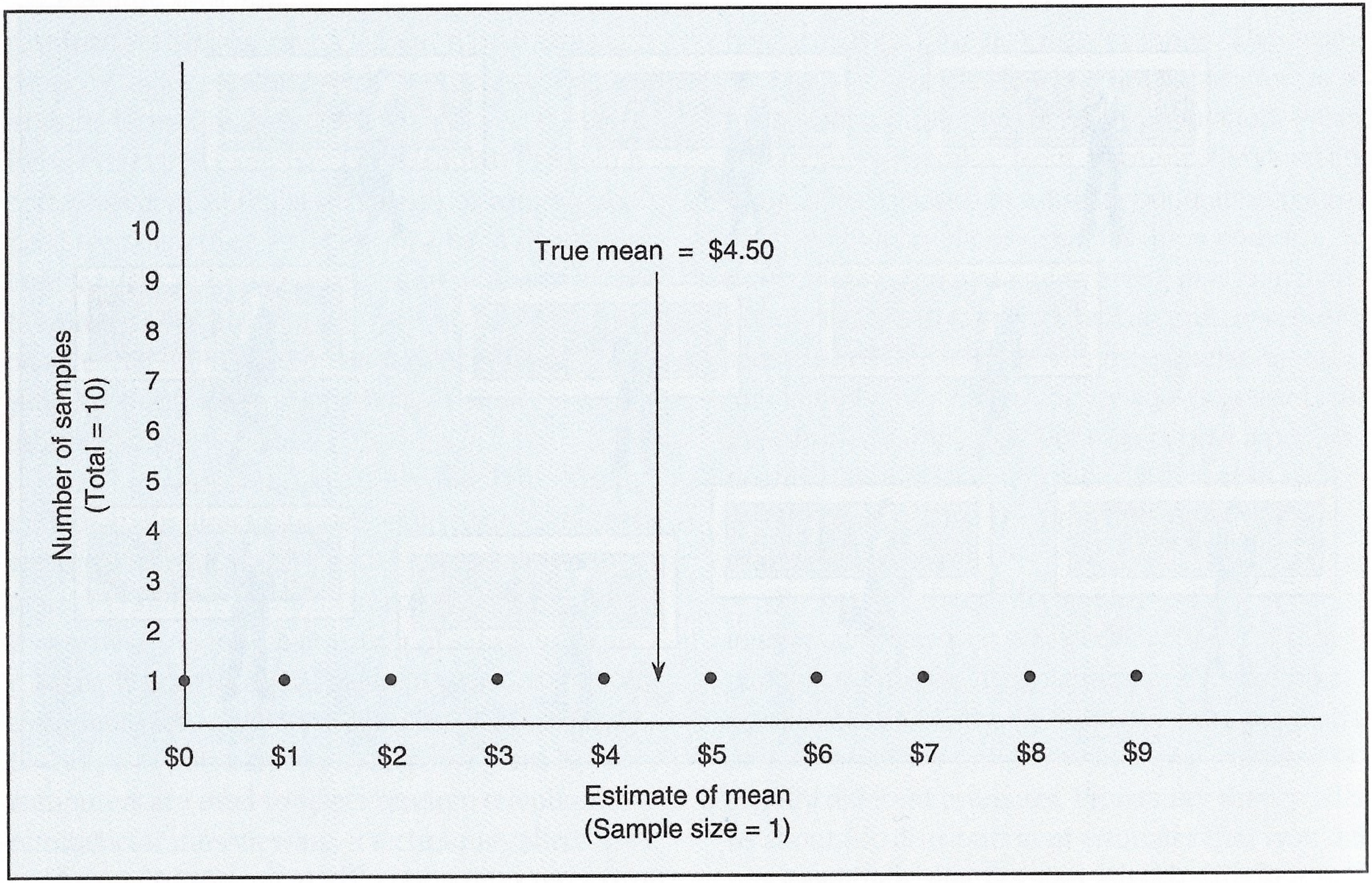


Another example: A population of 10 people with $0–$9



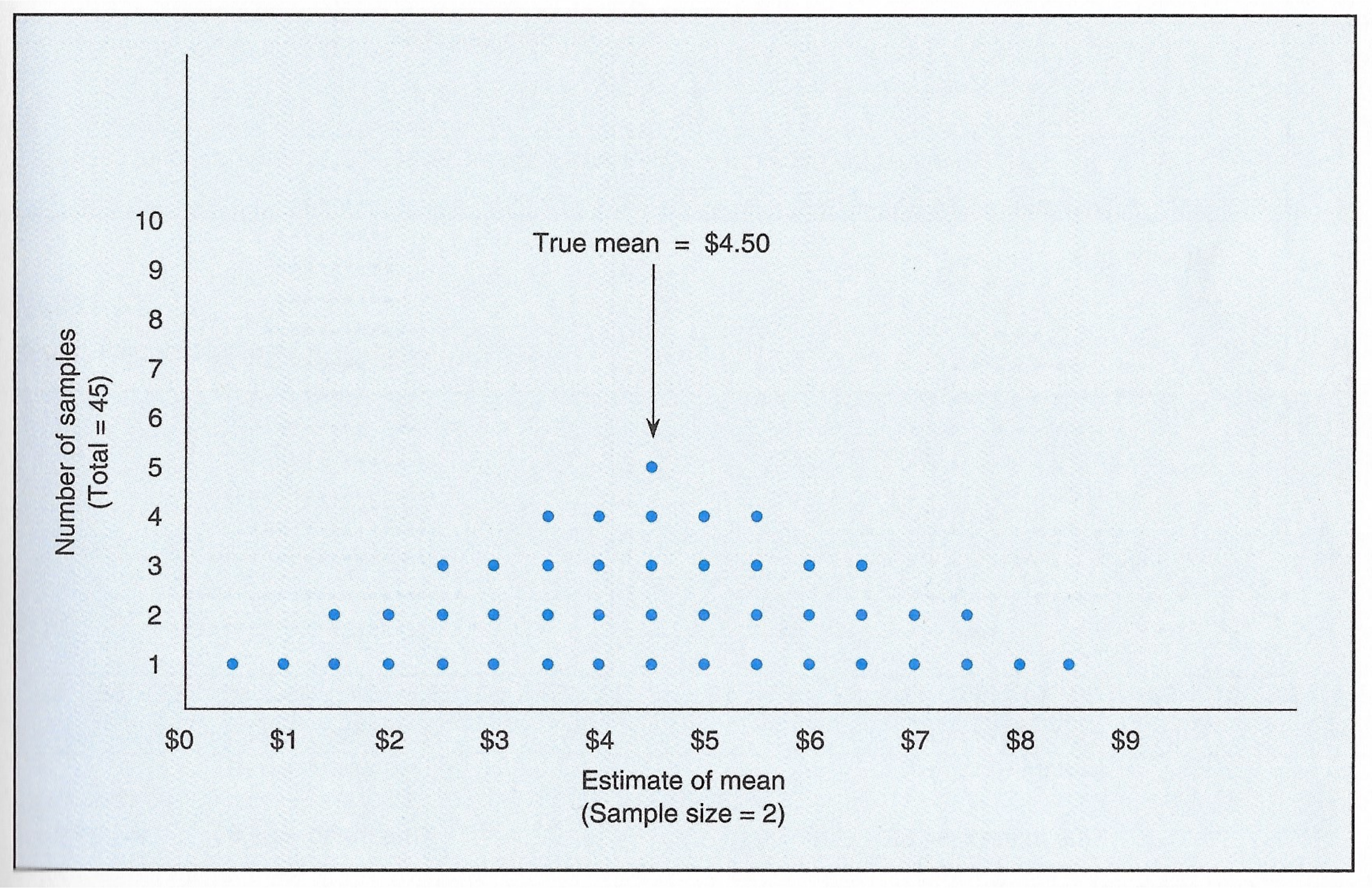
Source: Babbie 2001, p.187.

The sampling distribution (n=1)



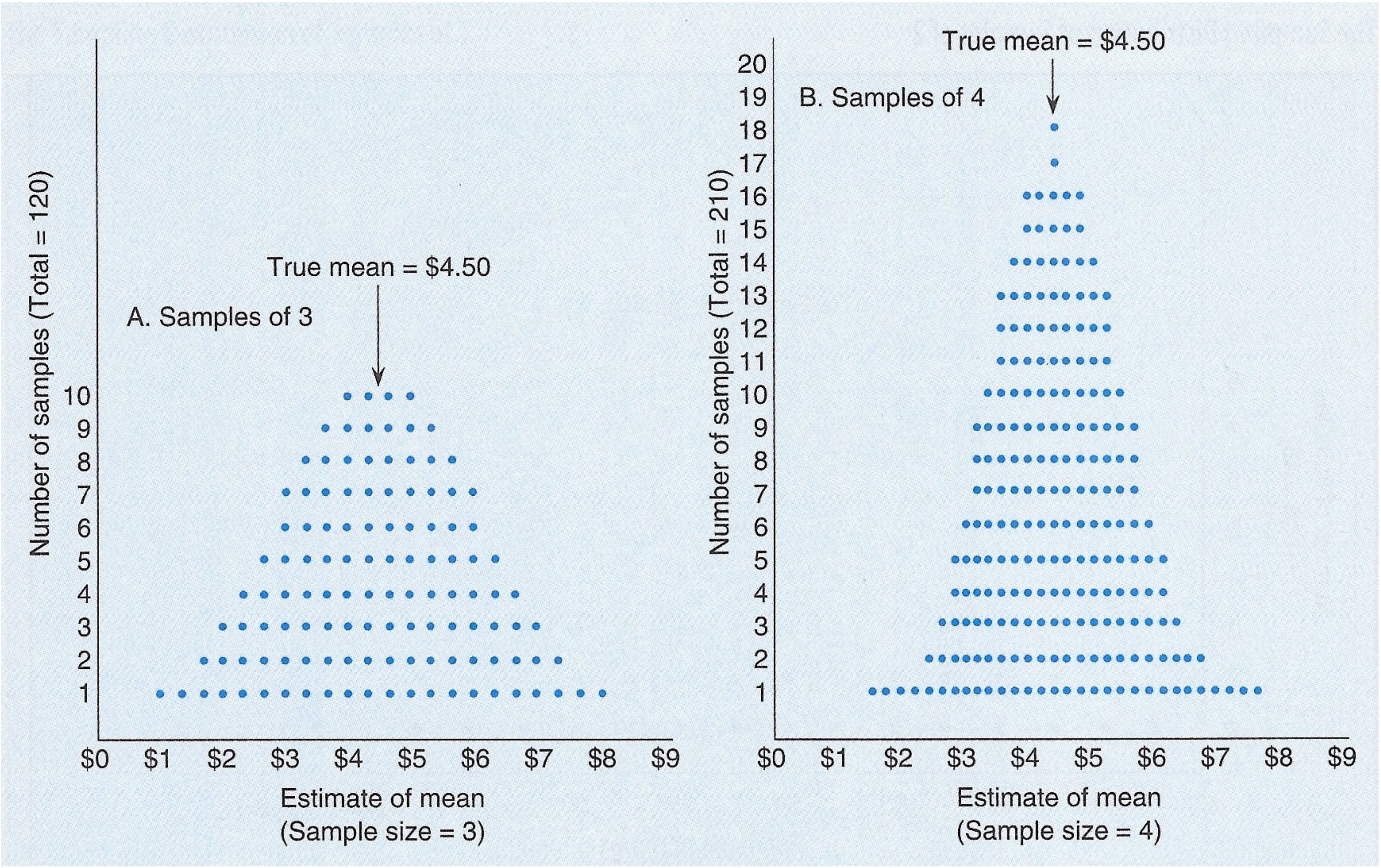
Source: Babbie 2001, p.188.

The sampling distribution (*n*=2)



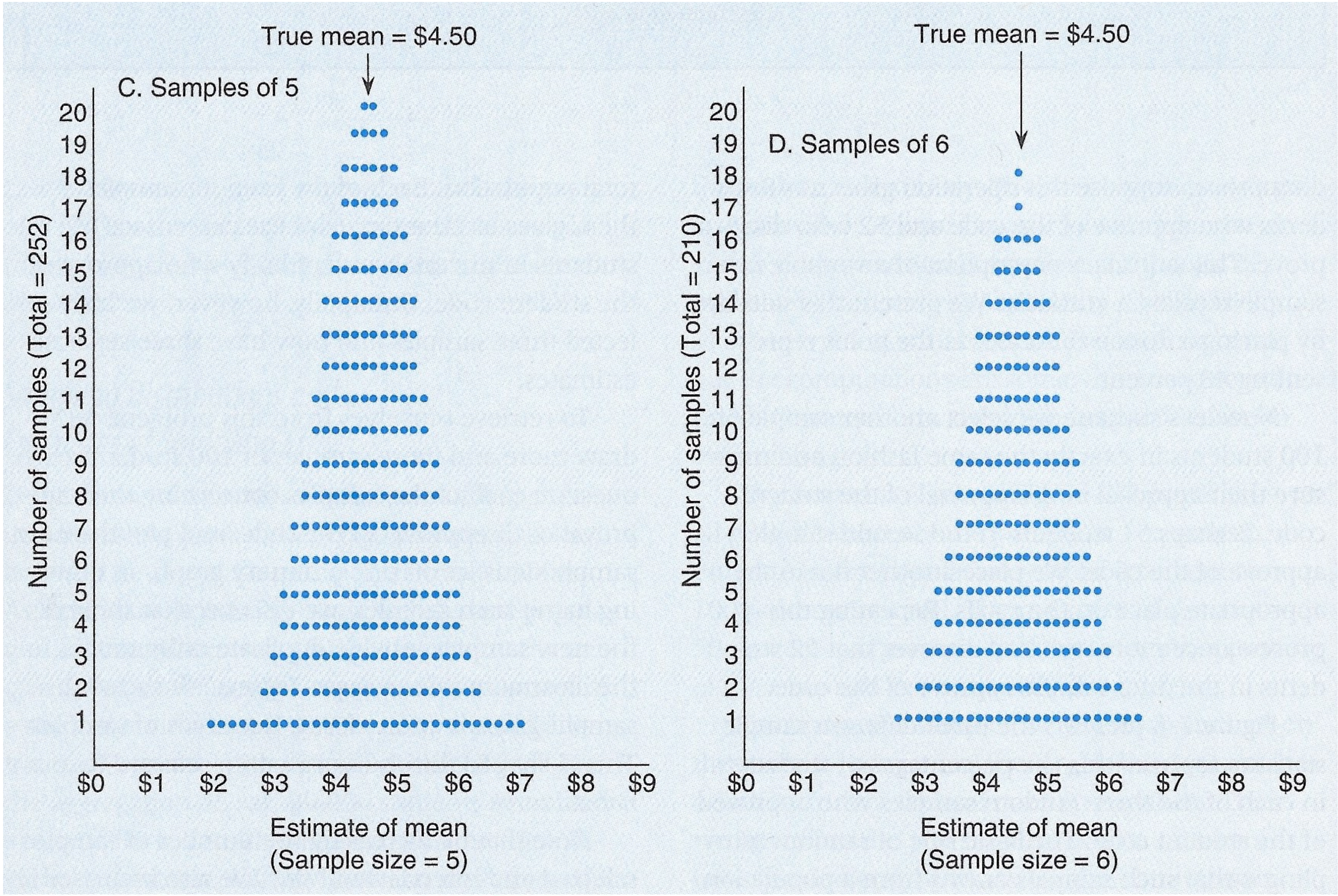
Source: Babbie 2001, p.189.

The sampling distribution



Source: Babbie 2001, p.190.

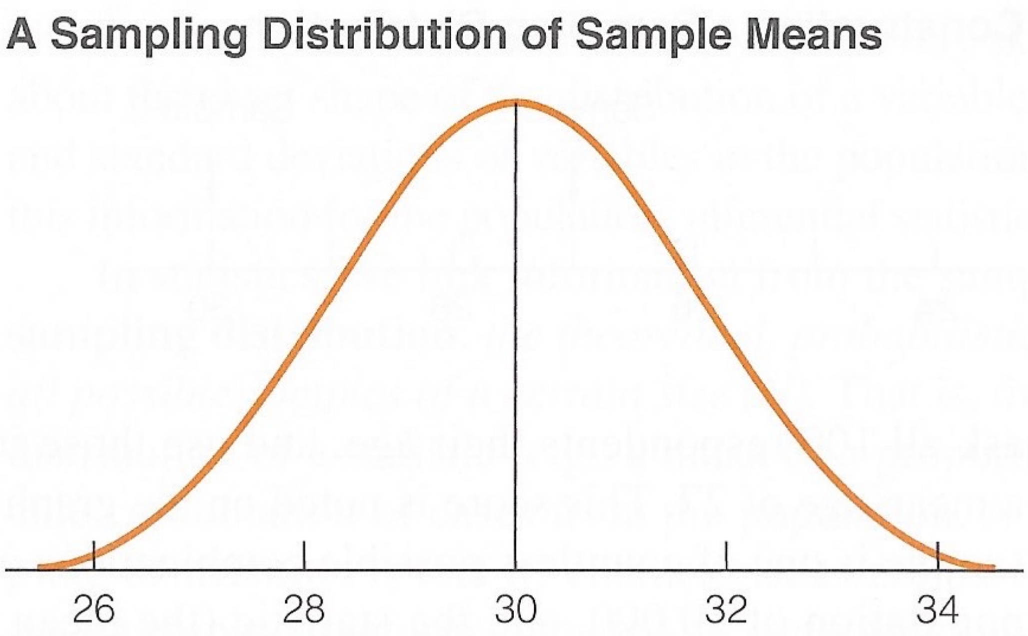
The sampling distribution



Source: Babbie 2001, p.190.

Properties of sampling distribution

- It has a mean ($\mu_{\bar{X}}$) equal to the population mean (μ)
- It has a standard deviation (standard error, $\sigma_{\bar{X}}$) equal to the population standard deviation (σ) divided by the square root of *n*
- It has a normal distribution



Source: Healey 2015, p.152.

First theorem

- Tells us the shape of the sampling distribution and defines its mean and standard deviation

- If repeated random samples of size *n* are drawn from a **normal population** with mean μ and standard deviation σ
  - Then, the sampling distribution of sample means will **have a normal distribution** with...
  - A mean: $\mu_{\bar{X}} = \mu$
  - A standard error of the mean: $\sigma_{\bar{X}} = \sigma/\sqrt{n}$

First theorem

- Begin with a characteristic that is normally distributed across a population (IQ, height)
- Take an infinite number of equally sized random samples from that population
- The sampling distribution of sample means will be normal

Central limit theorem

- If repeated random samples of size *n* are drawn from **any population** with mean μ and standard deviation σ
  - Then, as *n* becomes large, the sampling distribution of sample means will **approach normality** with...
  - A mean: $\mu_{\bar{X}} = \mu$
  - A standard error of the mean: $\sigma_{\bar{X}} = \sigma/\sqrt{n}$
- This is true for any variable, even those that are not normally distributed in the population
  - As sample size grows larger, the sampling distribution of sample means will become normal in shape

Central limit theorem

- The importance of the central limit theorem is that it removes the constraint of normality in the population
  - Applies to large samples ($n \geq 100$)

- If the sample is small ($n < 100$)
  - We must have information on the normality of the population before we can assume the sampling distribution is normal

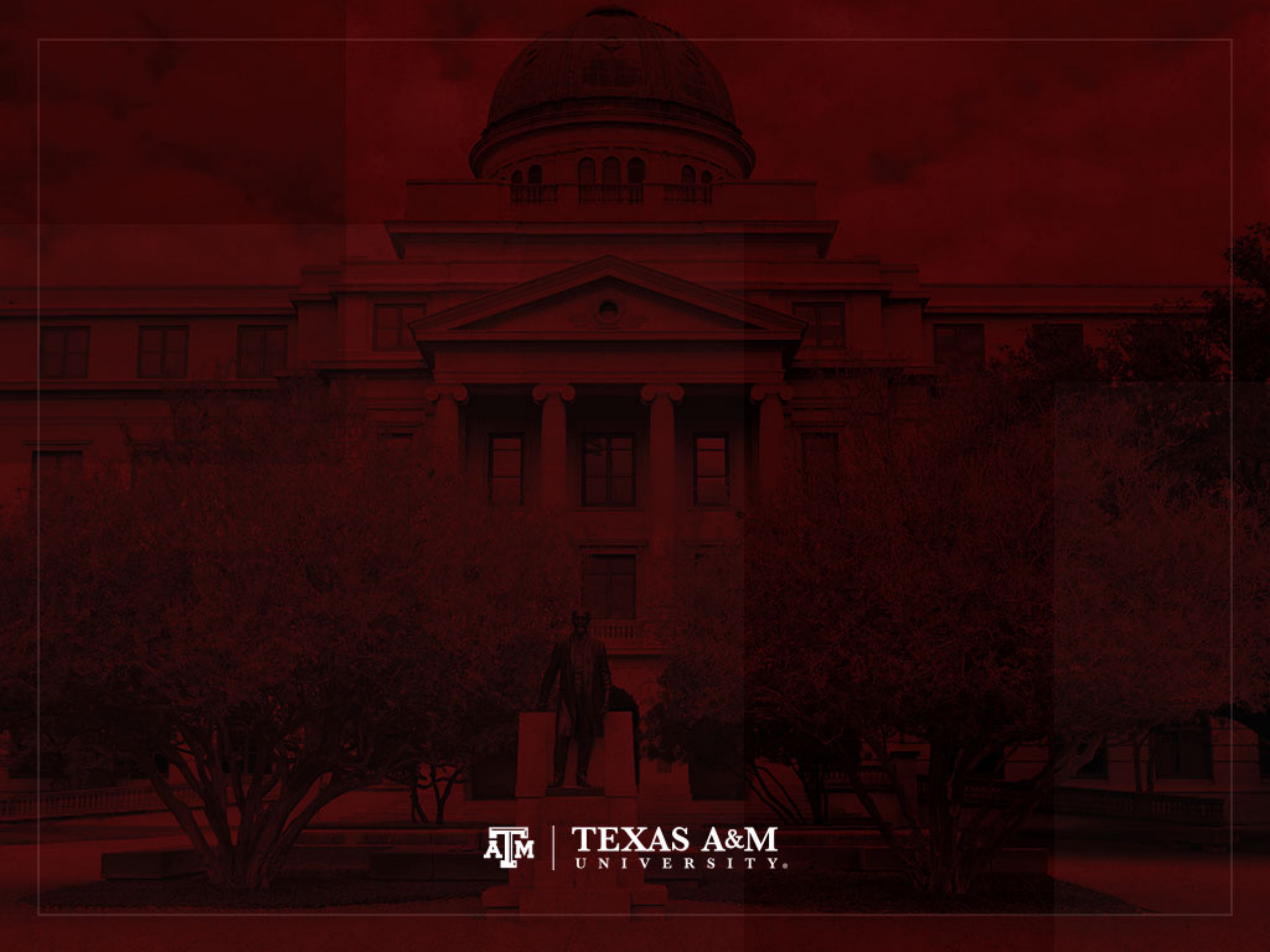
Additional considerations

- The sampling distribution is normal
  - We can estimate areas under the curve (Appendix A)
  - Or in Stata: `display normal(z)`
- We do not know the value of the population mean (μ)
  - But the mean of the sampling distribution ($\mu_{\bar{X}}$) is the same value as μ
- We do not know the value of the population standard deviation (σ)
  - But the standard deviation of the sampling distribution ($\sigma_{\bar{X}}$) is equal to σ divided by the square root of n

Symbols

Distribution	Shape	Mean	Standard deviation	Proportion
Samples	Varies	$\bar{X}$	s	P_s
Populations	Varies	μ	σ	P_u
Sampling distributions	Normal	$\mu_{\bar{X}}$		
of means		$\mu_{\bar{X}}$	$\sigma_{\bar{X}} = \sigma/\sqrt{n}$	
of proportions		μ_p	σ_p	

Source: Healey 2015, p.157.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