CHAPTER

13

The Life Table

HALLIE J. KINTNER

NATURE AND USE OF LIFE TABLES

Some of the measures of mortality discussed in Chapter 12 are associated with a statistical model known as a life table. A life table is designed essentially to measure mortality, but various specialists employ it in a variety of ways. Public health workers, demographers, actuaries, economists, and many others use life tables in studies of longevity, fertility, migration, and population growth, as well as in making projections of population size and characteristics and in studies of widowhood, orphanhood, length of married life, length of working life, and length of disability-free life. In its simplest form, an entire life table is generated from age-specific mortality rates, and the resulting values are used to measure mortality, survivorship, and life expectation. In other applications, the mortality rates in the life table are combined with other demographic data into a more complex model that measures the combined effect of mortality and changes in one or more socioeconomic characteristics (e.g., a table of working life, which combines mortality rates and labor force participation ratios and measures their combined effect on working life).

Life tables are, in essence, one form of combining mortality rates of a population at different ages into a single statistical model. They are principally used to measure the level of mortality of the population involved. One of their main advantages over other methods of measuring mortality is that they do not reflect the effects of the age distribution of an actual population and do not require the adoption of a standard population for acceptable comparisons of levels of mortality in different populations. Another is that a life table readily permits making mortality allowances for age cohorts, eliminating the burdensome task of compiling death statistics for age cohorts from annual death statistics by age, even when the latter are available.

TYPES OF LIFE TABLES

Life tables differ in several ways, including the reference year of the table, the age detail, and the number of factors comprehended by the table. We may distinguish two types of life tables according to the reference year of the table: the current or period life table and the generation or cohort life table. The first type of table is based on the experience over a short period of time, such as 1 year, 3 years, or an intercensal period, in which mortality has remained substantially the same. Commonly, the death statistics used for a current life table relate to a period of 1 to 3 years, and the population data used relate to the middle of that period (usually close to the date of a census). This type of table, therefore, represents the combined mortality experience by age of the population in a particular short period of time (treated synthetically or viewed cross-sectionally); it does not represent the mortality experience of an actual cohort. Instead, it assumes a hypothetical cohort that is subject to the age-specific death rates observed in the particular period. Therefore, a current life table may be viewed as a snapshot of current mortality. It is an excellent summary description of mortality in a year or a short period.

The second type of life table, the generation life table, is based on the mortality rates experienced by a particular birth cohort (e.g., all persons born in the year 1900). According to this type of table, the mortality experience of the persons in the cohort would be observed from their moment of birth through each consecutive age in successive calendar years until all of them die. Obviously, data over a long period of years are needed to complete a single table, and it is not possible wholly on the basis of actual data to construct generation tables for cohorts born in the 20th century. This type of table is useful for projections of mortality, for studies of mortality trends, and for the measurement of fertility and reproductivity. In general, unless otherwise specified, the term "life table" is used in this chapter to refer to a current life table.

Life tables are also classified into two types—complete (or unabridged) and abridged—according to the length of the age interval in which the data are presented. A complete life table contains data for every single year of age from birth to the last applicable age. An abridged life table contains data at intervals of 5 or 10 years of age for most of the age range. Demographers usually prepare the simpler abridged life table rather than the more elaborate complete life table. Tables with values for 5- or 10-year intervals are sufficiently detailed for most purposes, and the abridged table is less burdensome to prepare. Moreover, it is often more convenient to use. Tables 13.1 and 13.2 are illustrations of complete and abridged life tables, respectively. Occasionally, the basic values from a complete life table are presented only for every fifth age in order to economize on space.

We may also distinguish a conventional life table, which is concerned only with the general mortality experience of a cohort by age, from a multiple decrement table, which describes the separate and combined effects of more than one factor. Mortality is always involved. Multiple decrement tables are of several forms. The mortality factor may be applied in terms of component death rates (e.g., for causes of death), or mortality may be combined with changes in one or more socioeconomic characteristic(s) of the population. Multiple decrement tables describe the diminution of an original cohort through these factors (e.g., the attrition of the single population through mortality and marriage). Increment-decrement tables and multistate tables, elaborations of multiple decrement tables, describe the effect of accession to and withdrawal from the original cohort at various stages in its life history. Examples include a table of working life (which combines mortality rates and labor force participation ratios) and a nuptiality table (which combines mortality rates and data on the prevalence or incidence of marriage and divorce).

AVAILABILITY OF LIFE TABLES

The first recognized life table was published in 1693 by Halley. It was based on birth and death registration data for the city of Breslau during the years 1687 to 1691. The assumption adopted in the preparation of this table that the population of Breslau had remained stationary (i.e., that the total population and the numbers in each age and sex group did not change over many decades) was not entirely correct and, therefore, the resulting life table could not be regarded as being correct. Other life tables were prepared in the 17th and 18th centuries on the basis of limited data, but they were subject to necessary simplifying assumptions that rendered them inexact.

The first scientifically correct life table based on both population and death data classified by age was prepared by Milne and published in 1815. It was based on the mortality experience in two parishes of Carlisle, England, during the period 1779–1787. A large number of life tables have been published since then. In the early years, most of these pertained to European countries, particularly Scandinavian countries, but life tables are now available for most countries of the world and every continent is represented.

In the United States, official complete life tables have been prepared since 1900–1902 in connection with the decennial censuses of population. These tables are based on the registered deaths in the expanding death registration area (continental United States first being covered for the 1929–1931 table) in the 3-year period containing the census year. Hence, there is an unbroken decennial series of complete life tables covering the 20th century (U.S. Bureau of the Census, 1921, 1936, 1946; U.S. National Office of Vital Statistics, 1954; U.S. National Center for Health Statistics, 1964, 1975, 1985a, 1997a). There are some tables also for intercensal periods, as for 1901–1910, 1920–1929, and 1930–1939. An annual series of abridged life tables was started in 1945 and continued to the end of the 20th century. They are now published on a provisional basis for a given year just after the close of the year and on a revised basis in the annual volume on mortality statistics for the year or in current reports. These annual tables are based on the annual death registration and on postcensal estimates of population.

In addition to national tables, tables have been prepared from time to time for geographic areas varying in size from specific cities to geographic divisions and regions. Sets of life tables for all states have now been published in connection with the censuses of 1930 (whites only), 1940 (whites only), 1950 (nonwhite tables for states in the South region only), 1960 (race/ethnic group for selected states), 1970, 1980, and 1990 (U.S. National Office of Vital Statistics, 1948, 1956a; U.S. National Center for Health Statistics, 1966, 1977, 1990, 1998). Life tables for geographic divisions or regions corresponding to these state tables have also been published (U.S. National Office of Vital Statistics, 1956b; U.S. National Center for Health Statistics, 1965). For the first time, life tables for metropolitan and nonmetropolitan areas as a whole were published for 1959–1961 (U.S. National Center for Health Statistics, 1967). For some periods prior to 1940, separate tables were published for the urban and rural populations.

Special compilations of life tables or analytic studies of life tables were published in connection with the 1900, 1910, 1930, and 1940 censuses. The life tables included in these volumes covered the death registration states or the United States as a whole, specified individual states and cities, and the urban and rural parts of the death registration states in some cases. A general guide to the U.S. life tables for 1900 to 1959 has been published (U.S. National Center for Health Statistics, 1963).

TABLE 13.1 Complete Life Table for the Total Population of the United States: 1989–1991

Age Interval: Period of life between two exact ages (1) x to x + 1 (years)	Proportion dying: Proportion of persons alive at beginning of age interval dying during interval (2) q_x	Of 100,000 born alive: Number living at beginning of age interval (3) l_x	Of 100,000 born alive: Number dying during age interval (4) d_x	Stationary population: In the age interval (5) L_x	Stationary population: In this and all subsequent age intervals (6) T_x	Average remaining lifetime: Average number of years of life remaining at beginning of age interval (7) e_x	Age interval: Period of life between two ages (1) x to x + 1 (years)	Proportion dying: Period of persons alive at beginning of age interval dying during interval (2) q_x	Of 100,000 born alive: Number living at beginning of age interval (3) l_x	Of 100,000 born alive: Number dying during age interval (4) d_x	Stationary population: In the age interval (5) L_x	Stationary population: In this and all subsequent age intervals (6) T_x	Average remaining lifetime: Average number of years of life remaining at beginning of age interval (7) e_x
0–1	0.00936	100,000	936	99,258	7,536,614	75.37	28–29	0.00126	97,321	123	97,260	4,779,426	49.11
1–2	0.00073	99,064	72	99,028	7,437,356	75.08	29–30	0.00133	97,198	129	97,134	4,682,167	48.17
2–3	0.00048	98,992	48	98,968	7,338,328	74.13	30–31	0.00140	97,069	136	97,001	4,585,033	47.23
3–4	0.00037	98,944	37	98,926	7,239,360	73.17	31–32	0.00147	96,933	142	96,862	4,488,032	46.30
4–5	0.00030	98,907	30	98,892	7,140,434	72.19	32–33	0.00154	96,791	149	96,717	4,391,170	45.37
5–6	0.00027	98,877	27	98,863	7,041,542	71.22	33–34	0.00162	96,642	157	96,564	4,294,454	44.44
6–7	0.00025	98,850	25	98,838	6,942,634	70.23	34–35	0.00170	96,485	164	96,403	4,197,890	43.51
7–8	0.00023	98,825	23	98,814	6,843,796	69.25	35–36	0.00178	96,321	171	96,236	4,101,487	42.58
8–9	0.00020	98,802	20	98,792	6,744,983	68.27	36–37	0.00188	96,150	181	96,060	4,005,252	41.66
9–10	0.00018	98,782	18	98,773	6,646,191	67.28	37–38	0.00198	95,969	190	95,874	3,909,192	40.73
10–11	0.00016	98,764	16	98,756	6,547,418	66.29	38–39	0.00207	95,779	198	95,680	3,813,318	39.81
11–12	0.00016	98,748	16	98,740	6,448,662	65.30	39–40	0.00217	95,581	207	95,478	3,717,638	38.90
12–13	0.00022	98,732	22	98,721	6,349,922	64.31	40–41	0.00228	95,374	217	95,266	3,622,161	37.98
13–14	0.00032	98,710	32	98,694	6,251,201	63.33	41–42	0.00240	95,157	228	95,043	3,526,895	37.06
14–15	0.00047	98,678	46	98,655	6,152,507	62.35	42–43	0.00254	94,929	241	94,809	3,431,852	36.15
15–16	0.00063	98,632	62	98,601	6,053,852	61.38	43–44	0.00271	94,688	257	94,560	3,337,044	35.24
16–17	0.00077	98,570	76	98,532	5,955,251	60.42	44–45	0.00292	94,431	276	94,293	3,242,484	34.34
17–18	0.00089	98,494	88	98,450	5,856,719	59.46	45–46	0.00318	94,155	299	94,006	3,148,191	33.44
18–19	0.00096	98,406	94	98,359	5,758,269	58.52	46–47	0.00348	93,856	327	93,693	3,054,186	32.54
19–20	0.00101	98,312	99	98,263	5,659,910	57.57	47–48	0.00380	93,529	355	93,352	2,960,493	31.65
20–21	0.00104	98,213	102	98,162	5,561,647	56.63	48–49	0.00414	93,174	386	92,981	2,867,142	30.77
21–22	0.00109	98,111	107	98,058	5,463,485	55.69	49–50	0.00449	92,788	417	92,580	2,774,161	29.90
22–23	0.00112	98,004	110	97,949	5,365,428	54.75	50–51	0.00490	92,371	453	92,145	2,681,581	29.03
23–24	0.00114	97,894	112	97,838	5,267,479	53.81	51–52	0.00537	91,918	494	91,671	2,589,437	28.17
24–25	0.00116	97,782	113	97,726	5,169,641	52.87	52–53	0.00590	91,424	539	91,155	2,497,766	27.32
25–26	0.00117	97,669	114	97,612	5,071,915	51.93	53–54	0.00647	90,885	588	90,591	2,406,611	26.48
26–27	0.00119	97,555	116	97,497	4,974,303	50.99	54–55	0.00708	90,297	639	89,978	2,316,020	25.65
27–28	0.00121	97,439	118	97,380	4,876,806	50.05	55–56	0.00773	89,658	693	89,312	2,226,043	24.83

(*continues*)

TABLE 13.1 *(continued)*

Age Interval: Period of life between two exact ages (1) x to x + 1 (years)	Proportion dying: Proportion of persons alive at beginning of age interval dying during interval (2) q_x	Of 100,000 born alive: Number living at beginning of age interval (3) l_x	Of 100,000 born alive: Number dying during age interval (4) d_x	Stationary population: In the age interval (5) L_x	Stationary population: In this and all subsequent age intervals (6) T_x	Average remaining lifetime: Average number of years of life remaining at beginning of age interval (7) e_x	Age interval: Period of life between two ages (1) x to x + n (years)	Proportion dying: Period of persons alive at beginning of age interval dying during interval (2) q_x	Of 100,000 born alive: Number living at beginning of age interval (3) l_x	Of 100,000 born alive: Number dying during age interval (4) d_x	Stationary population: In the age interval (5) L_x	Stationary population: In this and all subsequent age intervals (6) T_x	Average remaining lifetime: Average number of years of life remaining at beginning of age interval (7) e_x
56–57	0.00844	88,965	751	88,590	2,136,731	24.02	83–84	0.08207	37,995	3,118	36,436	267,631	7.04
57–58	0.00926	88,214	817	87,806	2,048,142	23.22	84–85	0.08907	34,877	3,106	33,324	231,195	6.63
58–59	0.01019	87,397	891	86,952	1,960,336	22.43	85–86	0.09705	31,771	3,083	30,230	197,871	6.23
59–60	0.01120	86,506	969	86,022	1,873,385	21.66	86–87	0.10627	28,688	3,049	27,164	167,642	5.84
60–61	0.01223	85,537	1,046	85,014	1,787,363	20.90	87–88	0.11625	25,639	2,981	24,149	140,478	5.48
61–62	0.01328	84,491	1,122	83,930	1,702,349	20.15	88–89	0.12688	22,658	2,875	21,221	116,330	5.13
62–63	0.01439	83,369	1,200	82,769	1,618,419	19.41	89–90	0.13834	19,783	2,737	18,415	95,109	4.81
63–64	0.01560	82,169	1,282	81,528	1,535,650	18.69	90–91	0.15135	17,046	2,580	15,756	76,695	4.50
64–65	0.01691	80,887	1,368	80,203	1,454,122	17.98	91–92	0.16591	14,466	2,400	13,266	60,939	4.21
65–66	0.01827	79,519	1,453	78,793	1,373,919	17.28	92–91	0.18088	12,066	2,182	10,975	47,673	3.95
66–67	0.01967	78,066	1,536	77,298	1,295,127	16.59	93–94	0.19552	9,884	1,933	8,918	36,698	3.71
67–68	0.02121	76,530	1,623	75,719	1,217,829	15.91	94–95	0.21000	7,951	1,670	7,116	27,780	3.49
68–69	0.02297	74,907	1,721	74,047	1,142,110	15.25	95–96	0.22502	6,281	1,413	5,575	20,664	3.29
69–70	0.02499	73,186	1,829	72,272	1,068,064	14.59	96–97	0.24126	4,868	1,174	4,281	15,090	3.10
70–71	0.02727	71,357	1,946	70,384	995,792	13.96	97–98	0.25689	3,694	949	3,220	10,809	2.93
71–72	0.02979	69,411	2,068	68,377	925,408	13.33	98–99	0.27175	2,745	746	2,372	7,589	2.76
72–73	0.03251	67,343	2,189	66,249	857,031	12.73	99–100	0.28751	1,999	575	1,712	5,217	2.61
73–74	0.03534	65,154	2,303	64,003	790,783	12.14	100–101	0.30418	1,424	433	1,208	3,506	2.46
74–75	0.03824	62,851	2,403	61,650	726,780	11.56	101–102	0.32182	991	319	832	2,298	2.32
75–76	0.04126	60,448	2,494	59,201	665,131	11.00	102–103	0.34049	672	229	558	1,467	2.18
76–77	0.04455	57,954	2,582	56,663	605,930	10.46	103–104	0.36024	443	160	363	909	2.05
77–78	0.04819	55,372	2,668	54,038	549,267	9.92	104–105	0.38113	283	108	229	546	1.93
78–79	0.05239	52,704	2,761	51,324	495,229	9.40	105–106	0.40324	175	71	140	317	1.81
79–80	0.05723	49,943	2,858	48,514	443,905	8.89	106–107	0.42663	104	44	82	178	1.71
80–81	0.06277	47,085	2,956	45,607	395,391	8.40	107–108	0.45137	60	27	47	96	1.59
81–82	0.06885	44,129	3,038	42,610	349,784	7.93	108–109	0.47755	33	16	25	49	1.48
82–83	0.07535	41,091	3,096	39,543	307,174	7.48	109–110	0.50525	17	9	9	24	1.39

Source: U.S. National Center for Health Statistics. 1997. *U.S. Decennial Life Tables for 1989–91*. Vol. 1, no. 1, pp. 6–7.

TABLE 13.2 Abridged Life Table for the Total Population of the United States: 1989–1991

Age interval	Proportion dying	Of 100,000 born alive		Stationary population		Average remaining lifetime
Period of life between two ages (1) x to x + n year	Proportion of persons alive at beginning of age interval dying during interval (2)	Number living at beginning of age interval (3)	Number dying during age interval (4)	In the age interval (5)	In this and all subsequent age intervals (6)	Average number of years of life remaining at beginning of age interval (7)
	${}_nq_x$	l_x	${}_nd_x$	${}_nL_x$	T_x	e_x
0–1	0.009360	100,000	936	99,258	7,536,614	75.37
1–5	0.001888	99,064	187	395,814	7,437,356	75.08
5–10	0.001143	98,877	113	494,080	7,041,542	71.21
10–15	0.001337	98,764	132	493,566	6,547,418	66.29
15–20	0.004248	98,632	419	492,205	6,053,852	61.38
20–25	0.005539	98,213	544	489,732	5,561,647	56.63
25–30	0.006143	97,669	600	486,882	5,071,915	51.93
30–35	0.007706	97,069	748	483,546	4,585,033	47.23
35–40	0.009832	96,321	947	479,326	4,101,487	42.58
40–45	0.012781	95,374	1,219	473,970	3,622,161	37.98
45–50	0.018947	94,155	1,784	466,610	3,148,191	33.44
50–55	0.029371	92,371	2,713	455,538	2,681,581	29.03
55–60	0.045964	89,658	4,121	438,680	2,226,043	24.83
60–65	0.070356	85,537	6,018	413,444	1,787,363	20.90
65–70	0.102642	79,519	8,162	378,127	1,373,919	17.28
70–75	0.152879	71,357	10,909	330,661	995,792	13.96
75–80	0.221066	60,448	13,363	269,740	665,131	11.00
80–85	0.325242	47,085	15,314	197,520	395,391	8.40
85–90	0.463473	31,771	14,725	121,176	197,871	6.23
90–95	0.631526	17,046	10,765	56,031	76,695	4.50
95–100	0.773285	6,281	4,857	17,158	20,664	3.29
100+	1.000000	1,424	1,424	3,506	3,506	2.46

Source: Based on Table 13-1.

The United Nations *Demographic Yearbook* for 1996 is the most recent *Yearbook* to contain a comprehensive collection of national life tables. Values of the expectation of life, (1-year) mortality rates, and survivors from the two latest official complete life tables, largely for years in the period 1979–1995, are included. These values are shown for single ages up to 5 years and at every fifth age thereafter to 85 years. Similar tables in the 1985, 1980, 1974, 1967, 1966, 1961, 1957, and 1953 *Yearbooks* carry the life tables back to 1900. In addition, the latest value of the expectation of life is published every year. In the 1996 collection, values for the expectation of life are shown for 229 countries, but the other two life table functions are shown only for about 135 countries.

A compilation of life tables for a large number of countries spanning a wide range of time has been published by Keyfitz and Flieger (1968, 1990). Life tables for males and females for each country and year included in the compilation were derived electronically from official data on births, deaths, and population classified by age and sex, where such data were considered to be of satisfactory quality. While the first citation concerned only 29% of the world's population, the second citation refers to 152 countries. The later volume includes actual trends to 1985 and population projections to 2020. Sets of model life tables have been published by the United Nations (1982), Coale and Demeny (1966), and Coale, Demeny, and Vaughn (1983). These sets of tables correspond to values for life expectancy at birth varying generally by fixed intervals in years. The UN tables cover from 20 years to 73.9 years, mostly in intervals of 2.5 years. The Coale and Demeny tables relate to four different "regions" distinguished by the age pattern of mortality. The tables for females correspond to values for life expectancy at birth ranging from 20 to 80 years in intervals of 2.5 years; and the male tables are presented as companion tables. These tables are useful for estimating life table functions for countries for which not all the data necessary for the preparation of a life table are available. The World Health Organization annually publishes life expectancy by sex in the *World Health Statistics Annual.* The Population Reference Bureau frequently publishes life expectancy data in its *World Population Data Sheet*. Life tables for 229 countries are available from the U.S. Census Bureau's website.

ANATOMY OF THE LIFE TABLE

Life Table Functions

The basic life tables functions—${}_nq_x$, l_x, ${}_nd_x$, ${}_nL_x$, T_x, and e_x—can be observed in Tables 13.1 and 13.2. These six columns are generally calculated and published for every life table. However, in some cases, because of limitations of space, columns may be omitted. (For example, the United Nations publishes only q_x, l_x, and e_x in the *Demographic Yearbook*.) This is done without a significant loss of information because the functions are interrelated and some can be directly calculated from the others. In general, the mortality rate (${}_nq_x$) is the basic function in the table (i.e., the initial function from which all other life table functions are derived).

Alternative Interpretations

Life table functions are subject to two different interpretations depending on the interpretation given to the life table as a whole. In the more common interpretation, the life table is viewed as depicting the lifetime mortality experience of a single cohort of newborn babies, who are subject to the age-specific mortality rates on which the table is based. In the second interpretation, the life table is viewed as a stationary population resulting from the (unchanging) schedule of age-specific mortality rates shown and a constant annual number of births.

The Life Table as the Mortality Experience of a Cohort

Under the first interpretation, the life table model conceptually traces a cohort of newborn babies through their entire life under the assumption that they are subject to the current observed schedule of age-specific mortality rates. The cohort of newborn babies, called the radix of the table, is usually assumed to number 100,000. In this case, the interpretation of the life table functions in an abridged table would be as follows:

x to x + n The period of life between two exact ages. For instance, "20–25" means the 5-year interval between the 20th and 25th birthdays.

${}_nq_x$ The proportion of the persons in the cohort alive at the beginning of an indicated age interval (x) who will die before reaching the end of that age interval ($x + n$). For example, according to Table 13.2, the proportion dying in the age interval 20–25 is 0.005539—that is, out of every 100,000 persons alive and exactly 20 years old, 554 will die before reaching their 25th birthday. In other words, the ${}_nq_x$ values represent the probability that a person at his or her xth birthday will die before reaching his or her $x + n$th birthday.

l_x The number of persons living at the beginning of the indicated age interval (x) out of the total number of births assumed as the radix of the table. Again, according to Table 13.2, out of 100,000 newborn babies, 98,213 persons would survive to exact age 20.

${}_nd_x$ The number of persons who would die within the indicated age interval (x to $x + n$) out of the total number of births assumed in the table. Thus, according to Table 13.2, there would be 544 deaths between exact ages 20 and 25 to the initial cohort of 100,000 newborn babies.

${}_nL_x$ The number of person-years that would be lived within the indicated age interval (x to $x + n$) by the cohort of 100,000 births assumed. Thus, according to Table 13.2, the 100,000 newborn babies would live 489,732 person-years between exact ages 20 and 25. Of the 98,213 persons who reach age 20, the 97,669 who survive to age 25 would live 5 years each (97,669 × 5 = 488,345 person-years) and the 544 who die would each live varying periods of time less than 5 years, averaging about 21/2 years

$$544 \times 2.55 = 1387 \text{ person-years}$$

T_x The total number of person-years that would be lived after the beginning of the indicated age interval by the cohort of 100,000 births assumed. Thus, according to Table 13.2, the 100,000 newborn babies would live 5,561,647 person-years after their 20th birthday.

e_x The average remaining lifetime (in years) for a person who survives to the beginning of the indicated age interval. This function is also called the complete expectation of life or, simply, life expectancy. Thus, according to Table 13.2, a person who reaches his or her 20th birthday should expect to live 56.63 years more, on the average.

The interpretation of the functions in a complete life table is the same as in the abridged table except that the q_x, d_x, and L_x values relate to single-age intervals. The l_x, T_x, and e_x values have the same interpretation as in the abridged table because they are not "interval" values but pertain to exact age x.

The Life Table as a Stationary Population

An alternate interpretation of the life table is the one associated with the concept of a stationary population. A stationary population is defined as a population whose total number and distribution by age do not change with time. Such a hypothetical population would result if the number of births per year remained constant (usually assumed at 100,000) for a long period of time and each cohort of births experienced the current observed mortality rates throughout life. The annual number of deaths would thus equal 100,000 also, and there would be no change in the size of the population.

In this case, the interpretation of x to x + n, ${}_nq_x$ and e_x would be as previously indicated, but, given that births are constant at 100,000 per year and that the observed death rates at each age remain in effect for the life of the cohort, for the other life table functions, it would be as follows:

l_x The number of persons who reach the beginning of the age interval each year. According to Table 13.2, there would be 98,213 persons reaching exact age 20 every year.

${}_nd_x$ The number of persons that die each year within the indicated age interval. According to Table 13.2, there would be 544 deaths between exact ages 20 and 25 every year.

${}_nL_x$ The number of persons in the population who at any moment are living within the indicated age interval. According to Table 13.2, there would be 489,732 persons living between exact ages 20 and 25 in the population at any time.

T_x The number of persons in the population who at any moment are living within the indicated age interval and all higher age intervals. According to Table 13.2, there would be 5,561,647 persons over exact age 20 living in the population at any time.

Each interpretation has its particular applications. For example, the interpretation of the life table as the history of a cohort is applied in public health studies and mortality analysis, and in the calculation of survival rates for estimating population, net migration, fertility, and reproductivity. The interpretation of the life table as a stationary population is used in the comparative measurement of mortality and in studies of population structure.

Life Span and Life Expectancy

In measuring longevity, two concepts should be distinguished: *life span* and *life expectancy*. The first concept tries to establish numerically the age limit of human life (i.e., the age that human beings as a species could reach under optimum conditions). There is no known exact figure for the life span of human beings or any other species (Carey, 1997). However, by using the verified age of the longest lived individual, one operational definition of life span for a species can be constructed, namely, the maximum recorded age at death. Using this operational definition, the life span for humans appears to be just over 120 years (Olshansky, Carnes, and Cassel, 1990).

Life expectancy is the expected number of years to be lived, on the average, by a particular population at a particular time. Sufficiently accurate records have been available for some time for many countries from which estimates have been prepared. These estimates have generally come from a current life table, although in some instances they have been prepared from more limited data.

CONSTRUCTION OF CONVENTIONAL LIFE TABLES

General Considerations

The main concern in this section is with methods of constructing life tables where satisfactory data on births, deaths, and population are available. (Model life tables and indirect techniques of life table construction are often employed for statistically underdeveloped countries, as discussed in Chapter 22 and Appendix B.) Mathematical formulas and demographic procedures are discussed in detail, and techniques for manipulating the mortality data are presented. It should be observed that in every instance underlying these procedures, formulas, and techniques, there is an assumption that the data on births, deaths, and population are fully accurate. It is well known, however, that one of the most important aspects of the preparation of a life table is the testing of the data for possible biases and other errors. Most of the procedures used to check on the accuracy of the data have been discussed in previous chapters and will not be discussed again here. It suffices that the level of inaccuracy that can be tolerated depends mostly on the intended use of the life table.

One factor that is usually involved in the selection of the method to be used in the construction of a life table is the degree of adherence to the observed data that is desired. Full adherence to the observed values implies that the final life table functions will exhibit all the fluctuations in the observed data, whether these are due to real variations or to errors in the data. On the other hand, overgraduation (i.e., excessive "smoothing" of the data by mathematical methods) will eliminate or reduce true variations in the age pattern of mortality rates. Therefore, it must be decided whether the emphasis in the life table should be on its closeness to the actual data or on its presentation of the underlying mortality picture after fluctuations have been removed. In the statistically developed countries, as the quality and quantity of data have improved with the passage of time, fewer irregular fluctuations have been noticed in the observed mortality rates. Therefore, the tendency has been to place stronger emphasis on adherence to the data and less emphasis on eliminating or reducing the fluctuations.

The Complete Life Table

As was indicated before, a complete life table is a life table in which the values of the functions are shown for single years of age. This type of table either can be computed directly from the observed data or can be obtained by suitable interpolation of an abridged life table. In many instances, even when a complete life table is being constructed, the observed data are combined into age groups and then interpolated to obtain the single-year values. This pro-

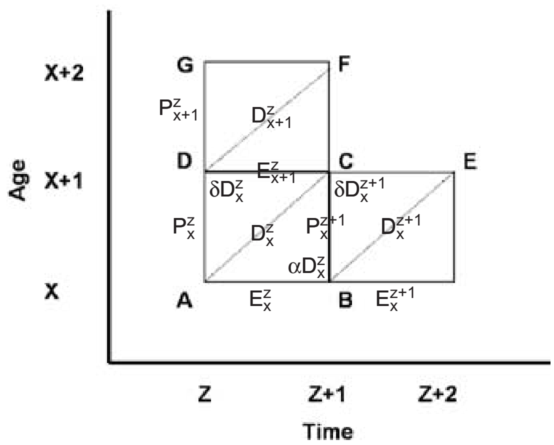

FIGURE 13.1 ••

cedure is generally used to smooth out artificial fluctuations in the data or to adjust for what are believed to be inconsistencies or errors in the data.

Basic Construction Problems

The construction of complete life tables can be considered in terms of three broad phases. First, the basic data on deaths, population, and births are checked for inconsistencies, biases, and other errors, and adjustments are made where necessary. Second, the death rates and mortality rates are computed and graduated (i.e., mathematically smoothed). Third, the remaining functions of the life table are calculated. As indicated previously, this chapter emphasizes the second and third steps.

Life tables are based on probabilities of dying, the probability that an individual alive at age x dies before reaching his or her next birthday (age $x + 1$). The Lexis diagram (Figure 13.1) shows the relation of the number attaining age x, the population in age x, and the deaths at age x. Individual lives move along diagonal lines from the origin up and to the right. In general, then,

D^z_x The number of deaths that occur during calendar year z among persons who have attained age x at last birthday (the area encompassed by points ABCD in Figure 13.1).

αD^z_x The number of persons who attained age x in the calendar year z and died before the end of the calendar year (the area encompassed by points ABC).

δD^z_x The number of persons alive at the beginning of calendar year z who were x years old last birthday and died before attaining age $x + 1$ (the area encompassed by points ADC).

P^z_x The number of persons alive at the beginning of calendar year z who are x years old last birthday (the line from point A to point D).

E^z_x The number of persons attaining age x in year z (the line from point A to point B).

Note that $\alpha D^z_x + \delta D^z_x = D^z_x$ and $E^z_{x+1} = P^z_x - \delta D^z_x$.

There are several alternative ways to compute mortality rates, q_x, from vital statistics and census data. One approach relates deaths in the parallelogram ABCE to lines connecting AB. Under the assumption of no migration, the mortality rate for ages above 1 would be

$$q_x = \frac{\alpha D^z_x + \delta D^{z+1}_x}{E^z_x} \tag{13.1}$$

The corresponding mortality rate for the first year of life would replace the denominator with the number of births in year z. However, these formulas involve the partial mortality experience of 2 consecutive calendar years rather than the experience of a single calendar year.

Another approach concerns deaths in ABCD and relates αD^z_x to lines connecting AB and δD^z_x to lines connecting AD. For ages above 1

$$q_x = 1 - \left(1 - \frac{\alpha D^z_x}{E^z_x}\right) \cdot \left(1 - \frac{\delta D^z_x}{P^z_x}\right) \tag{13.2}$$

The mortality rates could be based on the experience of a single calendar year if it is assumed that there is little change in mortality or in the seasonal birth pattern between the 2 years. In that case, the formulas for ages above 1 and for infants would be

$$q_x = 1 - \frac{P^{z+1}_x}{E^z_x} \cdot \frac{E^z_{x+1}}{P^z_x} \tag{13.3}$$

$$q_0 = 1 - \frac{P^{z+1}_0}{B^z} \cdot \frac{E^z_1}{P^z_0} \tag{13.4}$$

These formulas directly employ deaths in the single year z only, although deaths for prior years are required to determine the population at the beginning of the year at age under 1, or P^z_x in general.

It should be noted that these formulas require the registered deaths in every calendar year to be separated according to whether they occurred before or after the birthday anniversary in that year. When the death statistics are not available in this form, the usual approach is to estimate separation factors f_x from other sources or to assume that those observed in other populations apply in the present case.

Generally, separation factors above age 1 are assumed to be 0.5. Deaths under age 1 are concentrated at the beginning of the interval; about half of infant deaths in the United States occur in the first week of life and 65% happen within the first month. The average age at death for infants rises with increasing infant mortality levels. Preston, Keyfitz, and Schoen (1972) suggested the following approximation for the separation factor for those less than 1 year of age:

$${}_1a_0 = 0.07 + 1.7 \cdot {}_1m_0 \tag{13.5}$$

Given the separation factors, to calculate the q_x the deaths would be estimated as follows:

$$\delta D_x^z = f_x'' D_x^z \tag{13.6}$$

$$\alpha D_x^z = (1 - f_x'')D_x^z = f' D_x^z \tag{13.7}$$

The separation factor f_x' represents the proportion of deaths at a given age in a given year that occurred after the birthday anniversary in that year. In the case of infant deaths, f_x' represents the proportion of all infant death in a year that occurred to babies born in the year. Small errors in the separation factors do not materially affect estimates.

In the computation and graduation of the mortality rates, three different age segments are recognized as having peculiar problems and are treated separately. The first is the youngest age segment, which generally includes ages under 5. Where the quality of data is sufficiently good and the level of mortality is low, the first segment can be limited to the first year of life. The second segment generally covers the bulk of the table, from age 5 to age 85, where the most reliable data are found. The third segment covers the oldest ages, 85 years and older, and usually contains the data of highest uncertainty. It should be observed that these are somewhat arbitrary divisions. It is entirely possible or even advisable in some cases, depending on the nature of the observed data, further to subdivide some of these segments. The number of segments and their age limits should be determined based on an analysis of the data available.

Mortality at Ages Under 5 Years

The methods used to compute mortality risks at ages less than 5 years must necessarily be adapted to the type and quality of data available. As the quality and quantity of data improve, it is feasible to extend to ages 1 to 4 years the methods used in the main body of the life table. Owing to the distinct peculiarities of mortality during the first year of life, however, special procedures must be used to compute the death rates at ages less than 1 year.

On many occasions, the census counts are particularly inaccurate at the younger ages. It may be desirable or necessary, then, to design formulas or procedures to obtain the mortality rates at ages under 5 years on the basis of recorded births rather than on population counts. The most frequently used procedure employs the registered birth and death statistics. For the United States 1989–1991 decennial life table, mortality rates for those under 2 years were based on registered births and deaths while the companion rates for those 2 years and over were based on registered deaths and population counts (U.S. National Center for Health Statistics, 1997b).

The calendar-year infant mortality rate is often used to represent the mortality risk for the first year of life. This procedure partially substitutes cohort for period data. The conventional infant mortality rate is the number of deaths in infancy to a group of persons born in the same year. The calendar-year infant mortality rate is computed as the ratio of the number of deaths of those less than 1 year (in that year) to the number of live births in that year. So deaths in year z + 1 of infants born late in the year z are represented by deaths in year z to those born late in year z – 1.

If the vital statistics tabulations are sufficiently detailed, it is possible to use days, weeks, months, or quarter-years as units for constructing q_x for young ages. The United States 1989–1991 decennial life tables, for example, used 0 to 1 day, 1 to 7 days, 7 to 28 days, 28 to 365 days, and 1 to 2 years. The procedure assumes that births are uniformly distributed within these intervals.

It is not necessary to begin with mortality rates. At ages under 2 years, the United States decennial life tables did not begin with mortality rates but rather with observed and life table deaths. For each age interval 0 to 1 day, 1 to 7 days, 7 to 28 days, 28 to 365 days, and 1 to 2 years, the formula was

$${}_t d_x = \frac{l_0 \, {}_t D_x}{{}_t E_x}$$

where ${}_tD_x$ denotes the number of deaths occurring in 1989–1991 between exact ages x and $x + t$, and ${}_tE_x$ is a weighted count of births during 1987–1991. The denominator assumes that births are uniformly distributed over the year. The values of ${}_td_x$ were then used to calculate values of l_x up to age 2 years by successive application of the formula $l_{x+t} = l_x - d_x$.

For ages 1 to 4, other approximations of the mortality rates may be used when the census data are considered to be of acceptable accuracy and the death rates are low. A central death rate, m_x, is calculated from the observed deaths, D_x, and the census count, P_x, using the formula

$$m_x = \frac{D_x}{P_x} \tag{13.8}$$

In some instances, this formula is modified to improve its reliability by using an average population derived from the populations at three adjacent ages, as follows:

$$m_x = \frac{3D_x}{P_{x-1} + P_x + P_{x+1}}$$

where D_x denotes the number of deaths to those aged x. The population at risk of dying at age x during the 3-year period includes mainly members of three adjacent annual birth cohorts; hence the denominator sums the populations aged $x - 1$, x, and $x + 1$. This approach was used for the United States 1989–1991 decennial life table for ages 2 to 4. For example, deaths at age 2 during 1989–1991 were divided by the sum of the populations at ages 1, 2, and 3.

The central death rates are then converted to the needed mortality rates, or probabilities, by means of the formula

$$q_x = \frac{2m_x}{2+m_x} \tag{13.9}$$

This formula is based on the assumption that deaths between exact ages x and $x + 1$ occur, on the average, at age $x + \frac{1}{2}$, as, for example, when deaths at age x in a given year are rectangularly distributed by age and time interval. The formula for q_x shown may be derived from the basic formula for m_x and expresses this assumption more explicitly:

$$m_x = \frac{D_x}{P_x} \quad \text{and} \quad q_x = \frac{D_x}{P_x + .5D_x} \tag{13.10}$$

Then, dividing numerator and denominator by P_x,

$$q_x = \frac{m_x}{1+.5m_x} = \frac{2m_x}{2+m_x}$$

Mortality at Ages 5 to 84

The main segment of a life table is that covering ages 5 to 84. For this range we usually have the most accurate data on observed mortality. The goal is to obtain a smooth curve of death rates by age that joins age 5 with the previously computed rates for ages below 5. The death statistics and population data are combined into the conventional 5-year age groups: 5 to 9, 10 to 14, and so on. Grouping eliminates most of the problems associated with digit preference in age reporting and accomplishes part of the desired graduation, and this particular grouping has been found especially satisfactory. The death statistics employed would be the total of the deaths recorded in the observation period; usually 1, 2, or 3 years, and the population data would refer to the middle of the period.

There are two main approaches to obtaining mortality rates for single-year ages. One approach is to interpolate the population counts and registered deaths from the aggregated data to the corresponding single-year-of-age intervals. Then, q_x is calculated using

$$q_x = \frac{D_x}{P_x + .5 \cdot D_x} \tag{13.11}$$

The U.S. decennial life table for 1989–1991 used this approach for ages 5 to 94 (U.S. National Center for Health Statistics, 1997b).

The other main approach is to obtain central age-specific death rates from the grouped data using Equation (13.11), test for data anomalies and adjust the rates if necessary, calculate, ${}_nq_x$, and then graduate them to obtain single-year rates. Any abrupt change in the mortality curve is regarded as reflecting a possible anomaly in the data. Each possible anomaly is investigated in detail and, if necessary, adjustment procedures are developed for the data. These adjustment procedures vary considerably in detail; they are not generally subject to broad generalization because each set of data has it own peculiarities.

Graduation of Mortality Rates

Once the data have been adjusted, the calculation of the mortality rates for single years of age becomes an exercise in graduation techniques. A large number of graduation methods have been used in the past, and new ones are constantly being devised. However, it is important that the analyst select the method most appropriate for the data involved, taking into account limitations in time, personnel, mathematical skill, and possible use of the final life table. The selection of the interpolation formula to be used depends mostly on the desired balance between smoothness and closeness of fit to the data. In general, an improvement in smoothness can only be made at the expense of closeness of fit. The analyst must judge, according to his or her experience, the relative importance that should be given to each of the two opposing considerations.

The three most commonly used interpolation techniques to compute single-year q_x values from grouped (interval) values are (1) Sprague multipliers, (2) Beers multipliers, and (3) Karup-King multipliers. All three of these techniques were designed so that the sum of the interpolated single-year values for "countable items" (e.g., deaths) is consistent with the total number of countable items for the group (interval) as a whole. The U.S. decennial life tables for 1989–1991 used Beers multipliers (U.S. National Center for Health Statistics, 1997b).

Mortality at Ages 85 and Over

Population and death data at the oldest ages have always been considered to be of low accuracy. In most life tables prepared in the past, the statistics at these ages have been disregarded because of the low credibility that most demographers and actuaries assign to the basic registration and census data. Because the quality of these data has been improving, however, the tendency now is to retain as much of them as possible for constructing the life table. This can be done either by basing the final rates partially on the recorded data or by extending the range of the main portion of the life table beyond age 84—say to age 89 or 94.

In practice, it has been found that, regardless of the total volume of data and their accuracy, there is some point at the older ages beyond which arbitrary methods must be applied. At those very old ages, the data become either too scanty to be statistically reliable or they are regarded as invalid because of errors in age reporting and coverage in the census or death statistics. For practical purposes, any reasonable method can be used because the effect of the choice of rates at these ages on the life table functions at the younger ages would be relatively minor. The method selected, however, should produce a smooth juncture with the mortality rates in the main portion of the life table. It should also produce mortality rates that increase smoothly with advancing age.

One method used is to assume that mortality rates increase at the oldest ages by about the same percentage as, or, more realistically, a decreasing percentage of that found at the end of the main portion of the life table. Empirically, in the low mortality countries, the percentage of increase at the later ages has been found to be about 10% per year of age. This means that every mortality rate at the oldest ages would be 10% higher than the rate at the next previous age or a declining function of 10%. A second method is to fit a third degree polynomial to the last three acceptable mortality rates and to an assumed rate of unity at a very high age chosen arbitrarily (e.g., age 110). A third method is to fit a Gompertz-Makeham curve to the end values of the main portion of the life table.

A fourth method is to adopt a series of rates, based on mortality experience in other populations, that are believed to be acceptably accurate. In the 1989–1991 United States decennial life tables, death rates at ages 85 and over were based, at least in part, on the experience of the Medicare program (U.S. National Center for Health Statistics, 1997b). Medicare data were considered more accurate than conventional death rates, already described as suffering from age misreporting among the extreme elderly. Medicare death rates at ages 85 to 94 were blended with those based on census populations and registered deaths; rates for older ages were based entirely on Medicare data. The Medicare rates were based directly on data on deaths and Medicare enrollments for ages 66 to 105. The rates were then smoothed or graduated (see Appendix C for a discussion of "smoothing"). Graduated rates at the oldest ages were replaced by rates obtained by a method of extrapolation set forth in the Annual Social Security Trustees Reports. In this method for each sex, a minimum percentage increase from q_{x-1} to q_x was required. The level of q_x for females was not allowed to rise to a level higher than that for males of the same age and race/ethnic group.

Derivation of Other Life Table Functions

The other life table functions are calculated using standard procedures, once the single-year q_x's have been determined. The first values to be computed are the series of l_x's or number of survivors for single years of age. These values are obtained using the formula

$$l_x = (1 - q_{x-1})l_{x-1} \tag{13.12}$$

According to this formula, the number of survivors to a given age, l_x, equals the number reaching the previous age, l_{x-1}, times the probability of surviving from that age to the next, $(1\text{-}q_{x-1})$. The calculations for l_x can be "set in motion" by use of the radix of 100,000 for l_0. As examples, the calculations for Table 13.1 at ages 1, 2, and 37 would be as follows:

$$l_1 = (1 - q_0)\cdot l_0 = (1 - .00936)\cdot 100,000 = 99,064$$

$$l_2 = (1 - q_1)\cdot l_1 = (1 - .00073)\cdot 99,064 = 98,992$$

$$l_{37} = (1 - q_{36})\cdot l_{36} = (1 - .00188)\cdot 96,150 = 95,969$$

The d_x's or deaths are the second series of values to be calculated. These follow the formula

$$d_x = l_x - l_{x+1} \tag{13.13}$$

This means that the number of deaths at a given age equals the difference between the number surviving to that age and the number surviving to the subsequent age. The calculations for Table 13.1 at ages under 1, 1, and 36 would be as follows:

$$d_0 = l_0 - l_1 = 100,000 - 99,064 = 936$$

$$d_1 = l_1 - l_2 = 99,064 - 98,992 = 72$$

$$d_{36} = l_{36} - l_{37} = 96,150 - 95,969 = 181$$

The third series is the L_x's, the number of person-years lived by the cohort at a given age. These are computed using the approximation

$$L_x = f_x'' l_x + f_x' l_{x+1} = f_x'' l_x + (1 - f_x'') l_{x+1} \tag{13.14}$$

With the exception of age under 1, the L_x's are the average of l_x and l_{x+1}. The calculations for Table 13.1 at ages under 1, 1, and 36 would be as follows:

$$L_0 = .207 l_0 + .793 l_1 = .207(100,000) + .793(99,064) = 99,258$$

$$L_1 = 0.5 l_1 + 0.5 l_2 = 0.5(99,064) + 0.5(98,992) = 99,028$$

$$L_{36} = 0.5 l_{36} + 0.5 l_{37} = 0.5(96,150) + 0.5(95,969) = 96,060$$

The fourth series of values, the T_x's, the total number of person-years lived by the cohort after reaching age x, is computed by summing the L_x's for ages x and over to the end of the life table. Algebraically, we have

$$T_x = \sum_{y=x}^{y=\omega} L_y \tag{13.15}$$

where ω is the last age in the life table. For example, T_{85} is the sum of all the L_x's from L_{85} to L_{109} in the calculations for Table 13.1: $T_{85} = L_{85} + L_{86} + L_{87} + \ldots + L_{108} + L_{109} = 30,230 + 27,164 + 24,149 + \ldots + 25 + 9 = 197,871$. T_0 is the sum of the entire column of L_x's and represents the total number of person-years lived by the cohort in its lifetime.

The final series of values, e_x, expectation of future life, is computed using the formula

$$e_x = \frac{T_x}{l_x} \tag{13.16}$$

The calculations for Table 13.1 at ages 0 and 36 would be as follows:

$$e_0 = \frac{T_0}{l_0} = \frac{7,536,614}{100,000} = 75.37$$

(Hence, expectation of life at birth can be determined by inspection of T_0, with a shift of the decimal.)

$$e_{36} = \frac{T_{36}}{l_{36}} = \frac{4,005,252}{96,150} = 41.66$$

The Abridged Life Table

Specific Short-Cut Methods

The most fundamental step in life table construction is to convert the observed age-specific death rates into their corresponding mortality rates, or probabilities of dying. As we have seen, in a complete life table, the basic formula for this transformation is

$$q_x = \frac{2m_x}{2+m_x} \tag{13.17}$$

where m_x is the observed death rate at a given age and q_x is the corresponding probability of dying. As stated earlier, this formula is based on the assumption that deaths between exact ages x and $x + 1$ are rectangularly distributed by age and time interval. One of the key features of the various shortcut methods described below is the procedure for making this basic transformation from ${}_nm_x$ to ${}_nq_x$ when the data are grouped. Another difference in the methods is in the way the stationary population is derived. We will describe four shortcut methods here: the Reed-Merrell method, the Greville method, the Keyfitz-Frauenthal method, and the method of reference to a standard table.

The Reed-Merrell Method

Although it has now largely been replaced by other methods, the Reed-Merrell (Reed and Merrell, 1939) method was for many years one of the most frequently used shortcut procedures for calculating an abridged life table. In this method the mortality rates are read off from a set of standard conversion tables showing the mortality rates associated with various observed central death rates. The standard tables for ${}_3m_2$, ${}_5m_x$, and ${}_{10}m_x$ were prepared on the assumption that the following exponential equation holds:

$${}_nq_x = 1 - e^{-n\cdot{}_nm_x - an^3{}_nm_x^2} \tag{13.18}$$

where n is size of the age interval, ${}_nm_x$ is the central death rate, a is a constant, and e is the base of the system of natural logarithms. Reed and Merrell found that a value of $a = 0.008$ would produce acceptable results. The conversion of ${}_nm_x$'s to ${}_nq_x$'s by use of the Reed-Merrell tables is usually applied to 5-year or 10-year data, but special age groups are employed at both ends of the life table. At the younger ages the most frequently used groupings are (1) ages under 1, 1, and 2 to 4 or (2) ages under 1 and 1 to 4. Conversion tables for these ages or age groups, as well as for ${}_5m_x$ and ${}_{10}m_x$ were worked out by Reed and Merrell; they are reproduced in Appendix A.

For example, the death rate for the age group 55 to 59 (${}_5m_{55}$) observed in the United States in 1991, .009263, would be converted to the 5-year mortality rate (${}_5q_{55}$), .04534, by using the Reed-Merrell table of values of ${}_5q_x$ associated with ${}_5m_x$. We look up .009263 in the ${}_5m_x$ column, read off the corresponding ${}_5q_{55}$ value, interpolating as required. The conversion tables are used to derive ${}_5q_x$ from ${}_5m_x$ for all 5-year age groups from 5 to 9 on. At the higher ages, the mortality rate for the open-end group (e.g., 85 years old and over) is evidently equal to one because the life table ends at the age where there are no more survivors.

Once the mortality rates have been calculated, the construction of the abridged life table continues with the computation of each entry in the survivor column, l_x, and the death column, ${}_nd_x$, along standard lines, using the formulas

$$l_{x+n} = (1 - {}_nq_x)l_x \tag{13.19}$$

$${}_nd_x = l_x - l_{x+n} \tag{13.20}$$

All three shortcut methods described in this section follow the same procedure in deriving l_x and ${}_nd_x$.

In the calculation of the next life table function, ${}_nL_x$, each of the three methods to be discussed follows a different procedure. In the Reed-Merrell method, T_x values are directly determined from the l_x's for ages 10 and over, or 5 and over, by use of the following equations:

$$T_x = -.20833l_{x-5} + 2.5l_x + .20833l_{x+5} + 5\sum_{a=1}^{\infty} l_{x+5\alpha} \tag{13.21}$$

if the age intervals in the table are 5-year intervals, and

$$T_x = 4.16667l_x + .8333l_{x+10} + 10\sum_{\alpha=1}^{\infty} l_{x+10\alpha} \tag{13.22}$$

if the age intervals in the table are 10-year intervals. These equations are based on the assumption that the area under the l_x curve between any two ordinates is approximated by the area under a parabola through these two ordinates and the preceding and following ordinates (Formula 13.21) or the following ordinate only (Formula 13.22).

For the ages under 10, Reed and Merrell note that L_x may be determined directly from the following linear equations:

$$L_0 = .276l_0 + .724l_1 \tag{13.23}$$

$$L_1 = .410l_1 + .590l_2 \tag{13.24}$$

$${}_4L_1 = .034l_0 + 1.184l_1 + 2.782l_5 \tag{13.25}$$

$${}_3L_2 = -.021l_0 + 1.384l_2 + 1.637l_5 \tag{13.26}$$

$${}_5L_5 = -.003l_0 + 2.242l_5 + 2.761l_{10} \tag{13.27}$$

L_0 should be determined from l_o and l_1 by use of separation factors appropriate for each situation, however. ${}_nL_x$ for ages 10 and over may be derived by taking the differences between the T_x's, and e_x is computed as the ratio of T_x to l_x.

An illustration of the application of the Reed-Merrell method is given in Table 13.3, which shows the calculation

TABLE 13.3 Calculation of Abridged Life Table for Males in Urban Colombia, 1993, by the Reed-Merrell Method

Age interval (exact ages, x to x + n)	$_nm_x$	$_nq_x$	$_nd_x$	l_x	$\sum_{\alpha=0}^{\infty} l_{x+5\alpha}$	T_x	$_nL_x$	e_x
0–1[3]	0.0275	0.025485	2,549	100,000	[1]	6,486,253	98,155[2]	64.86
1–4[3]	0.0015	0.005848	570	97,451	[1]	6,388,098	388,305[3]	63.88
5–9[4]	0.0006	0.002996	290	96,881	1,248,435	5,999,793	483,892[4]	61.57
10–14	0.0007	0.003494	338	96,591	1,151,554	5,516,162	482,418	57.11
15–19	0.0037	0.018343	1,766	96,253	1,054,963	5,033,744	477,352	52.30
20–24	0.0059	0.029103	2,750	94,487	958,710	4,556,392	465,721	48.22
25–29	0.0056	0.027642	2,536	91,737	864,223	4,090,671	452,249	44.59
30–34	0.0052	0.025691	2,292	89,201	772,486	3,638,422	440,239	40.79
35–39	0.0055	0.027155	2,360	86,909	683,285	3,198,183	428,637	36.80
40–45	0.0054	0.026667	2,255	84,549	596,376	2,769,546	417,181	32.76
45–49	0.0067	0.032988	2,715	82,294	511,827	2,352,365	404,928	28.58
50–54	0.0088	0.043120	3,431	79,579	429,533	1,947,437	389,820	24.47
55–59	0.0139	0.067320	5,126	76,148	349,954	1,557,617	368,711	20.46
60–64	0.0213	0.101433	7,204	71,022	273,806	1,188,906	338,112	16.74
65–69	0.0338	0.156455	9,985	63,818	202,784	850,794	295,191	13.33
70–74	0.0514	0.228672	12,310	53,833	138,966	555,603	239,142	10.32
75–79	0.0781	0.327397	13,594	41,523	85,133	316,461	173,617	7.62
80–84	0.1129	0.438558	12,248	27,929	43,610	142,844	44,407	5.11
85 and over[5]	0.1593	1.000000	15,681	15,681	15,681	98,437[5]	98,437	6.28

[1] Entries not required for the calculations desired.
[2] $_1L_0 = .276\, l_0 + .724\, l_1$
[3] $_4L_1 = .034\, l_0 + 1.184\, l_1 + 2.782\, l_5$
[4] $_5L_5 = -.003\, l_0 + 2.242\, l_5 + 2.761\, l_{10}$
[5] $T_{85} = {}_\infty L_{85} = l_x / {}_\infty m_x$ (= 15681/0.1593)
Source: Observed age-specific death rates from United Nations, *Demographic Yearbook*, 1996, Table 26.

of an abridged life table for urban Colombia in 5-year age intervals for 1993 from the observed central death rates from the United Nations' *Demographic Yearbook*. In this case, the conversion tables are employed to obtain $_nq_x$ at all ages and use is made of the tables for m_0, $_4m_1$, and $_5m_x$. The steps are as follows:

1. Read off $_nq_x$ values corresponding to $_nm_x$ values from the appropriate conversion table in Appendix A.
2. Derive the l_x and $_nd_x$ columns by the following steps:
   a. Multiply q_0 (.025485) by the radix l_0 (100,000) to obtain d_0 (2,549).
   b. Subtract d_0 from l_0 to get l_1 (97,451).
   c. Continue multiplying the successive values of l_x by the corresponding $_nq_x$ values to get $_nd_x$ and subtracting the successive values of $_nd_x$ from l_x to get l_{x+n}.
3. Sum the values of l_x from the end of life to age x.

$$\left(= \sum_{\alpha=0}^{\infty} l_{x+5\alpha} \right)$$

4. Substitute these sums and the indicated l_x values in equation (13.21) to get T_x. For example, T_{25} is obtained from Equation (13.21) as follows:

$$T_{25} = -.20833(94,487) + 2.5(91,737) + .20833(89,201) + 5(772,486) = 4,090,671$$

5. Derive $_nL_x$ for the ages under 10 years by use of Equations (13.23), (13.25), and (13.27). The separation factors given in Equation (13.23) have been used for convenience although 0.85 and 0.15 would have been more realistic choices.

Greville's Method

A method suggested by T. N. E. Greville (1943) converts the observed central death rates to the needed mortality rates by the use of the formula

$${}_nq_x = \frac{{}_nm_x}{\frac{1}{n} + {}_nm_x\left[\frac{1}{2} + \frac{n}{12}({}_nm_x - \log_e c)\right]} \tag{13.28}$$

where c comes from an assumption that the $_nm_x$ values follow an exponential curve. $\text{Log}_e c$ could be assumed to be about 0.095. Using this method with the observed death rate of .009263 for $_5m_{55}$ (United States, 1991) cited in the preceding page, would also lead to a mortality rate of .045341 as follows:

TABLE 13.4 Calculation of Abridged Life Table for Males in Urban Colombia, 1993, by the Keyfitz-Frauenthal Method

Age interval (exact ages, x to x + n)	n	${}_n\mathbf{m}_x$	${}_n\mathbf{P}_x$	$({}_n\mathbf{P}_{x-n} - {}_n\mathbf{P}_{x+n})({}_n\mathbf{m}_{x+n} - {}_n\mathbf{m}_{x-n})/48\,{}_n\mathbf{P}_x$	${}_n\mathbf{q}_x$	$\mathbf{l}_x$	${}_n\mathbf{d}_x$	${}_n\mathbf{L}_x$	$\mathbf{T}_x$	$\mathbf{e}_x$
0–1	1	0.0275	218,599		0.025485	100,000	2,549	98,155	6,529,903	65.30
1–4	4	0.0015	1,045,612		0.005848	97,451	570	388,305	6,431,748	66.00
5–9	5	0.0006	1,280,393		0.002996	96,881	290	483,892	6,043,443	62.38
10–14	5	0.0007	1,289,824	0.0000102	0.003545	96,591	342	481,788	5,559,551	57.56
15–19	5	0.0037	1,077,176	0.0000238	0.018447	96,249	1,775	476,277	5,077,763	52.76
20–24	5	0.0059	1,053,570	0.0000017	0.029077	94,474	2,747	465,284	4,601,486	48.71
25–29	5	0.0056	1,032,450	–0.0000012	0.027606	91,727	2,532	452,341	4,136,202	45.09
30–34	5	0.0052	967,247	–0.0000006	0.025662	89,195	2,289	440,237	3,683,861	41.30
35–39	5	0.0055	769,837	0.0000019	0.027134	86,906	2,358	428,590	3,243,624	37.32
40–45	5	0.0054	623,693	0.0000126	0.026700	84,548	2,257	416,968	2,815,034	33.30
45–49	5	0.0067	456,546	0.0000376	0.033127	82,291	2,726	404,315	2,398,066	29.14
50–54	5	0.0088	381,569	0.0000675	0.043369	79,565	3,451	388,550	1,993,751	25.06
55–59	5	0.0139	284,877	0.0001134	0.067669	76,114	5,151	366,585	1,605,201	21.09
60–64	5	0.0213	257,531	0.0001748	0.101810	70,963	7,225	335,034	1,238,616	17.45
65–69	5	0.0338	176,277	0.0004468	0.157376	63,738	10,031	291,060	903,582	14.18
70–74	5	0.0514	131,927	0.0006518	0.229148	53,707	12,307	234,252	612,522	11.40
75–79	5	0.0781	83,108	0.0012880	0.327626	41,400	13,564	168,663	378,270	9.14
80–84	5	0.1129	48,381	–0.0027950	0.423353	27,836	11,784	108,841	209,607	7.53
85 and over		0.1593	X	X	1.000000	16,052	16,052	100,766	100,766	6.28

Source: Observed age-specific death rates from United Nations, *Demographic Yearbook*, 1996, Table 26.

$$
{}_5q_{55} = \frac{.009263}{\dfrac{1}{5} + .009263\left[\dfrac{1}{2} + \dfrac{5}{12}(.009263 - .095)\right]}
$$

$$
{}_5q_{55} = \frac{.009263}{.204300}
$$

$$
{}_5q_{55} = .04534
$$

The derivation of ${}_5q_{55}$ by this method requires several columns in a manual calculation, but it may be programmed for direct calculation by computer on the basis of the ${}_5m_x$'s and the two constants, n and $\log_e c$. In Greville's method, the central death rates in the life table and in the observed population are assumed to be the same, and the desired value of ${}_nL_x$ is calculated by the use of

$$
{}_nL_x = \frac{{}_nd_x}{{}_nm_x} \tag{13.29}
$$

For the last age interval, that is, the interval with the indefinite upper age limit, the usual approximation for ${}_\infty L_x$ is

$$
{}_\infty L_x = \frac{l_x}{{}_\infty m_x} \tag{13.30}
$$

The Keyfitz-Frauenthal Method

Keyfitz and Frauenthal (1975) suggested the following procedure for converting the annual central age-specific death rate to the life table ${}_nq_x$:

$$
{}_nq_x = 1 - \exp\left[-n\cdot\left({}_nm_x + \frac{({}_nP_{x-n} - {}_nP_{x+n})({}_nm_{x+n} - {}_nm_{x-n})}{48\,{}_nP_x}\right)\right] \tag{13.31}
$$

where P is the observed population in the age interval. For example, the observed death rate for ${}_5m_{55}$ (U.S., 1991) cited in the preceding section would be converted to a mortality rate of .04553 as follows:

$$
{}_5q_{55} = 1 - \exp\left[-5\cdot\left((.009263) + \frac{(14,093,824 - 10,423,513)\cdot(.014319 - .005758)}{48\cdot(11,644,495)}\right)\right]
$$

The desired value of ${}_nL_x$ is calculated as

$$
{}_nL_x = \frac{n(l_x - l_{x+n})}{\ln l_x - \ln l_{x+n}}\left[1 + \frac{n}{24}({}_nm_{x+n} - {}_nm_{x-n})\right] \tag{13.32}
$$

An illustration of the application of the Keyfitz-Frauenthal method is given in Table 13.4, which shows the calculation of an abridged life table for urban Colombia in 5-year age intervals for 1993. The steps are as follows:

1. Set down n, the width of each age interval.
2. Calculate differences between alternate populations and between alternate death rates, and multiply them together, to obtain the numerator.
3. Calculate the denominator.
4. Obtain value of exponential term.
5. Calculate ${}_nq_x$ as 1 – (4).
6. Obtain ${}_nL_x$. Derive ${}_nL_x$ for the ages under 5 years by use of Equations (13.23) and (13.25). The separation factors given in Equation (13.23) have been used for

convenience although 0.85 and 0.13 would have been more realistic choices (see p. 285).

7. Calculate T_x and e_x in usual manner. Save ${}_\infty L_x$ by $l_x \div {}_\infty m_x$.

Method of Reference to a Standard Table

A fourth method that is used frequently in routine calculations bases the conversion of the observed ${}_nm_x$ to the life table ${}_nq_x$ on the relation that exists in a complete life table between the observed ${}_nm_x$ and the life table ${}_nq_x$ (U.S. National Office of Vital Statistics, 1947). Because this method obtains the new table by reference to a standard table, it should only be used when mortality in both tables is of a comparable level.

A simple application of this concept assumes that in each age interval the relation of ${}_nq_x$ to observed ${}_nm_x$ shown by the standard table applies to the table under construction. This relation was used in the construction of the annual U.S. abridged tables for 1946 to 1996. New standard tables were adopted and the factors employed were modified when new decennial life tables became available. The value ${}_ng_x$ is computed for the standard life table on the basis of the observed central death rate (${}_nm_x$) and the mortality rate (${}_nq_x$), using the formula

$$ {}_ng_x = \frac{n}{{}_nq_x} - \frac{1}{{}_nm_x} \tag{13.33} $$

where ${}_ng_x$ represents the average number of years lived in the age group by those dying in the age group (Sirken, 1966). The index ${}_ng_x$ lies between 0 and n. This value is assumed to apply to the new life table at the same age. The required mortality rates in the new life table are computed using the formula

$$ {}_nq_x = \frac{n \cdot {}_nm_x}{1 + {}_ng_x \cdot {}_nm_x} \tag{13.34} $$

where ${}_nm_x$ refers to the observed ${}_nm_x$ values applicable to the new table under construction. In the previous example, if the 1989–1991 U.S. life table was used as the standard, the value of, ${}_5g_{55}$ would be equal to 2.97741. The observed death rate of .009263 for ages 55 to 59 in the United States in 1991 would be converted to the mortality rate for 1991, .045392, as follows:

$$ {}_5q_{55} = \frac{5(.009263)}{1 + 2.195846(.009263)} $$

$$ {}_5q_{55} = \frac{.046315}{1.020340} $$

$$ {}_5q_{55} = .045392 $$

For the calculation of ${}_nL_x$ in the abridged table, one may apply the simple relationship ${}_nL_x / (l_x + l_{x+n})$ from the standard table (U.S. National Office of Vital Statistics, 1947). The annual abridged U.S. tables made since 1954 employ another relationship between l_x and ${}_nL_x$, involving a factor we may designate as ${}_nG_x$ (Sirken, 1966). The value of ${}_nG_x$, representing the distribution of deaths in the interval x to $x + n$, is obtained from the standard table by the formula

$$ {}_nG_x = \frac{nl_x - {}_nL_x}{{}_nd_x} \tag{13.35} $$

This value is assumed to apply to the new table, and it is used in the formula

$$ {}_nL_x = nl_x - {}_nG_x \cdot {}_nd_x \tag{13.36} $$

to obtain the desired value of ${}_nL_x$ in the new table. The value for the open-ended interval, ${}_\infty L_x$, is determined by a special formula. A factor r_x is computed for the standard table as follows:

$$ r_x = \frac{{}_\infty m_x \cdot {}_\infty L_x}{l_x} \tag{13.37} $$

where ${}_\infty m_x$ is the observed central death rate for the open-ended interval. This factor is applied in the new table by using the formula

$$ {}_\infty L_x = \frac{l_x r_x}{{}_\infty m_x} \tag{13.38} $$

The method of reference to a standard table is particularly useful and convenient for constructing an annual series of life tables. The ${}_ng_x$ and ${}_nG_x$ factors can be calculated once from a complete life table in the initial year and used repeatedly for each year until a new complete table is prepared.

Comparison of Methods

A comparison of the mortality rates for the United States in 1991 derived by the four methods described is shown in Table 13.5. It may be observed that the results are very similar. Tables 13.3 and 13.4 may be compared for the difference in results when the Reed Merrell method and the Keyfitz-Frauenthal method are applied to urban Colombia in 1993. It should be noted that in Greville's method the central death rates in the life table (${}_nm_x^l$) are required to agree with the central death rates observed in the population (${}_nm_x^p$). This is not the case in the Reed-Merrell method or in the method of reference to a standard table, however. There is no generally agreed upon requirement that these two sets of central death rates should be equal in numerical value. In fact, it can be maintained that because the distribution of the life table population is different from the distribution of the actual population in any age interval, the two rates should be different. A method has been proposed by Keyfitz to compute a life table that would agree with the observed data after adjustment for the differences in the age distribution of the populations (Keyfitz, 1966; 1968). This method is complex and is beyond the scope of this book.

GENERATION LIFE TABLES

In previous sections of this chapter, various methods for preparing current life tables have been discussed. In every instance, the basic data for these methods involve a census

TABLE 13.5 Comparison of Mortality Rates for the Total United States Population, 1991, Computed by Four Different Abridged Life Table Methods

Age interval (exact ages, x to x + n)	${}_ng_x$ Standard[1]	${}_nm_x$ Observed[2]	Computed ${}_nq_x$			
			Reed- Merrell	Greville	Reference to standard table	Keyfitz-Frauenthal
1–5	422.203800	0.000474	0.001856	0.002367	0.001975	
5–10	−103.093000	0.000215	0.001074	0.001074	0.001099	
10–15	−45.045000	0.000258	0.001289	0.001289	0.001305	0.001293
15–20	8.575279	0.000890	0.004441	0.004441	0.004416	0.004432
20–25	3.345601	0.001101	0.005491	0.005491	0.005485	0.005483
25–30	4.249223	0.001230	0.006133	0.006133	0.006118	0.006125
30–35	−1.625560	0.001541	0.007678	0.007678	0.007724	0.007676
35–40	0.308471	0.001977	0.009840	0.009840	0.009879	0.009853
40–45	3.585178	0.002536	0.012606	0.012606	0.012566	0.012664
45–50	1.963739	0.003805	0.018859	0.018859	0.018884	0.019011
50–55	2.246376	0.005758	0.028412	0.028411	0.028422	0.028554
55–60	2.195846	0.009263	0.045341	0.045340	0.045392	0.045346
60–65	2.278723	0.014319	0.069283	0.069282	0.069333	0.069135
65–70	2.294695	0.021368	0.101741	0.101739	0.101846	0.101717
70–75	2.352904	0.032051	0.148948	0.148946	0.149017	0.149153
75–80	2.421006	0.048068	0.215454	0.215461	0.215286	0.216034
80–85	2.497600	0.075754	0.319215	0.319274	0.318507	0.320953

[1] Computed from the life table for the United States, 1981, total population, which is used as the standard table. The ${}_ng_x$ factors at ages below 50 seem irregular and would be expected to approximate 2.5. However, where death rates are low, the computed ${}_nq_x$ is relatively insensitive to large variations in ${}_ng_x$.

[2] Ratio of deaths to population observed in the United States in 1991.

of the population (or accurate estimates of the population distributed by age and sex), the deaths recorded in the year of the census or in a number of years around the census year, and the births in a few years just prior to the census year. Life tables prepared from these data portray the mortality experience of the population observed during the relatively short period to which the data on deaths apply.

Because life tables are mathematical models that trace a cohort of lives from birth to death according to an assumed series of mortality rates, it would be logical to try to base the life table on the mortality rates experienced by an actual birth cohort. For this purpose, it is necessary to have collected data on an annual basis for many years before a life table could be prepared. For example, a generation life table for the birth cohort of 1900 would employ the observed death or mortality rates for infancy in 1900, for age 1 in 1901, for age 2 in 1902, and so on until the latest available rates at the highest ages are obtained; the remaining rates would have to be obtained by projection. Table 13.6 presents selected values from a generation life table for the cohort of white females born in 1900 in the United States, prepared by Bell, Wade, and Goss (1992).

A sufficient period of time has elapsed since mortality data have been systematically recorded for a number of generation life tables to have been prepared. For example, Jacobson (1964) prepared life tables for white males and females born in the United States in 1840 and every 10th year thereafter through 1960, on the basis of the mortality rates experienced in each year of the life of these cohorts and projected mortality rates. Earlier, Dublin and Spiegelman (1941), Dublin, Lotka, and Spiegelman (1949), and Spiegelman (1957) developed or described such tables. Generation life tables can be used to compute generation reproduction rates, to study life expectancy historically, to project mortality, and to make estimates of orphanhood (Gregory, 1965). A series of such tables could represent the development of mortality and life expectancy of real cohorts over time and improve the basis for analyzing the relation between the earlier mortality of a cohort and its later experience. In view of the general tendency for mortality to fall, the expectation of life at birth in a generation life table tends to be higher than in the current table for the starting year of the generation life table. A comparison of life expectations at birth in two such tables reflects the average improvement in mortality for the actual cohort over its lifetime. Conversely, current life tables typically understate the probable life expectancy to be observed in the future for persons at a given age because mortality is expected to continue declining. The generation life table for the cohort of U.S females born in 1900, for example, shows a life expectation at birth of 58.3 years (Bell *et al.*, 1992). In contrast, the current life table for 1900 shows a life expectation at birth of 49.0 years. The actual experience of the 1900 cohort added nearly 10 years to its life expectation at birth.

TABLE 13.6 Selected Values from the Generation Life Table for the Cohort of Females Born in the United States in 1900

Calendar year	Exact age (x)	Mortality rate (q_x)	Number surviving (l_x)	Average remaining lifetime (e_x)
1900	0	.11969	100,000	58.3
1901	1	.05470	88,031	65.2
1905	5	.00624	82,532	65.4
1910	10	.00172	81,108	61.5
1915	15	.00105	80,255	57.1
1920	20	.00221	78,484	53.4
1925	25	.00239	76,611	49.6
1930	30	.00234	74,814	45.7
1935	35	.00217	73,192	41.7
1940	40	.00231	71,500	37.6
1945	45	.00239	69,792	33.5
1950	50	.00283	67,819	29.4
1955	55	.00411	65,034	25.4
1960	60	.00439	62,177	21.6
1965	65	.00711	57,755	18.0
1970	70	.00987	52,055	14.7
1975	75	.14062	44,735	11.7
1980	80	.19618	35,959	8.9
1985	85	.29205	25,457	6.6
1990	90	.43457	14,394	4.7
1995	95	.60400	5,700	3.4
2000	100	.76175	1,358	2.5
2005	105	.87040	176	1.9
2010	110	.94318	10	1.4

Rates after 1985 are projected.

Source: F. C. Bell, A. Wade, S. Goss, "*Life Tables for the United States Social Security Area*, 1900–2080" *Actuarial Study* No. 107, Office of the Actuary, U.S. Social Security Administration. SSA Pub. No. 11-11536. August, 1992, pp. 54–55.

The analysis of cohort mortality could serve as an improved basis for projecting mortality. One approach used at present is to trace the mortality experience of a series of cohorts from a group of current life tables prepared in the past for as many years back as is considered reliable. The mortality curves for these cohorts are incomplete to various degrees, but once the cohorts have reached the older ages, the mortality curves can be projected over their remaining lifetime in various ways. Currently, the general practice, however, is to project mortality on an age-specific basis rather than on a cohort basis—that is, to analyze the series of rates for the same age group and to project the rates for each age group separately.

As stated earlier, the preparation of a generation life table requires compilation of data over a considerable period of time on an annual basis and also the projection of some or many incomplete cohorts. It is, therefore, a burdensome task to prepare many such tables "manually," but the use of a computer reduces this burden considerabley. (The basic "input" for an annual series of generation life tables, for example, is the matrix of observed central death rates by single ages and single calendar years.) The fact that the basic data for a single table pertain to many different calendar years means that the time reference of the generation life table is indefinite. The indefiniteness of the time reference is more pronounced for cohort life tables than it is for cohort measures of fertility, for example, which have a more concentrated incidence by age.

One characteristic of the generation life table may be viewed either as an advantage or as a disadvantage, depending on one's interest. A generation life table can involve the combination of mortality rates that are significantly different in nature, owing to the improvement in mortality over a long period of time. For example, the health conditions today at the turn of the 21st century are significantly different from those existing at the turn of the 20th century and the corresponding mortality patterns are dissimilar. On the other hand, this type of table does reflect the actual combination of changing health conditions and intracohort influences to which particular cohorts were subject. A current table reflects the influence of a more unitary set of health conditions and a single mortality pattern.

INTERRELATIONS, COMBINATION, AND MANIPULATION OF LIFE TABLE FUNCTIONS

Most demographers are only infrequently faced with the task of constructing a complete, or even an abridged, life table. Many do often have the task of manipulating in various ways available life tables or life table functions. This may involve the calculation of a missing function for a given table, the combination of functions from different tables, or the combination of entire tables.

Interrelations of Functions

The manipulation of life table functions is aided by a review of how these functions are interrelated. All the functions in a life table are dependent on other functions in the table, but usually the q_x function is regarded as being independent of all other functions. A particular q_x value is clearly independent of all other q_x values at either earlier or later ages. A particular l_x, d_x, or L_x value is dependent only on the values of q_x at age x and earlier ages. A particular T_x value is dependent on values of L_x at age x and later ages, and hence on the entire column of q_x values. A particular value of e_x is dependent, in effect, only on the values of q_x at age x and all later ages. Although e_x is computed from T_x and l_x, which are dependent on earlier values of q_x, the absolute values of the T_x's and l_x's are "washed out" in the calculation of e_x. Implicitly, in the derivation of e_x, the q_x's for age x and later ages are weighted by the percentage distribution of the L's that they generate. Because of the sharp drop from q_0 to q_1, e_1 is commonly higher than e_0; thereafter, e_x tends to fall steadily. At low levels of q_0, e_0 begins to exceed e_1.

In some cases, values are presented for only some of the life table functions. Some or all of the others may be desired. Let us consider the case where, as in the UN *Demographic Yearbook*, we are given 1-year values of q_x, l_x, and e_x at every fifth age and we want $_5L_x$ values for computing 5-year survival rates (ratios of $_5L_x$'s; see later). We apply the following relationships:

$$e_x = \frac{T_x}{l_x}$$

$$\therefore T_x = e_x l_x$$

$$\text{and } {}_5L_x = T_x - T_{x+5}$$

The $_5q_x$'s corresponding to the l_x values at every fifth age are calculated by $1 - \frac{l_{x+5}}{l_x}$ and the $_5d_x$'s corresponding to the l_x values are calculated by $l_x - l_{x+5}$.

Given the same basic information as just noted, we may wish to secure the values for the individual ages of some function. In that case, any of the standard interpolation formulas may be applied. We may interpolate q_x or l_x to single ages and obtain single values of d_x as a by-product. Or we may derive T_x at every fifth age as described in the preceding paragraph and then apply one of the interpolation formulas to secure T_x at single ages. Single values for e_x may be secured by direct interpolation or by interpolating the l_x function to single ages and completing the table. (Interpolation procedures are discussed more fully in Appendix C.)

Combination of Tables and Functions

First, let us consider the case where life tables for two populations are available and we should like to have a single life table for the combined population. Suppose, for example, we want to derive a single table for the general population from separate life tables for the male and female populations. This can be done by weighting the q_x's from the male and female tables in accordance with the distribution of the population by sex at each age. A new table may now be recomputed from the combined q_x's and an assumed radix of 100,000. If only one of the other functions is wanted, the values from the separate tables may be combined on the basis of population weights in only a few scattered ages because the shifts from age to age in the distribution of the population by sex could introduce unacceptable fluctuations by age in these functions for the combined population, and the resulting values for different functions may not be consistent with one another. (Note that these functions would not all receive the same population weights, e.g., l_x would be weighted by the population at age x, and T_x and e_x would be weighted by the population for ages x and over, the weights in each case reflecting the ages covered by the measure.) Only the use of a constant weighting factor over all ages can prevent fluctuations by age in the values of a function. If an entire column for a function is wanted, therefore, it is best to recompute the values for the function by starting with the weighted q_x's.

A different problem is represented by the need for separate male and female life tables whose values bear a realistic absolute relation to one another. The tables for males and females as originally computed do not bear a consistent "additive" relationship to one another because births of boys and girls do not occur in equal numbers, as is implied by the radixes of the separate life tables for males and females. To make the tables additive, it is necessary to inflate the l_x, d_x, L_x, and T_x values of the male life table by the sex ratio at birth in the actual population. Thus, the stationary male population at each age adjusted for absolute comparability with the female stationary population is derived by

$$\frac{B^m}{B^f} \cdot L_x^m \tag{13.39}$$

and the stationary male population of all ages, adjusted for combination with the total female stationary population, is obtained by

$$\frac{B^m}{B^f} \cdot T_0^m \tag{13.40}$$

The birthrate (and also the death rate) for the male and female life tables taken in combination is given by

$$\frac{\frac{B^m}{B^f} l_0 + l_0}{\frac{B^m}{B^f} T_0^m + T_0^f} \tag{13.41}$$

The ratio of children under 5 to women of childbearing age in the stationary population, a measure known as the replacement quota, is obtained by

$$\frac{\frac{B^m}{B^f} \cdot L_{0-4}^m + L_{0-4}^f}{L_{15-49}^f} \tag{13.42}$$

We have been describing the procedures by which we may arrive at life table values for a total population consistent in absolute level with the life table values for the separate component male and female populations. Even if the stationary populations are properly combined and reduced to the level of a single table with a radix of 100,000 by applying the factor

$$\frac{100{,}000}{\frac{B^m}{B^f} 100{,}000 + 100{,}000} \tag{13.43}$$

to the absolute functions in each table, the resulting d_x, l_x, L_x, and T_x would not agree exactly with the values derived directly from the weighted q_x's. Nor will the e_x or q_x values or the derived survival rates agree. Figures for e_0 consistent with the derivation of a combined table just described may be derived from the reciprocal of the formula given earlier for the birthrate:

$$\frac{\frac{B^m}{B^f}T_0^m + T_0^f}{\frac{B^m}{B^f}l_0 + l_0} \qquad (13.44)$$

The q_x's can be obtained as the ratio of d_x to l_x:

$$\frac{\frac{B^m}{B^f}d_x^m + d_x^f}{\frac{B^m}{B^f}l_x^m + l_x^f} \qquad (13.45)$$

The directly computed table takes account of the actual distribution of the population by sex at each age, while the method described links the two tables only by the sex ratio at birth. The differences between these two combined tables tend to be quite small, even inconsequential, however, because the proportions of the sexes approximate equality and have only a small variation over much of the age distribution. On the other hand, the two combined tables for the white and nonwhite populations of the United States would show more pronounced differences because the proportions of these groups in the population are quite different and vary more by age. To convert the basic tables for the white and nonwhite populations into additive form, the factor for adjusting the nonwhite table (B_N/B_T) is about .20; and to derive the combined table with a radix of 100,000, the factor is about 100,000 / 120,000, or .83.

STATISTICAL ANALYSIS OF LIFE TABLES

Frequently it is difficult to see whether the differences between two life tables are significant or merely reflect chance fluctuations. Sampling variation is especially important when the life tables are based on sample surveys or follow-up observational studies, but it also affects life tables based on vital rates from large populations, such as countries. In this case, deaths are viewed as a random sample of the superpopulation of deaths that have occurred over time and might occur. In recognition of this stochastic variation, the U.S. National Center for Health Statistics publishes standard errors for the probabilities of dying and life expectancies in decennial life tables.

Statistical tests are based on an interpretation of the count of deaths in a population as a random variable in the observed population. Chiang (1968) assumed that ${}_nD_x$,the count of actual deaths in an age-sex group, is a binomial random variable in ${}_nP_x$ persons (trials) with fixed probability of dying ${}_nq_x$. Based on this assumption, the expected number of deaths in a sample of persons is ${}_nP_x \cdot {}_nq_x$, The variance of the number of deaths is $Var({}_nD_x) = {}_nP_x \cdot {}_nq_x \cdot {}_np_x$, where p is the probability of surviving or 1-q_x.

This leads to an estimate for the variance of the age-specific probability of dying:

$$Var\ {}_nq_x = \frac{{}_nq_x^2(1 - {}_nq_x)}{{}_nD_x} \qquad (13.46)$$

The variance of life expectancy is

$$Var\ e_x = \frac{\sum_{a=x}^{\omega-n} l_a^2\left[e_{a+n} + n - {}_na_a\right]^2 Var({}_nP_a)}{l_x^2} \qquad (13.47)$$

Statistical tests for differences in life expectancy at age x between two populations i and j is

$$Z = \frac{e_{x,i} - e_{x,j}}{\left[(Var(e_{x,i}) + Var(e_{x,j}))^{.5}\right]} \qquad (13.48)$$

For example, Hummer, Rogers, Nam, and Ellison (1999) used nationally representative data from the National Health Interview Survey-Multiple Cause of Death linked file to examine the association of religious attendance and sociodemographic factors with overall mortality. They calculated life expectancy at age 20 for both sexes and all race/ethnic groups. They examined whether e_{20} differs significantly between those attending religious services once per week, whose e_{20} was 61.912 years, and those attending services more than once per week, whose e_{20} was 62.925 years. The statistical test for differences in e_{20} between the two groups is

$$Z = \frac{62.925 - 61.912}{\sqrt{(.0095524)^2 + (.0066757)^2}} = 86.92$$

The value of the test statistic exceeds the critical value of 1.96, so we can conclude that this difference is statistically different from zero. Therefore, persons who attend religious services more than once per week have significantly different life expectancy from those who attend only once per week.

Because life tables for the United States and the states are based on a large number of deaths, the standard errors are rather small. Stochastic variation is not the only source of error for life table functions, and it is generally thought to be smaller than measurement errors, such as age misreporting.

APPLICATIONS OF LIFE TABLES IN POPULATION STUDIES

Inasmuch as the measurement of mortality is involved in many types of demographic studies, the life table model and life table techniques as special tools for measuring mortality can be applied in a wide variety of population studies. The life table is a tool in the analysis of fertility, reproductivity, migration, and population structure; in the estimation and projection of population size, structure, and change; and in the analysis of various social and economic characteristics of the population, such as marital status, labor force status, family status, and educational status. In many of these cases, the standard life table functions are combined with probabilities for other types of contingent events (e.g., first-marriage rates, rates of entry into the labor force) in order to obtain new measures.

Mortality Analysis

It is impractical to discuss here all of the many types of analyses of mortality that could be performed on the basis of life table values. The most frequently used procedures involve comparisons between populations of life expectancy at birth and at various ages, of proportions surviving to various ages, and of mortality rates at various ages.

Measurement of the Level of Mortality, Survivorship, and Life Expectancy

The most common general use of a life table is to measure the level of mortality of a given population. As such, it offers some new measures that eliminate some of the problems involved in the use of existing standard measures. For example, the difference in the crude death rates of two populations, as was indicated in the previous chapter, is affected by the difference in the age composition of the populations. However, standardized rates have the disadvantage of depending on the particular selection of a standard population. Life tables have the advantage that the overall death rate of the life table (m^l) represents the result of the weighting of the age-specific life table death rates (m_x^l) by a population (L_x) generated by the series of observed death rates themselves. However, because the weighting scheme involves a variable population distribution from table to table, there is still an issue of comparability between the overall life table death rates. As will be described later, the life table also has the special advantage of providing measures of mortality that automatically are structured in cohort form. Hence, it eliminates the usually laborious task of compiling death statistics according to birth cohorts, even where this is possible. Often, insufficient basic data are available for doing this for a particular country and a life table is the only means of making a necessary allowance for the mortality of age cohorts.

The *expectation of life at birth* is the life table function most frequently used as an index of the level of mortality. It also represents a summarization of the whole series of mortality rates for all ages combined, as weighted by the life table stationary population. In fact, the reciprocal of the expectation of life, $1/e_0$, is equivalent to the "crude" death rate, m^l, of the life table population, as can be seen from the following derivation:

$$m^1 = \text{Total number of deaths/Total population}$$
$$= \frac{l_0}{T_0} = \frac{1}{\frac{T_0}{l_0}} = \frac{1}{e_0} \qquad (13.49)$$

For example, the death rate in the 1989–1991 U.S. life table is calculated from Table 13.1 as follows:

$$m^1 = \frac{l_0}{T_0} = \frac{100{,}000}{7{,}536{,}614} = .01327, \text{ or} \qquad (13.50)$$

$$m^1 = \frac{1}{e_x} = \frac{1}{75.37} = .01327 \qquad (13.51)$$

The same formulas give the "crude" birthrate, f^l, of the life table population, and the growth rate of this population (r^l) is, of course, zero:

$$f^l = \frac{l_0}{T_0} = \frac{1}{e_0} = m^l \qquad (13.52)$$

$$\therefore r^l = f^l - m^l = 0 \qquad (13.53)$$

Because the infant mortality rate strongly affects the expectation of life at birth, the *expectation of life at age 1* has been suggested as a comparative measure of the general level of mortality of a population, perhaps in conjunction with the infant mortality rate. Another life table function frequently used is the *expectation of life at age 65*. This value measures mortality at the older ages, the ages where most of the deaths in the developed countries currently occur. (Much of the measured change here may reflect inadequacies in the underlying data.) Other life table values used to measure mortality are the *probability of surviving from birth to the 65th birthday*.

$$_{65}p_0 = \frac{l_{65}}{l_0} \qquad (13.54)$$

and the age to which half of the cohort survives—that is, the *median age at death* of the initial cohort assumed in the life table. The median age at death is the age corresponding to l_x value 50,000 in a life table with a radix of 100,000.

Illustrative changes in the mortality record of white males in the United States during the 20th century, as measured by life table values, are shown in Table 13.7. It suggests that significant improvement has been recorded during the first year of life. The variation in these measures is suggested by the figures for several countries, which are shown in Table 13.8.

Measures based on the life table have been used in the intensive analysis of vital statistics. Naturally, the type of vital statistics most frequently analyzed with life table

TABLE 13.7 Change in the Mortality of White Males in the United States According to Various Life Table Measures: 1900 to 1996 [Life tables for periods before 1929–31 relate to the Death Registration States]

Period	Expectation of life at birth	Expectation of life at age 1	Expectation of life at age 65	Probability of surviving from birth to age 65	Median age at death of initial cohort
1900–02	48.23	54.61	11.51	.392	57.2
1909–11	50.23	56.26	11.25	.409	59.3
1919–21	56.34	60.24	12.21	.507	65.4
1929–31	59.12	62.04	11.77	.530	66.4
1939–41	62.81	64.98	12.07	.583	68.7
1949–51	66.31	67.41	12.75	.635	70.7
1959–61	67.55	68.34	12.97	.658	71.4
1969–71	67.94	68.33	13.02	.663	71.5
1979–81	70.82	70.70	14.26	.724	74.2
1989–91	72.72	72.35	15.24	.760	76.1
1996	73.90	73.40	15.80	.779	77.2
Increase, 1900–2 to 1996	25.67	18.79	4.29	.387	20.0
Increase, 1949–51 to 1996	7.59	5.99	3.05	.144	6.5

Source: Various official United States life tables.

TABLE 13.8 Mortality of Females According to Various Life Table Measures, for Selected Countries: Around 1990

Country and year	Expectation of life at- Birth	Age 1	Age 65	Probability of surviving from birth to age 65	Median age at death of initial cohort
Argentina, 1990–2	75.59	76.30	17.26	.826	80.3
Bangladesh, 1988	55.97	61.50	11.98	.566	68.6
Canada, 1992	80.89	80.36	19.88	.886	83.8
China, 1990	70.49	71.92	14.74	.756	75.8
Egypt, 1991	66.39	58.65	12.87	.702	72.6
India, 1986–90	58.90	62.60	12.90	.572	68.6
Japan, 1994	82.98	82.29	20.97	.919	>85
Moldova, 1994	69.79	70.18	13.19	.717	73.6
Peru, 1990–95	66.55	70.65	14.59	.727	77.3
Philippines, 1991	66.70	68.70	13.70	.692	73.6
Russian Federation, 1994	71.18	71.29	14.58	.741	75.4
Zimbabwe, 1990	62.00	65.10	13.30	.613	70.7

Source: United Nations *Demographic Yearbook, 1996*, Tables 32 and 33.

measures is mortality because these measures were originally designed for precisely that application, and that application is usually the simplest. The analysis of mortality could cover variations in time and between population groups by age, sex, ethnic group, race, marital status, geographic residence, and other demographic characteristics that may be regarded as being associated with the overall level of mortality. Table 13.9 shows, for example, the variation in time, by age, of the relative mortality of white males and females in the United States in terms of life table mortality rates. Although mortality of white males has improved tremendously during the 20th century in the United States (Table 13.7), the improvement has been greater for white females (Table 13.9).

TABLE 13.9 Mortality Rate of White Females as Percent of the Mortality Rates of White Males in the United States for Selected Ages, as shown by various life tables: 1900 to 1996 [Life tables for periods before 1929–31 relate to the Death Registration States]

Period	Exact age 0	1	10	20	30	40	50	60	70	80
1900–02	83	90	90	93	97	88	87	88	91	91
1909–11	83	92	87	86	91	79	81	84	91	93
1919–21	80	90	85	101	105	90	91	88	92	95
1929–31	80	89	77	87	91	78	75	78	84	90
1939–41	79	89	70	68	79	72	66	67	78	87
1949–51	77	89	67	45	63	62	55	56	68	82
1959–61	76	88	67	35	54	57	50	48	58	77
1969–71	76	87	73	34	49	57	52	45	51	68
1979–81	78	84	89	32	39	55	53	50	50	61
1989–91	77	89	88	36	35	45	57	56	56	61
1996	82	77	66	33	41	49	59	60	63	72

Note: Ratios of q_x per 100.
Source: Various official United States life tables.

Model life tables may be used to evaluate the quality of mortality data for statistically underdeveloped populations and to establish the validity of recorded differences in the patterns of mortality of these populations. The mortality data for a country in earlier historical periods or for some segment of the current population may appear to be so defective as to raise a question whether the corresponding life tables provide an accurate representation of the patterns of mortality actually experienced. Zelnik (1969), for example, compared the age patterns of mortality in official U.S. life tables for whites and blacks from 1900–1902 to 1959–1961 with Coale-Demeny model life tables to examine the quality of the U.S. official life tables for blacks (see Chapter 22).

Analysis of Fertility, Reproductivity, and Age Structure

Life table measures and techniques have also been used to analyze other vital phenomena in addition to mortality. The procedures usually involve the combination of mortality with the specific vital rates that are being analyzed. These procedures have particular application in studies of fertility and reproductivity. For example, age-specific fertility rates are combined with survival rates from life tables to calculate the net reproduction rate (Chapter 17). The mean length of a generation, which is simply the average age of mothers at the birth of their daughters, is another measure based on both fertility rates and life table survival rates.

The life table is an important instrument for the analysis of population dynamics and age structure. Studies of the relation of growth rates, birthrates, and death rates to age structure depend heavily on the use of life tables, particularly to indicate the effects on age structure of various levels of mortality and to develop the "stable" age distributions corresponding to various levels of fertility (Chapter 23). The stationary population distributions of life tables correspond to given levels of mortality and a zero growth rate.

Calculation of Life Table Survival Rates

Survival rates are defined in terms of two ages, and hence two time references, the initial age and date and the terminal age and date. Both age references are equally applicable, but survival rates are more commonly identified symbolically in terms of the initial ages than in terms of the terminal ages. Survival rates express survival from a younger age to an older age, but they can be used to restore deaths to a population. Survival rates used to reduce a population for deaths are multiplied against the initial population; survival rates used to restore deaths are divided into the terminal population in a reverse calculation:

$$\text{Forward survival: } {}_5P_x^t \cdot {}_5s_x^5 = {}_5E_{x+5}^{t+5} \tag{13.55}$$

$$\text{Reverse survival: } \frac{{}_5P_x^t}{{}_5s_x^5} = {}_5E_{x-5}^{t-5} \tag{13.56}$$

where the elements represent the initial or terminal populations, the 5-year survival rate, and the expected later or earlier populations. Normally, "revival" rates, which directly express "revival" from an older to a younger age, are not used.

The most common form of survival rate employed in population studies is for a 5-year age group and a 5-year time period. The general formula is

$${}_5s_x^5 = \frac{{}_5L_{x+5}}{{}_5L_x} \tag{13.57}$$

According to the 1989–1991 U.S. life table (Table 13.2), the proportion of the population 45 to 49 years old that will survive 5 years is calculated as follows:

$${}_5s_{45}^5 = \frac{455{,}538}{466{,}610} = .97627$$

The proportion of the population 75 years and over that will live another 10 years is

$$\begin{aligned} {}_\infty s_{75}^{10} &= \frac{{}_\infty L_{85}}{{}_\infty L_{75}} \quad \text{or} \quad \frac{T_{85}}{T_{75}} \\ &= \frac{197{,}871}{665{,}131} = .29749 \end{aligned} \tag{13.58}$$

Survival rates for population age groups are computed from the L_x values of the life table, using the L_x value for the initial age group as the denominator and the L_x value for the terminal age group as the numerator.

A complete life table readily permits calculation of survival rates for single ages for 1 year or any other time period. A 1-year survival rate for a single age is represented by $s_x = \frac{L_{x+1}}{L_x}$. The proportion of the population 64 years of age on a given date that will survive to the same date in the following year, on the basis of the 1989–1991 complete U.S. life table (Table 13.1), is

$$s_{64} = \frac{L_{65}}{L_{64}} = \frac{78{,}793}{80{,}203} = .98242$$

The proportion of 65-year-olds who are expected to live another 10 years is

$$s_{65}^{10} = \frac{L_{75}}{L_{65}} = \frac{59{,}201}{78{,}793} = .75135$$

Survival rates involving birthdays are computed using the l_x values. The proportion of newborn babies who will reach their fifth birthday is

$$\frac{l_5}{l_0} = \frac{98{,}877}{100{,}000} = .98877$$

The proportion of 60-year-olds that will reach their 65th birthday is

$$\frac{l_{65}}{L_{60}} = \frac{79{,}519}{85{,}014} = .93536$$

The proportion of infants born in a year who will survive to the end of that year (when the cohort is under 1 year of age) is

$$\frac{L_0}{l_0} = \frac{99,258}{100,000} = .99258$$

while the proportion of the newborn infants who will reach their first birthday is

$$\frac{l_1}{l_0} = \frac{99,064}{100,000} = .99064$$

If survival from birth to a given age interval is wanted, then the survival rate is $\frac{{}_nL_x}{nl_0}$ or, for a 5-year age group, $\frac{{}_5L_x}{5l_0}$. Here n, or 5, cohorts of 100,000 births are at risk. For survival from birth to age interval 30 to 34, we have

$$\frac{483,546}{500,000} = .96709$$

Survival rates involving 5-year age groups may also be computed using values of L_x or l_x for the midpoint age. The survival rate from birth to age interval 30 to 34 may be approximated by

$$\frac{l_{32.5}}{l_0} = \frac{L_{32}}{l_0} = \frac{96,717}{100,000} = .96717$$

This approximation has little effect on the survival rate, particularly in the ages up through young adulthood.

Survival rates for parts of a calendar year may be calculated by interpolating between the L_x's. A $\frac{3}{4}$-year survival rate from a complete life table is computed as follows (except at age under 1):

$$\frac{\frac{1}{4}L_x + \frac{3}{4}L_{x+1}}{L_x} \tag{13.59}$$

For age 45,

$$\frac{\frac{1}{4}(94,006) + \frac{3}{4}(93,693)}{94,006} = .9975$$

For ages under 1, life tables giving L_x values by months of age or other subdivisions of age under 1 should be used. Alternatively, special factors may be derived from statistics for infant deaths in a manner similar to the way the separation factors for all infant deaths were derived.

Use of Life Table Survival Rates

In the use of life table survival rates in population studies, decisions have to be made regarding the selection of the life table and life table survival rates most appropriate for a particular problem. Where a life table is not available for the particular year or period, but for prior and subsequent years or for the initial and terminal years of the period, special survival rates may have to be computed on the basis of the available life tables. Commonly, for example, life tables are available for the census years, but we are interested in measuring population changes or net migration for the intercensal period. Survival rates appropriate for this purpose may be derived by (1) calculating the required survival rates from each of the two tables and (2) averaging them. This assumes that mortality changes occurred evenly over the intercensal period. We should also consider whether they were sufficiently great to justify the additional calculations.

In other cases, there may be a question of adjusting rates for various geographic, racial, ethnic, or other socioeconomic categories. Differences in age-specific death rates for various geographic, ethnic, or socioeconomic categories should be examined, when they can be computed, to determine whether life table survival rates for the general population should be adjusted for these differences. For example, the convergence of state mortality for each sex-race in the United States group to the national level has been so pronounced over the past several decades that, for most purposes, it is not necessary to employ separate life tables for each state in making population projections for states. National life tables for each sex-race group are adequate.

Survival rates for the general population can be adjusted directly for geographic or socioeconomic variations by use of observed central death rates for the various categories, as follows: (1) take the complement of the general survival rate; (2) calculate an adjustment factor for the particular geographic or socioeconomic category in terms of central death rates, (3) multiply the adjustment factor by the complement of the general survival rate, and (4) take the complement of the result in step 3. For example, if ${}_5s_x^5$ is a 5-year survival rate for ages x to x + 4 and ${}_5m_x$ is a central death rate at ages x to x + 4 for the general population, we may develop survival rates for the single population as follows:

(1) $1 - {}_5s_x^5$ (a type of 5-year mortality rate)
(2) $({}_5m_x + {}_5m_{x+5})$ for the single population ÷ $({}_5m_x + {}_5m_{x+5})$ for the general population
(3) $= (1)\cdot(2)$
(4) $= (1) - (3)$

Sometimes the evidence will not justify making this type of adjustment in the general rates or suggest that it is unnecessary.

Migration may be a troublesome element in making a correct estimate of deaths by means of life table survival rates for populations that are not closed (i.e., affected by migration). Application of survival rates to the initial population tends to understate deaths for a population with net in-migration and to overstate deaths for a population with net out-migration. A reverse procedure for applying the survival rates has the opposite effect (Chapters 18 and 19).

In many cases, official life tables are available but they are based on seriously incomplete statistics on deaths and are not satisfactory for most uses. In these cases, and in others where life tables are not available for the country, one may have to decide whether to construct a life table, "borrow" a life table from another country, or employ a model life table. Commonly, the countries that lack life tables simply do not have adequate death statistics for constructing such tables. In some cases, one may construct a table by use of population census data and "stable population techniques," but it is more practical and convenient to "borrow" another country's table or, preferably, to adopt a model table appropriate to the population under study. The selection of a model table and use of stable population techniques are discussed in Chapter 22 and Appendix B.

The estimates of survivors will differ depending on the age interval employed in computing the survival rates. (We consider this question apart from the age detail required in the estimates of survivors.) With few exceptions, 5-year survival rates are sufficiently detailed to take account of the important variations in the estimates of survivors. For most calculations, 10-year age intervals will be adequate, however. The use of survival rates with a given age interval implies that the actual population has the same distribution by age as the life table population in this interval. For this reason, survival rates for very broad age spans should be avoided except in rough calculations. Hence, the terminal open-ended interval should relate to a relatively limited age span containing only a small percentage of the total population. Terminal group 65 and over is to be avoided except for very "young" populations (e.g., Syria, Guatemala). On the other hand, for countries with relatively old populations (e.g., France, Great Britain), a terminal group of 75 and over may be unsatisfactory.

While life table survival rates represent the ratio of l_x or L_x values to one another, the actual level of survival rates is affected only by the q_x's in the age range to which the survival rates apply. Hence, life table survival rates for various populations may properly be combined on the basis of the population distribution at these ages. For example, 5-year survival rates with initial ages 40 to 44, for the white and nonwhite populations of the United States, would be combined on the basis of the distribution of the population in these race groups at ages 40 to 44 in order to obtain a survival rate for both populations combined.

National Census Survival Rates

Another means of allowing for mortality, national census survival rates, employs life table concepts but does not involve the actual use of life tables. National census survival rates are particularly applicable in the measurement of internal net migration, but they are also used in measuring or evaluating the level of mortality and in constructing life tables, especially for countries lacking adequate vital statistics. National census survival rates essentially represent the ratio of the population in a given age group in one census to the population in the same birth cohort at the previous census. Normally, then, census survival rates pertaining to the children born in the decade are not computed. An illustration of the method of computation of census survival rates for several countries and a comparison with life table survival rates are presented in Table 13.10. National census survival rates measure mortality essentially, but they are affected by the relative accuracy of the two census counts employed in deriving them. Underlying their use are certain basic assumptions. They are that the population is a closed one (i.e., that there was no migration during the intercensal period) and that there has been no abnormal influence on mortality (e.g., war). Census survival rates may be rather irregular, often fluctuating up and down throughout the age scale and exceeding unity in some ages. Note particularly the rates for Botswana in Table 13.11. The more inconsistent in accuracy are the data from the two censuses, the more erratic the census survival rates are.

In the use of national census survival rates to measure internal net migration for geographic subdivisions of countries, the fact that the rates incorporate both the effect of mortality and relative net census error is considered an advantage. The reasoning is that because net migration is obtained as a residual and the census survival rates "incorporate" the effect of the relative accuracy of the census counts in addition to mortality, the estimates of net migration are unaffected by the errors in the census data. National census survival rates for the United States have been used both in the estimation of net migration of age, sex, and race groups, for states and counties in historical studies, and in the evaluation of the levels of mortality shown in the official life tables.

LIFE TABLES WITH MULTIPLE DECREMENTS

Conventional life tables represent the reduction of the life table cohort by mortality alone. Life tables with multiple decrements describe how attrition from more than one factor reduces the life table cohort. A double decrement table is a type of multiple decrement life table in which there are two forms of exit from the initial cohort, one of which is mortality and the other is some change in social or economic status. One example is a nuptiality (marriage formation) table, which follows a cohort of never-married persons as they are exposed to marriage rates and death rates. Another example of a double decrement table is one that follows a cohort of first

TABLE 13.10 National Census Survival Rates for Males for Botswana, Ireland, Turkey, and Uruguay, 1980–1990

Age (years)		Census survival rate, 1980–1990						Life table rate, survival Uruguay 1984–1986
		Botswana						
		Population						
		1980	1990	Rate. (2) ÷ (1) =	Ireland	Turkey	Uruguay	
At first census	At second census	(1)	(2)	(3)	(4)	(5)	(6)	(7)
0–4	10–14	86,069	88,615	1.029581	0.988389	1.178368	0.949638	0.962900
5–9	15–19	74,301	71,704	0.965048	0.957009	1.041048	0.975587	0.995636
10–14	20–24	58,709	53,038	0.903405	0.778674	0.905234	1.137193	0.993229
15–19	25–29	42,972	44,203	1.028647	0.723915	0.971136	1.041833	0.989527
20–24	30–34	32,646	35,568	1.089506	0.876978	0.953246	1.000395	0.987296
25–29	35–39	26,498	29,592	1.116764	0.954542	1.022280	1.147528	0.985492
30–34	40–44	20,327	22,412	1.102573	0.962540	0.996185	1.16505	0.980693
35–39	45–49	16,826	17,813	1.058659	0.961294	0.944353	1.103631	0.970599
40–44	50–54	15,600	15,529	0.995449	0.936017	0.896453	1.006115	0.952333
45–49	55–59	13,575	12,231	0.900994	0.921369	0.896584	0.932286	0.919222
50–54	60–64	11,424	10,008	0.876050	0.870831	0.840947	0.904868	0.873166
55–59	65–69	10,090	8,297	0.822299	0.831721	0.759667	1.068891	0.815967
60–64	70–74	8,477	6,537	0.771145	0.723513	0.641406	1.203539	0.734393
65 and over	75 and over	21,220	13,070	0.615928	0.384943	0.402123	1.147139	0.625499

Source: Basic data from United Nations, *Demographic Yearbook*, 1996, Table 34; 1994, Table 7; 1993, Table 26; 1988, Tables 7, 26; 1983, Table 7.

marriages as they as exposed to divorce and mortality. Multiple decrement tables allow for several decremental factors (including death). An example of a multiple decrement table is a cause-of-death life table, which subdivides the conventional life table into component tables for the causes of death. This type of table provides information about the population eventually dying of each cause and their average age at death as well as the probability that a person will eventually die from that cause.

Anatomy of Multiple Decrement Tables

The basic features of multiple decrement tables can be observed in Table 13.11. Cause-of-death life tables consist of one or more (component) conditional tables relating to those ultimately dying from a particular cause of interest (or from a set of such causes) and another table relating to those dying from all other causes (those not of interest). Table 13.11 consists of two conditional tables, one for malignant neoplasms and the other for all other causes of death. This set of tables divides the basic life table functions that were shown in the abridged life table for the United States in Table 13.2. The basic life table functions—${}_nq_x$, l_x, ${}_nd_x$, ${}_nL_x$, T_x, and e_x—can be observed in each conditional table. For cause-of-death life tables, the conditional life tables are mutually exclusive and additive. Each death is represented in only one d_x column. The sum of life table deaths across all ages and conditional tables represents the life table deaths attributed to all causes combined. The two tables display the most salient aspect of cause-specific mortality.

The multiple decrement table conceptually traces a cohort of newborn babies through their entire life under the assumption that they are subject to the current observed schedule of age-specific mortality rates from different causes of death. The radix (cohort of newborn babies) is split among the component tables according to the cause of death. Panel A of Table 13.11 includes only 22,024 babies who eventually die from malignant neoplasms. Panel B of Table 13.11 begins with only the 77,973 babies who eventually die from other causes. The panels of the multiple decrement life table are linked in that together they represent what happens to the initial cohort. Therefore, the 22,024 who ultimately die from malignant neoplasms and the 77,973 who eventually die from other causes sum to the 100,000 radix of the conventional life table.

The interpretation of the life table functions in these abridged tables is as follows:

x to x + n The period of life between two exact ages. For instance, "20–25" means the 5-year interval between the 20th and 25th birthdays.

${}_nq_x$ The proportion of the persons in the cohort alive at the beginning of an indicated age interval (x) who will die from a particular cause before reaching the end of that age interval (x + n). For example, according to Table

TABLE 13.11 Multiple Decrement Life Table for Malignant Neoplasms and Other Causes, United States 1989–91

		Of 100,000 born alive		Stationary population			Of 100,000 born alive who eventually die from cause c
Period between two exact ages, x to x+n (years)	Proportion of persons alive at beginning of age interval dying during interval from cause c	Number living at beginning of age interval who eventually die from cause c	Number dying during age interval from cause c	Population in this age interval who will eventclly die from cause	In this and all subsequent age intervals	Average number of years of life remaining at beginning of age interval for those who will eventually die from cause c	Number living at beginning of age interval
A. Malignant Neoplasms							
	$_nq_{x,neoplasms}$	$l_{x,\ neoplasms}$	$_nd_{x,neoplasms}$	$_nL_{x,neoplasms}$	$_nT_{x,\ neoplasms}$	$e_{x,neoplasms}$	$l^*_{x,neoplasms}$
0–1	0.00002	22,024	2	22,022	1,587,922	72.10	100,000
1–5	0.00014	22,021	13	88,053	1,565,900	71.11	99,990
5–10	0.00017	22,008	16	109,999	1,477,847	67.15	99,929
10–15	0.00016	21,992	15	109,919	1,367,849	62.20	99,854
15–20	0.00021	21,976	20	109,830	1,257,929	57.24	99,785
20–25	0.00028	21,956	28	109,710	1,148,099	52.29	99,692
25–30	0.00043	21,928	42	109,534	1,038,389	47.35	99,566
30–35	0.00080	21,886	77	109,235	928,855	42.44	99,374
35–40	0.00152	21,808	146	108,677	819,620	37.58	99,022
40–45	0.00291	21,662	277	107,618	710,943	32.82	98,360
45–50	0.00572	21,385	539	105,578	603,325	28.21	97,100
50–55	0.01046	20,846	966	101,816	497,748	23.88	94,654
55–60	0.01732	19,880	1,553	95,519	395,932	19.92	90,267
60–65	0.02628	18,327	2,248	86,017	300,413	16.39	83,217
65–70	0.03600	16,079	2,863	73,239	214,397	13.33	73,009
70–75	0.04728	13,217	3,374	57,648	141,157	10.68	60,011
75–80	0.05613	9,843	3,393	40,731	83,510	8.48	44,691
80–85	0.06310	6,450	2,971	24,822	42,778	6.63	29,286
85–90	0.06504	3,479	2,066	12,228	17,957	5.16	15,796
90–95	0.06054	1,412	1,032	4,482	5,729	4.06	6,413
95–100	0.05107	380	321	1,100	1,247	3.28	1,727
100+	1.00000	60	60	147	147	2.46	271
B. Other Causes							
	$_nq_{x,other}$	$l_{x,\ other}$	$_nd_{x,other}$	$_nL_{x,other}$	$_nT_{x,\ other}$	$e_{x,other}$	$l^*_{x,other}$
0–1	0.0093400	77,973	934	77,156	5,947,292	76.27	100,000
1–5	0.0017400	77,039	174	307,738	5,870,136	76.20	98,802
5–10	0.0009700	76,865	97	384,083	5,562,398	72.37	98,579
10–15	0.0011700	76,768	117	383,548	5,178,315	67.45	98,455
15–20	0.0039900	76,651	399	382,258	4,794,768	62.55	98,305
20–25	0.0051600	76,252	516	379,970	4,412,510	57.87	97,793
25–30	0.0055800	75,736	558	377,285	4,032,540	53.24	97,131
30–35	0.0067100	75,178	671	374,213	3,655,255	48.62	96,415
35–40	0.0080100	74,507	801	370,533	3,281,043	44.04	95,555
40–45	0.0094100	73,706	941	366,178	2,910,510	39.49	94,528
45–50	0.0124500	72,765	1,245	360,713	2,544,333	34.97	93,321
50–55	0.0174700	71,520	1,747	353,233	2,183,620	30.53	91,724
55–60	0.0256800	69,773	2,568	342,445	1,830,388	26.23	89,484
60–65	0.0376900	67,205	3,769	326,603	1,487,943	22.14	86,190
65–70	0.0529900	63,436	5,299	303,933	1,161,340	18.31	81,356
70–75	0.0753400	58,137	7,534	271,850	857,408	14.75	74,560
75–80	0.0997000	50,603	9,970	228,090	585,558	11.57	64,898
80–85	0.1234200	40,633	12,342	172,310	357,468	8.80	52,112
85–90	0.1265800	28,291	12,658	109,810	185,158	6.54	36,283
90–95	0.0973300	15,633	9,733	53,833	75,348	4.82	20,049
95–100	0.0453600	5,900	4,536	18,160	21,515	3.65	7,567
100+	1.0000000	1,364	1,364	3,355	3,355	2.46	1,749

Source: U.S. National Center for Health Statistics, 1999.

TABLE 13.12 Actual Deaths and Life Table Deaths for Malignant Neoplasms and Other Causes, United States: 1989–1991

Exact ages	Actual deaths: Total $_nD_x$	Actual deaths: Neoplasms $_nD_{x,\text{ neoplasms}}$	Actual deaths: All other causes $_nD_{x,\text{other}}$	Life table deaths: Total $_nd_x$	Life table deaths: Neoplasms $_nd_{x,\text{ neoplasms}}$	Life table deaths: All other causes $_nd_{x,\text{other}}$
0–1	114,810	274	114,536	936	2	934
1–5	21,444	1,545	19,899	187	13	174
5–10	12,238	1,774	10,463	113	16	97
10–15	13,599	1,581	12,018	132	15	117
15–20	46,609	2,258	44,351	419	20	399
20–25	63,099	3,226	59,873	544	28	516
25–30	79,018	5,578	73,440	600	42	558
30–35	101,279	10,482	90,797	748	77	671
35–40	117,550	18,118	99,432	947	146	801
40–45	135,123	30,758	104,364	1,219	277	942
45–50	156,378	47,214	109,164	1,784	539	1,245
50–55	201,256	71,674	129,582	2,713	966	1,747
55–60	295,958	111,499	184,459	4,121	1,553	2,568
60–65	465,044	173,739	291,305	6,018	2,248	3,770
65–70	651,048	228,339	422,709	8,162	2,863	5,299
70–75	787,101	243,437	543,664	10,909	3,374	7,535
75–80	904,394	229,612	674,782	13,363	3,393	9,970
80–85	904,222	175,431	728,791	15,314	2,971	12,343
85–90	737,820	103,545	634,276	14,725	2,066	12,659
90–95	447,208	42,868	404,340	10,765	1,032	9,733
95–100	175,745	11,607	164,138	4,857	321	4,536
100+	37,506	1,571	35,935	1,424	60	1,364
Total, all ages				100,000	22,024	77,976

Source: Published and unpublished data from the U.S. National Center for Health Statistics.

13.11 panel A, the proportion dying from malignant neoplasms in the age interval 20 to 25 is 0.00028. That is, out of every 100,000 persons alive and exactly 20 years old, 28 will die from malignant neoplasms before reaching their 25th birthday. In other words, the $_nq_x$ values represent the probability that a person at his *x*th birthday will die from malignant neoplasms before reaching his or her $x + n$th birthday.

l_x The number of persons living at the beginning of the indicated age interval (x) out of the total number of births assumed as the radix of the table who will ultimately die from a particular cause. Again, according to Table 13.11, out of 22,024 newborn babies who eventually die from malignant neoplasms, 21,956 persons would survive to exact age 20.

$_nd_x$ The number of persons who would die from a particular cause within the indicated age interval (x to x + n) out of the 100,000 births assumed in the combined tables. Thus, according to Table 13.11, there would be 28 deaths from malignant neoplasms between exact ages 20 and 25 to the initial cohort of 100,000 newborn babies.

$_nL_x$ The number of person-years that would be lived within the indicated age interval (x to x + n) by the cohort who will ultimately die from malignant neoplasms. Thus, according to Table 13.11, the 100,000 newborn babies would live 109,710 person-years between exact ages 20 and 25. Of the 21,956 persons who reach age 20 before eventually dying from malignant neoplasms, the 21,928 who survive to age 25 would live 5 years each ($21{,}928 \times 5 = 109{,}640$ person-years) and the 28 who die would each live varying periods of time less than 5 years, averaging about $2\frac{1}{2}$ years ($28 \times 2.5 = 70$ person-years)

T_x The total number of person-years that would be lived after the beginning of the indicated age interval by the cohort of those who eventually die from malignant neoplasms. Thus, according to Table 13.11, the 22,024 newborn babies who ultimately die from malignant neoplasms would live 1,148,099 person-years after their 20th birthday.

e_x The average remaining lifetime (in years) for a person who survives to the beginning of the indicated age interval for those ultimately dying from a particular cause. Thus, according to Table 13.11, persons who reach their 20th birthday and who eventually die from malignant neoplasms should expect to live 52.29 years more, on the average.

The multiple decrement table is a way to partition the conventional life tables into conditional tables for each cause of death. The conditional tables are linked to the conventional life table by several relationships. At all ages, the number of survivors and person-years in the conventional life table is the respective sum over the conditional tables. That is,

$$l_x = \sum_c l_{x,c} \quad \text{and} \quad {}_nL_x = \sum_c {}_nL_{x,c}$$

Furthermore, the life expectancy for a population is the average of the multiple decrement life expectancies for the separate causes of death, weighted by the proportions dying of each cause:

$$e_x = \sum \frac{(l_{x,c}e_{x,c})}{l_x}$$

For example, life expectancy at birth in Table 13.2 is 75.37, which is the weighted sum of life expectancy at birth for those who eventually die from malignant neoplasms (72.10) and those who eventually die from other causes (76.27), where the weights are the proportions of the 100,000 births who eventually die from each cause. So

$$e_0 = \left(\frac{22{,}024}{100{,}000}\right)\cdot 72.10 + \left(\frac{77{,}976}{100{,}000}\right)\cdot 76.27$$

Table 13.11 shows life table deaths for malignant neoplasms and other causes for the United States in 1989–1991, using the life table deaths by cause (${}_nd_{x,c}$) series of Table 13.12. We have added the column l_x^*, which rescales the l_x series to the radix 100,000. The rescaling is not necessary but simplifies comparison of rates of population attrition for each cause.

Construction of Multiple Decrement Tables

Double-decrement and multiple-decrement tables may be constructed either on the basis of age-specific probabilities of the occurrence of the events (occurrence/exposure rates) or on the basis of prevalence ratios, obtained usually from censuses or surveys. When prevalence ratios are used, the life table stationary population is distributed into different statuses according to the prevalence of those statuses in the actual observed population.

The construction of multiple decrement tables is described in terms of cause-of-death life tables. As mentioned, multiple decrement cause-of-death life tables subdivide total life table deaths into the different causes or groups of causes of death. In constructing a multiple decrement table, one first constructs a conventional life table using age-specific probabilities of dying for all causes combined. Then, from the counts of actual deaths by cause, the proportion of deaths due to each cause is computed. Next, ${}_nd_x$, life table deaths between ages x and x + n, are separated into c cause-of-death subcategories ${}_nd_{x,1}$, ${}_nd_{x,2}$, ... ${}_nd_{x,c}$. This subdivision of life table deaths by cause is made on the basis of the actual distribution of deaths by cause at each age:

$${}_nd_{x,c} = {}_nd_x \left(\frac{{}_nD_{x,c}}{{}_nD_x}\right) \tag{13.60}$$

Table 13.12 shows the actual deaths and life table deaths for the 1989–1991 U.S. life table. Total deaths have been divided into two subcategories, malignant neoplasms and other causes. The actual deaths are the averages from 1989 to 1991. The column for total life table deaths comes from Table 13.2. Beginning at age under 1 year, we allocate total life table deaths into life table deaths by cause proportionally based on the actual deaths:

$$d_{0,neoplasms} = d_0 \cdot \frac{D_{0,neoplasms}}{D_0} = 936 \cdot \left(\frac{274}{114{,}810}\right) = 2$$

$$d_{0,other} = d_0 \cdot \frac{D_{0,other}}{D_0} = 936 \cdot \left(\frac{114{,}536}{114{,}810}\right) = 934$$

All life table deaths are included in the allocation. For example, the life table deaths at age under 1 by cause sum to the total life table deaths for age under 1 (934 + 2 = 936).

The life table deaths partitioned by cause are used to construct life tables conditional on dying from that cause. The ${}_nd_{x,c}$ series can be used to construct the conditional life table showing the survival distribution for individuals who eventually die of one cause by defining the initial population $l_{0,c}$ to be

$$l_{0,c} = {}_\omega d_{0,c} = \sum_{x=0}^{\omega-n} {}_nd_{x,c} \tag{13.61}$$

The sum of the ${}_nd_{x,c}$ terms across all ages represents all life table deaths for the *c*th cause. Table 13.11 indicates that the sum of all life table deaths from malignant neoplasms is 22,024; this becomes the radix for the conditional life table for malignant neoplasms.

Table 13.12 shows the summations of the ${}_nd_{x,c}$ terms, which yield the totals, $l_{0,neoplasms} = 22{,}024$ and $l_{0,other} = 77{,}976$. The reader can confirm that ${}_\omega d_0 = l_0 = 100{,}000$ = sum of $l_{0,c}$. The $l_{0,c}$ estimates tell us that, of 100,000 persons born, 22% would eventually die from malignant neoplasms and 78% would die from other causes, at the 1989–1991 survival probabilities.

We estimate the number of survivors for the cause at subsequent ages, $l_{x,c}$, by subtraction of the ${}_nd_{x,c}$ terms. That is,

$$l_{x+n,c} = l_{x,c} - {}_nd_{x,c} \tag{13.62}$$

The $l_{x,c}$ series is exactly the l_x series of the conventional life table, except that it describes the survival experience of persons eventually dying of a particular cause c.

The life table deaths for each cause and age group are used to calculate the cause-specific probability of dying. These are also called probabilities of death from specified causes and represent the probability that an individual will die of cause c after surviving to age x but before reaching age x + n.

$$ {}_nq_{x,c} = \frac{{}_nd_{x,c}}{l_x} \tag{13.63} $$

For example, the probability that an individual aged 20 will die from malignant neoplasms before age 25 is

$$ {}_5q_{20,neoplasms} = \frac{28}{98,213} = .000285 $$

The sum of the life table deaths for cause c is the number who eventually die of cause c from among the births that begin the life table (l_0). The probability that an individual eventually dies of the cth cause is

$$ {}_\omega q_{0,c} = \sum_{a=0}^{\omega-n} \frac{{}_nd_{a,c}}{l_0} = \frac{{}_\omega d_{0,c}}{l_0} \tag{13.64} $$

Life expectancy for each conditional table is constructed according to the formulas for constructing abridged life tables with the substitution of the conditional terms ${}_nl_{x,c}$ and ${}_nL_{x,c}$ for the conventional life table quantities ${}_nl_x$ and ${}_nL_x$.

Cause-Elimination Life Tables

Life tables have been computed under the assumption that a specific decrement (for instance, a cause of death or group of causes of death) is eliminated, that is, that it is impossible to die from the eliminated cause (Greville, 1948; U.S. National Center for Health Statistics, 1968, 1985b, 1999). Such cause-elimination life tables describe the hypothetical situation that a cause of decrement has been eliminated. This is the same as assuming that the probability of dying from that cause is zero (or the probability of surviving form it is 1.0). Note that while deaths from a cause are assumed to disappear, the disease itself is not assumed to go away.

Two important cautions must be observed in interpreting cause-elimination life tables. First, they provide no information about future longevity even though they can be useful in formulating assumptions about future longevity. They simply give us additional insight into the current cause pattern of mortality. Second, they are theoretical constructs in that they hypothesize shifts in mortality that do not occur independently and hence do not realistically represent the actual gains from eliminating a cause. It is likely that the gains from eliminating a cause would, in actual experience, be much smaller than shown in the table, as explained below.

Cause-elimination life tables are usually based on the assumption that eliminating one cause of death has no effect on the risk of dying from the remaining causes. The potential gain in life expectation as measured by cause-elimination life tables is constrained by the fact that individuals saved would be immediately subject to the death rates from other causes, usually major causes in later life. This phenomenon is called "competing risks." Because death from the cause eliminated would often have occurred at the older ages, few additional years would be added if the individual survived and died shortly after from another cause. The various causes of decrement are assumed to act independently of each other in the present case. If the causes are assumed not to be independent, it is still possible to develop a cause-eliminated life table if the interconnectedness can be modeled explicitly (Manton and Stallard, 1984).

Anatomy of Cause-Elimination Life Tables

Table 13.13 presents an abridged life table for the total population of the United States in 1989–1991, prepared on the assumption that malignant neoplasms (cancer) are eliminated as a cause of death. A series of such life tables, considered in comparison with a life table for all causes combined, provides various measures of the relative importance of the different causes of death. The cause-elimination life table includes the basic life table functions—${}_nq_x$, l_x, ${}_nd_x$, ${}_nL_x$, T_x, and e_x, as well as three additional functions.

The interpretation of the life table functions in an abridged life table would be as follows:

x to x + n The period of life between two exact ages

${}_nq_x^{(-i)}$ The proportion of the persons in the cohort alive at the beginning of an indicated age interval (x) who will die before reaching the end of that age interval (x + n) assuming that malignant neoplasms have been eliminated as a cause of death. Comparison of Tables 13.13 and 13.2 indicates that when malignant neoplasms have been eliminated as a cause of death, the proportion dying in the age interval 20 to 25 falls from .00554 to .00526.

$l_x^{(-i)}$ The number of persons living at the beginning of the indicated age interval (x) out of the total number of births posited as the radix of the table, assuming that malignant neoplasms have been eliminated. According to Table 13.13, out of 100,000 newborn babies, 98,282 persons would survive to exact age 20 assuming that malignant neoplasms have been eliminated. Note that the number of survivors to age 20 is higher in Table 13.13 (which assumes that no one dies from malignant neoplasms), than in Table 13.2, (which assumes that no causes of death have been eliminated).

${}_nd_x^{(-i)}$ The number of persons who would die within the indicated age interval (x to x + n) out of the 100,000 births, assuming no deaths from neoplasms. Of the 98,282 survivors to age 20, 517 are expected to die before age 25. Again comparing Tables 13.2 and 13.13, the number of deaths between exact ages 20 and 25 to the initial cohort of 100,000 newborn babies would fall from 544 to 517 if malignant neoplasms were eliminated.

TABLE 13.13 Abridged Life Table Eliminating Malignant Neoplasms as a Cause of Death, for the Total Population of the United States: 1989–1991

Age interval	Proportion dying	Of 100,000 born alive		Stationary population		Average remaining lifetime			
Period of life between two exact ages stated in years	Proportion of persons alive at beginning of age interval dying during interval	Number living at beginning of age interval	Number dying during age interval	In this age interval	In this and all subsequent age intervals	Average number of years of life remaining at beginning of age interval	Probability of eventually dying of specified cause	Gain in life expectancy eliminating specified cause	Gain in life expectancy for those who would have died
x to x + n	$_nq_x^{(-i)}$	$l_x^{(-i)}$	$d_x^{(-i)}$	$_nL_x^{(-i)}$	$T_x^{(-i)}$	$e_x^{(-i)}$	Ψ_x^i	$g_x^{(-i)}$	$\gamma_x^{(-i)}$
0–1	0.00934	100,000	934	99,260	7,872,401	78.72	0.22023	3.36	15.25
1–5	0.00175	99,066	174	395,855	7,773,142	78.46	0.22229	3.39	15.24
5–10	0.00096	98,893	95	494,206	7,377,287	74.60	0.22257	3.38	15.20
10–15	0.00117	98,798	116	493,766	6,883,081	69.67	0.22266	3.38	15.16
15–20	0.00405	98,682	400	492,499	6,389,315	64.75	0.22280	3.37	15.13
20–25	0.00526	98,282	517	490,144	5,896,816	60.00	0.22355	3.37	15.08
25–30	0.00572	97,765	559	487,464	5,406,672	55.30	0.22451	3.37	15.03
30–35	0.00691	97,206	672	484,413	4,919,208	50.61	0.22546	3.37	14.95
35–40	0.00834	96,534	805	480,736	4,434,795	45.94	0.22641	3.36	14.84
40–45	0.00989	95,729	946	476,396	3,954,059	41.30	0.22713	3.33	14.64
45–50	0.01327	94,783	1,257	470,980	3,477,663	36.69	0.22712	3.25	14.33
50–55	0.01900	93,526	1,777	463,492	3,006,684	32.15	0.22568	3.12	13.81
55–60	0.02890	91,748	2,652	452,556	2,543,192	27.72	0.22173	2.89	13.04
60–65	0.04467	89,097	3,980	436,062	2,090,636	23.46	0.21426	2.57	11.99
65–70	0.06790	85,117	5,780	411,798	1,654,573	19.44	0.20221	2.16	10.69
70–75	0.10827	79,337	8,590	376,115	1,242,775	15.66	0.18522	1.71	9.23
75–80	0.17008	70,747	12,033	324,472	866,659	12.25	0.16283	1.25	7.66
80–85	0.27173	58,714	15,954	254,085	542,187	9.23	0.13699	0.84	6.11
85–90	0.41446	42,760	17,722	168,451	288,102	6.74	0.10950	0.51	4.65
90–95	0.59446	25,037	14,884	84,814	119,651	4.78	0.08285	0.28	3.37
95–100	0.74997	10,154	7,615	28,428	34,836	3.43	0.06057	0.14	2.33
100+	1.00000	2,539	2,539	6,408	6,408	2.52	0.04189	0.06	1.45

Source: U.S. National Center for Health Statistics. R. N. Anderson, 1999. "United States Life Tables Eliminating Certain Causes of Death." *U.S. Decennial Life Tables for 1989–91*, vol. 1, no. 4. Hyattsville, MD: U.S. National Center for Health Statistics.

$_nL_x^{(-i)}$ The number of person-years that would be lived within the indicated age interval (x to x + n) by the cohort. Thus, according to Table 13.13, the 100,000 newborn babies would live 490,144 person years between exact ages 20 and 25.

$T_x^{(-i)}$ The total number of person-years lived after the beginning of the indicated age interval. Thus, according to Table 13.13, the 100,000 newborn babies would live 5,896,816 person-years after their 20th birthday.

$e_x^{(-i)}$ The average remaining lifetime (in years) for a person who survives to the beginning of the indicated age interval, assuming that malignant neoplasm has been eliminated as a cause of death. Thus, according to Table 13.13, a person who reaches his or her 20th birthday should expect to live, on average, 60 more years.

Ψ^i The probability of eventually dying of the specific cause, malignant neoplasms in this case. According to Table 13.13, 22% of live births would die from malignant neoplasms. Note that this type of probability was discussed previously with multiple decrement life tables.

$g^{(-i)}$ The gain in life expectancy from eliminating a specified cause. If malignant neoplasms were eliminated as a cause of death, then 3.36 years would be added to life expectancy at birth for the total U.S. population in 1989–1991. This column represents the difference in e_0 between Tables 13.13 and 13.2.

$\gamma^{(-i)}$ The gain in life expectancy for those who would have died if the cause not been hypothetically eliminated. According to Table 13.13, those who would have died from malignant neoplasms can expect to live, on average, 15.25 additional years at birth.

Construction of Cause-Eliminated Life Table

Cause-elimination life tables are usually constructed in association with cause-of-death tables. They also use information about the actual distribution of deaths by cause at each age. The first step is the calculation of the probabilities of survival with the ith cause eliminated, ${}_np_x^{(-i)}$ by the exponential formula (Chiang, 1968; Greville, 1948).

$$ {}_n p_x^{(-i)} = {}_n p_x^{\left(1-\frac{{}_n D_{x,c}}{{}_n D_x}\right)} \tag{13.65}$$

This formula employs the probability of surviving (${}_np_x$) from the corresponding life table for all causes combined and the actual distribution of deaths by cause at each age.

Then the age-specific probabilities of death if the ith cause of death is eliminated are calculated as

$$ {}_n q_x^{(-i)} = 1 - {}_n p_x^{(-i)} \tag{13.66}$$

The number of survivors at each age assuming that the ith cause is eliminated $l_x^{(-i)}$ are calculated successively starting with $l_0^{(-i)} = 100{,}000$ by the formula

$$ l_{x+n}^{(-i)} = {}_n p_x^{(-i)} l_x^{(-i)} \tag{13.67}$$

The number of persons living in the stationary population in the age interval x to x + n assuming elimination of the ith cause is estimated using

$$ {}_n L_x^{(-i)} = (n - {}_n f_x)\cdot l_n^{(-i)} + {}_n f_x \cdot l_{x+n}^{(-i)} \tag{13.68}$$

with ${}_nf_x$ computed from the life table for all causes combined as

$$ {}_n f_x = \frac{n l_x - {}_n L_x}{l_x - l_{x+n}} \tag{13.69}$$

This procedure assumes that the average number of years lived by those who die within the age interval is the same in the life table eliminating the ith cause as in the life table for all causes combined.

The number of persons in the stationary population in the oldest age group, $L_\omega^{(-i)}$ is estimated by

$$ L_\omega^{(-i)} = T_\omega^{(-i)} = \frac{e_\omega \cdot l_\omega^{(-i)}}{1 - {}_n r_{\omega-n}^i} \tag{13.70}$$

where e_ω comes from the life table for all causes combined and ${}_nr^i_{\omega-n}$ denotes the proportion of deaths observed in the last age interval attributable to the ith cause of death.

With the value of $T_\omega^{(-i)}$ available, values of $T_x^{(-i)}$ for successively younger ages were calculated by

$$ T_x^{(-i)} = T_{x+n}^{(-i)} + {}_n L_x^{(-i)} \tag{13.71}$$

Finally, life expectancy at each age assuming the ith cause is eliminated is obtained by

$$ e_x^{(-i)} = \frac{T_x^{(-i)}}{l_x^{(-i)}} \tag{13.72}$$

The probability that an individual eventually dies of the cth cause is

$$ {}_\omega q_{x,c} = \sum_{a=x}^{\omega-n} \frac{{}_n d_{a,c}}{l_x} = \frac{{}_\omega d_{x,c}}{l_x} = \psi_x^i \tag{13.73}$$

For any age, the gain in life expectancy from eliminating a specific cause of death is the difference in life expectancy from two life tables—the cause-elimination life table and the life table for all causes combined:

$$ g_x^{(-i)} = e_x^{(-i)} - e_x \tag{13.74}$$

For the gain in life expectancy for those who would have died from the ith cause of death,

$$ \gamma_x^{(-i)} = \frac{e_x^{(-i)} - e_x}{\psi_x^i} \tag{13.75}$$

Increment-Decrement Tables

Increment-decrement tables are a type of multiple decrement table that allows for both increments and decrements in the initial cohort, such as labor force entry and exit, school enrollment and withdrawal, and marriage, divorce, and widowhood. When data and computer facilities are limited, they are typically constructed from age-specific prevalence ratios.

While conventional increment-decrement tables allow both entries and exits in the table, these appear as net entries and net exits for age intervals over broad age bands in the table. For example, tables of working life constructed by this method assume a unimodal curve of labor force participation, reaching a maximum in young adulthood and then falling to zero in older ages. The table may show only net entries into the labor force until about age 35 and only net exits from the labor force thereafter.

Increment-decrement life tables are constructed by disaggregating a conventional life table into those in the "state" (e.g., in the labor force) and those "not in the state" (e.g., not in the labor force). To achieve this, observed prevalence ratios (e.g., labor force participation ratios) are applied to the stationary population (${}_nL_x$) of the basic life table. The occurrence/exposure rates of net entry or net exit at each age have to be derived indirectly. Various techniques may be employed in developing them. Application of prevalence ratios in single years of age to the stationary population yields the stationary population in the state (e.g., in the labor force) in single years. The survivors at each age in the state ${}^1l_x$ and not in the state ${}^0l_x$ can be calculated by interpolation of the stationary population. The implicit rates of labor force entry and exit can be derived after removing the effect of mortality. An increment-decrement table may also be constructed by employing occurrence/exposure rates. These rates may be derived from census reports or from surveys on current status and status at an earlier date. Cross-sectional data from a single census measuring the changes in the proportions over single ages can also be used to derive the rates.

Increment-decrement tables provide some types of information not available from conventional tables and double-decrement tables. Using tables of working life as illustration, we can obtain such measures as the proportion of the cohort ever entering the labor force, the average remaining years of active life, the rate of accessions to the labor force and retirements at each age, and the mean ages of entry into the labor force and retirement.

Increment-decrement tables are limited because they cannot model all possible transitions between the states under consideration. For this reason, multistate tables have largely replaced increment-decrement tables. Unlike increment-decrement tables, multistate tables of working life are not limited to a unimodal curve of labor force participation and can handle those individuals who enter the labor the force, exit, and then reenter it.

MULTISTATE LIFE TABLES

Multistate life tables are a major extension of life tables. Both conventional life tables and multiple decrement life tables model departures, over time, from a cohort of live births. Conventional life tables concern two states—life and death—while multiple decrement models allow for more than two states—life and various causes of death, for example. Multistate models, in contrast, allow not only for movements between life (an active state) and death (an absorbing state) but also for all possible movements among various types of active states. For example, a multistate nuptiality model allows individuals to move from being unmarried to married, from married to divorced, from divorced to remarried, and from remarried to divorced. Because of this flexibility, multistate models have had a broad range of applications, including marital status, labor force behavior, interregional migration, and health status.

Multistate models relate to “states”, which typically include death and various categories for the living. States are mutually exclusive and discrete. There are several types of states. Absorbing states (like death) only permit entries. Transient states allow both entries and exits. In addition, some states (like never married) permit only exits. Together, the states are the elements of the “state space.”

Mathematically, multistate models are a type of Markov process. Markov processes assume that transition probabilities depend only on age (or duration in state) and current state. Furthermore, the probabilities are assumed independent of the previous state.

Anatomy of Multistate Models

Unlike conventional tables or multiple decrement tables, multistate models do not have a standard table format with a small set of measures shown in every analysis. Instead, measures are selectively presented according to the requirements of the analysis. Many measures have counterparts in conventional and multiple decrement life tables. Output measures include lifetime transition probabilities, proportion of the population dying while in each state, expected duration of stay in each state (e.g., marriage), and number of transitions to each state per person.

Construction of Multistate Models

Multistate life tables are constructed from transition probabilities between the states analyzed in the table. There are different ways of deriving the basic transition probabilities. They may be obtained from a survey with a retrospective question on the previous year’s status. Alternatively, the appropriate transitions, including deaths, may be secured from a longitudinal (panel) survey. The panel data for two dates can be used both to validate retrospective data from a single survey and to provide the matrix of required transition data. The appropriate transitions can also be calculated from census population counts and vital statistics. The construction of a multistate table is illustrated in this chapter with a table of working life.

Working-Life Tables

Working-life tables model the worklife history of a hypothetical cohort assumed to experience currently prevailing labor force ratios. Multistate models of working life describe labor force attachment as a dynamic process. Members of the population are viewed as entering and leaving the labor market repeatedly during their lifetimes, with nearly all participating for some period during their lives. Tables of working life are useful in understanding the mechanisms and implications of changes in the labor force. These tables provide an indication of the average number of working years to be expected after a given age by all persons or by persons in the labor force attaining that age. In addition, the tables provide information on age-specific rates of accession to, and separation from, the labor force. These measures are used in legal proceedings to estimate work years lost and earnings foregone by individuals whose careers have been truncated by death. They are often used by governments for estimating manpower replacement needs for industry and assessing the economic implications of changes in economic activity ratios and the age-structure of the population.

The construction of a working-life table for U.S. males in 1979–1980, developed by Smith (1982, 1986), is shown as Table 13.14. The major source of data for constructing this table is the Current Population Survey (CPS), the nationwide monthly household survey sponsored by the U.S. Bureau of Labor Statistics. Because individuals are interviewed during each of four successive months and

State at time 1, age x	State at time 2, age x+1			
	Total	In labor force	Not in labor force	Dead
In labor force	Group A	Actives	Exits	Deaths of actives
Not in labor force	Group B	Entrants	Inactives	Deaths of inactives

FIGURE 13.2 Labor force flows for the 1979–1980 Tables of Working Life

again in the same 4 months of the following year, CPS records can be matched so that each person's status at the beginning and end of a 12-month interval can be compared. To construct the table of working life shown as Table 13.14, individuals' labor force status in a given month of 1979 was matched to that information in the same month of 1980. If labor force status changed between the two reference dates, then labor force transitions were registered. Surviving respondents were classified as "active" or "inactive" if their status was identical at both dates, and as "entrants" or "exits" if their status changed. The state space of this multistate model consists of three states: two labor force states (active and inactive) and death. Four transitions among the states are possible, two between the labor force statuses and two between each of these and death. Life table calculations are carried out on single-year-of-age data. Life table calculations are based on exact ages, but the survey data have a slightly different age reference. In the survey, when the average person claims to have a certain age x, that person's age is actually halfway between exact age x and exact age x + 1. Therefore, the survey data have to be recentered to exact ages before developing the life table functions. Mortality rates were assumed to be identical for all persons of a given age, regardless of labor force status. The number of persons expected to die between interviews was estimated using a standard mortality schedule.

As an example of the process involved in constructing a working life table, we first calculate a transition matrix for each age by setting up a 2 × 2 contingency table of labor force status in 1980 by labor force status in 1979, as shown in Figure 13.2. Age-specific transition probabilities (p_x, shown in columns 2 to 6 in Table 13.14) indicate the likelihood that an individual of a given exact age and labor force status will be in each of three possible states (active, inactive, or dead) 1 year later. For each age and labor force status, the transition probabilities sum to 1. For example, at age 16,

$$p_x^d + {}^{i}p_x^i + {}^{i}p_x^a = .00130 + .70257 + .29613 = 1.00000$$

For each combination of age and labor force status, the transition probabilities are computed as row percentages from tables, such as is shown in Figure 13.2. For example, the probability of exiting the labor force beginning at age x is

$$ {}^{a}p_x^i = \frac{Exits_x}{Group\ A_x} \tag{13.76}$$

where p is the transition probability, the prefixed superscript a refers to the state at time 1, the suffixed superscript i refers to the state at time 2, and Group A_x refers to the row total from Figure 13.2.

Age-specific rates of transfer between statuses m_x (columns 7 to 9 in Table 13.14) are the number of transfers from state 1 to state 2 between exact ages x and $x + 1$ per thousand cohort members age x in the stationary population. These transfer rates allow for multiple status changes by individuals during the year. Transfer rates are computed from transition probabilities as follows:

$$ {}^{a}m_x^i = \frac{4 \cdot {}^{a}p_x^i}{(1+{}^{a}p_x^a)(1+{}^{i}p_x^i) - ({}^{a}p_x^i \cdot {}^{i}p_x^a)} \tag{13.77}$$

For example, the rate of separation from the labor force at age 17 is

$$ {}^{a}m_x^i = \frac{4 \cdot {}^{a}p_x^i}{(1+{}^{a}p_x^a)(1+{}^{i}p_x^i) - ({}^{a}p_x^i \cdot {}^{i}p_x^a)} $$
$$ = \frac{4 \cdot .22043}{(1+.77808) \cdot (1+.61255) - (.22043 \cdot .38597)} = .31692 $$

The probability of moving into and out of the labor force is relatively high at some ages, such as 16 and 17, when many start their careers with temporary summer jobs. Although some individuals will make changes repeatedly during the year, only their final transition is included in the year-to-year comparisons. Their other changes are lost.

The survivors from an initial cohort of 100,000 births, l_x, are in column 10 of Table 13.14. They are calculated from the survivors in the preceding age $x - 1$ and the probability of surviving from age $x + 1$ to age x:

TABLE 13.14 Working-Life Table for Men, United States: 1979–1980

Exact age (1)	Probability of transition between specific states during age interval x to x + 1: Living to dead p_x^d (2)	Inactive to Inactive $^ip_x^i$ (3)	Inactive to Active $^ip_x^a$ (4)	Active to Inactive $^ap_x^i$ (5)	Active to active $^ap_x^a$ (6)	Age-specific rates of transfer per 1000 persons in initial status during age interval x to x + 1: Mortality m_x^d (7)	Labor force accession $^im_x^a$ (8)	Voluntary labor force separation $^am_x^i$ (9)
16	.00130	.70257	.29613	.26233	.73637	.00126	.61967	.36095
17	.00148	.61255	.38597	.22043	.77808	.00149	.55491	.31692
18	.00165	.58417	.41418	.17233	.82602	.00165	.58722	.24433
19	.00177	.55755	.44068	.14453	.85370	.00177	.62430	.20475
20	.00189	.53996	.45815	.12034	.87777	.00189	.64607	.16970
21	.00200	.52949	.45851	.09781	.90019	.00200	.85514	.13678
22	.00206	.51802	.47991	.08162	.91632	.00207	.66891	.11376
23	.00208	.50424	.49369	.07061	.92731	.00208	.68945	.09861
24	.00205	.49676	.49919	.05970	.93825	.00205	.69448	.06305
25	.00202	.49823	.49976	.05036	.94763	.00202	.69103	.06963
26	.00197	.49443	.50360	.04307	.95496	.00197	.69456	.05942
⋮								
65	.02820	.90426	.06754	.24122	.73058	.02860	.08239	.29425
66	.03043	.90424	.06533	.24690	.72267	.03090	.08005	.30255
67	.03293	.90565	.06142	.25232	.71475	.03348	.07554	.31033
68	.03577	.90603	.05820	.25232	.71191	.03642	.07167	.31072
69	.03893	.90693	.05414	.25343	.70764	.03970	.06679	.31262
70	.04238	.90671	.04891	.25038	.69724	.04330	.06062	.32276
71	.04603	.90957	.04440	.25865	.69532	.04711	.05505	.32073
72	.04979	.90948	.04073	.26201	.68820	.05106	.050570	.32619
73	.05359	.90651	.03990	.27557	.67084	.05507	.05028	.34723
74	.05750	.90943	.03307	.27980	.66270	.05920	.04179	.35356
75	.06161	.91349	.02478	.22977	.70850	.06357	.03037	.28162

Exact age	Survivors in each status to exact age x per 100,000 persons born — Labor force status: Total l_x (10)	Active ^al_x (11)	Inactive ^il_x (12)	Number of status transfers within surviving population during age interval x to x + 1: Labor force entries $^it_x^a$ (13)	Voluntary labor force exits $^at_x^i$ (14)	Deaths by labor force status: Total t_x^d (15)	Active $^at_x^d$ (16)	Inactive $^it_x^d$ (17)
16	97,823	46,923	50,900	28,496	18,694	123	65	58
17	97,700	56,660	41,040	21,827	18,474	145	87	58
18	97,555	59,926	37,629	20,534	15,272	161	103	58
19	97,394	65,085	32,309	18,645	13,809	172	119	53
20	97,222	69,801	27,421	16,354	12,188	183	136	48
21	97,039	73,833	23,206	13,992	10,338	194	151	43
22	96,845	77,335	19,510	12,016	8,962	200	163	37
23	96,644	80,226	16,418	10,466	8,023	201	169	32
24	96,443	82,499	13,944	8,967	6,930	198	171	26
25	96,246	84,366	11,880	7,618	5,927	194	172	22
26	96,052	85,885	10,167	6,562	5,140	189	171	19
⋮								
65	70,376	24,686	45,690	3,830	6,739	1,965	655	1,330
66	68,391	21,121	47,270	3,812	5,971	2,081	610	1,471
67	66,310	18,351	47,959	3,627	5,340	2,184	576	1,607
68	64,126	16,062	48,064	3,428	4,707	2,294	552	1,742
69	61,833	14,232	47,801	3,152	4,202	2,407	534	1,874
70	59,426	12,649	46,777	2,806	3,834	2,518	514	2,004
71	56,908	11,107	45,801	2,486	3,346	2,619	491	2,128
72	54,288	9,756	44,532	2,220	2,982	2,703	467	2,236
73	51,585	8,528	43,057	2,123	2,772	2,764	440	2,325
74	48,821	7,439	41,382	1,695	2,426	2,807	407	2,400
75	46,013	6,298	39,715	1,176	1,654	2,835	373	2,462

(*continues*)

TABLE 13.14 (*continued*)

	Person years lived in each status during age x			Person years lived in each status beyond exact age x		
Age	Total L_x (18)	Active L_x^a (19)	Inactive L_x^i (20)	Total T_x (21)	Active T_x^a (22)	Inactive T_x^i (23)
16	97,762	51,792	45,970	5,430,730	3,820,429	1,610,301
17	97,628	58,293	39,335	5,332,968	3,768,638	1,564,330
18	97,475	62,506	34,969	5,235,340	3,710,345	1,524,995
19	97,308	67,443	29,865	5,137,865	3,647,839	1,490,026
20	97,130	71,817	25,313	5,040,557	3,580,395	1,460,162
21	96,941	75,583	21,358	4,943,427	3,508,578	1,434,849
22	96,744	78,780	17,964	4,848,486	3,432,995	1,413,491
23	96,544	81,363	15,181	4,749,742	3,354,215	1,395,527
24	96,345	83,433	12,912	4,653,196	3,272,852	1,380,348
25	96,149	85,125	11,024	4,556,853	3,189,419	1,367,434
⋮						
65	69,384	22,903	46,481	1,002,062	180,527	841,535
66	67,351	19,736	47,615	832,678	137,624	795,054
67	65,218	17,207	48,011	865,327	117,887	747,440
68	62,960	15,147	47,833	800,109	100,681	699,428
69	60,629	13,440	47,189	737,129	85,533	851,596
70	58,166	11,878	46,288	676,500	72,093	604,407
71	55,597	10,431	45,166	618,334	60,216	558,118
72	52,936	9,142	43,794	562,737	49,784	512,953
73	50,203	7,963	42,220	509,801	40,642	169,159
74	47,417	6,869	40,548	459,596	32,659	426,939
75	44,596	5,873	38,723	412,181	25,791	385,390

Expectation of active and inactive life by current labor force status

	Total population			Currently active in labor force		Currently inactive in labor force	
Exact age	Life expectancy e_x (24)	Active years remaining e_x^a (25)	Inactive years remaining e_x^i (26)	Active years remaining ${}^ae_x^a$ (27)	Inactive years remaining ${}^ae_x^i$ (28)	Active years remaining ${}^ie_x^a$ (29)	Inactive years remaining ${}^ie_x^i$ (30)
16	55.5	39.1	16.5	39.8	15.7	38.3	17.2
17	54.6	38.6	16.0	39.3	15.3	37.7	16.9
18	53.7	38.0	15.6	38.7	15.0	37.1	16.6
19	52.8	37.5	15.3	38.1	14.7	36.4	16.3
20	51.8	36.8	15.0	37.4	14.5	35.7	16.1
21	50.9	36.2	14.8	36.7	14.3	35.0	16.0
22	50.0	35.4	14.6	35.9	14.1	34.2	15.8
23	49.1	34.7	14.4	35.1	14.0	33.4	15.7
24	48.2	33.9	14.3	34.3	13.9	32.6	15.6
25	47.3	33.1	14.2	33.5	13.8	31.8	15.6
26	46.4	32.3	14.1	32.7	13.8	30.9	15.5
⋮							
65	14.2	2.3	12.0	4.1	10.1	1.2	13.1
66	13.6	2.0	11.6	3.9	9.7	1.0	12.6
67	13.0	1.8	11.3	3.8	9.3	0.8	12.2
68	12.5	1.6	10.9	3.6	8.9	0.7	11.8
69	11.9	1.4	10.5	3.4	8.5	0.5	11.4
70	11.4	1.2	10.2	3.2	8.1	0.4	11.0
71	10.9	1.1	9.8	3.1	7.8	0.3	10.5
72	10.4	0.9	9.4	2.8	7.5	0.2	10.1
73	9.9	0.8	9.1	2.6	7.3	0.1	9.7
74	9.4	0.7	8.7	2.2	7.2	0.1	9.3
75	9.0	0.6	8.4	1.7	7.2	0.0	8.9

Source: S. J. Smith, 1986, February. *Worklife Estimates: Effects of Race and Education*. Bulletin 2254, U.S. Bureau of Labor Statistics.

$$l_x = l_{x-1} \cdot (1 - p_{x-1}^d) \qquad (13.78)$$

Columns 11 and 12 in Table 13.14 represent the number of survivors from the initial cohort remaining in a labor force status for each age. Survivors are allocated between states using the transfer rates. The number of survivors in each state at age x is the number in that state at the previous age $x - 1$ plus persons entering that state minus those exiting that state and those who died while in the state of interest. For example, the number of inactives at age x is

$${}^i l_x = {}^i l_{x-1} + ({}^a L_{x-1} \cdot {}^a m_{x-1}^i) - ({}^i L_{x-1} \cdot {}^i m_{x-1}^a) - ({}^i L_{x-1} \cdot {}^i m_{x-1}^d) \quad (13.79)$$

The number of survivors in each state at age x can be restated in terms of the number of survivors from the previous age and the number of transfers among states, shown in columns 13 through 17 in Table 13.14. Continuing with the example of the number of inactives at age x,

$${}^i l_x = {}^i l_{x-1} + {}^a t_{x-1}^i - {}^i t_{x-1}^a - {}^i t_{x-1}^d \qquad (13.80)$$

For example, the number of inactives at age 20 is

$$\begin{aligned} {}^i l_{20} &= {}^i l_{19} + {}^a l_{19}^i - {}^i l_{19}^a - {}^i l_{19}^d \\ &= 32{,}309 + 13{,}809 - 18{,}645 - 53 = 27{,}421 \end{aligned}$$

Assuming that deaths and labor force entries and exits are evenly distributed throughout the year, the total number in the stationary population alive at midyear is half of the sum of the stationary population at the beginning and end of that interval:

$$L_x = \frac{l_x + l_{x+1}}{2},\ L_x^i = \frac{{}^i l_x + {}^i l_{x+1}}{2},\ \text{or } L_x^a = \frac{{}^a l_x + {}^a l_{x+1}}{2}$$

The number in each state at midyear, computed in this way, is shown in columns 18 through 20 of Table 13.14. This figure is also known as the number of person-years lived by the group in any state as it passes through a given age.

Columns 21 through 23 in Table 13.14 are the sum of the person years (of total life, active life, inactive life) from age x to the end of the table. For example, the sum of the person-years lived in inactive status from age x to the end of the table is

$$T_x^i = \sum_{x=x}^{x=\omega} L_x^i$$

Columns 24 through 30 of Table 13.14 show many measures of the expectation of life for the total population. The population-based expectancy of working life is the average number of years to be spent in the labor force above exact age x for each person reaching that age. This is an overall measure for the total cohort and can be derived for every age. The average working life expectancy for the total population at age x is

$$e_x^a = \frac{T_x^a}{l_x}$$

For example, the average working life expectancy for the total population at age 66 is

$$e_{66}^a = \frac{T_{66}^a}{l_{66}} = \frac{137{,}624}{68{,}391} = 2.0$$

Other life expectancies also can be calculated, but Table 13.14 does not present all the information that is required. The labor force-based expectancy of working life refers to the average number of years to be spent in the labor force above a given exact age for each person in the labor force at that age. It is the ratio of total number of years in the labor force from age x onward to the number of survivors in the labor force at age x. For example, the average working life expectancy for those currently working is

$${}^a e_x^a = \frac{{}^a T_x^a}{{}^a l_x}$$

However, ${}^a T_x^a$ is not shown in Table 13.14, which follows a cohort of individuals through a lifetime of labor force entries and exits to estimate average remaining working life for all persons in the cohort. It is possible to follow other, more specific cohorts through their lifetime using the same general procedure. First, a cohort is identified, such as those who entered the labor force at a specific age. Then, survivors of the original cohort are subjected to the transfer rates appropriate to their current age and status to derive the survivors at each age over the lifetime. This set of survivors by age is converted into person-years of activity and inactivity in the labor force for that group during the age interval. The person-years are then summed across all ages and then averaged over persons alive and in the given status at the initial age. Figure 13.3 demonstrates some of these calculations by following a cohort of men at initial age 16.

Because of the high variances of the transition probabilities, resulting mainly from the detailed cross-classification of the sample data, and because of the problems of heterogeneity and age dependency, multivariate analysis may be used to improve the stability and other qualities of the data (i.e., regression analysis with covariates may be applied to the original rates).

THE LIFE TABLE IN CONTINUOUS NOTATION

Up to this point, this chapter has presented life tables using discrete notation, both because it is easier for most people to grasp and because it facilitates applications. This section introduces the life table in continuous notation because much of demographic theory rests on the use of this notation in life table analysis.

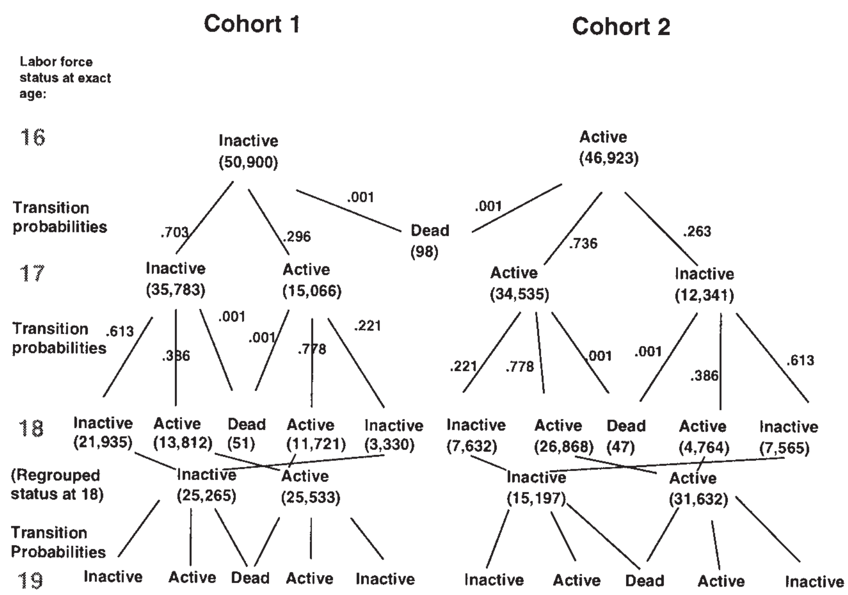

FIGURE 13.3 Selected portion of the labor force status-specific markov chain for men, initial age 16

In this formulation, age is viewed as a continuous variable rather than a discrete variable. The notation for the basic life table functions changes. In the discrete formulation, the basic life table functions were indexed with age shown as a subscript. When continuous notation is used, they are indexed with age shown in parentheses to indicate the value at which the function is evaluated. For example, the number of survivors at age x is l_x in the discrete formulation but l(x) in continuous notation.

In the discrete formulation, time was indexed in terms of years, but with continuous notation, time can be broken into smaller increments. For example, the number of survivors in a 5-year age interval can be viewed as the sum of the number of survivors in 5 single-year age groups or 10 half-year age groups. As the age group interval becomes narrower, eventually the difference between an interval and a point disappears. So continuous notation uses an integral ($\int$) rather than a summation sign (Σ) to represent sums.

For example, the number of person-years lived in the age interval x to $x + n$ is

$$ {}_nL_x = \lim_{k\to 0}\sum_{a=x}^{x+n-k} {}_kL_a = \int_{a=x}^{x+n} l(a)da \qquad (13.81)$$

In continuous notation, life expectancy is

$$ e_x = \frac{\int_{a=x}^{\omega} l(a)da}{l_x} \qquad (13.82)$$

The hazard rate or the force of mortality (or instantaneous death rate) is defined as

$$ h(t) = \mu(x) = -\frac{l'(x)}{l(x)} = -\frac{d}{dx}\ln l(x) \qquad (13.83)$$

Increasing the hazard rate corresponds to decreasing survival time.

The number of survivors at each age can be defined as the integral of its hazard function,

$$ l(x) = \exp\left[-\int_0^x \mu(a)da\right] \qquad (13.84)$$

where l(0) is set to 1.

It is possible to estimate the force of mortality from life table survivors at age x. For example, Jordan (1967, p. 18) uses the following approximation:

$$ \mu(x) = -\frac{1}{l(x)}\frac{d}{dx}l(x) \approx \frac{(d_{x-1}+d_x)}{2lx} = \frac{(l_{x-1}-l_{x+1})}{2lx}$$

Another approach is to define a cumulative force of mortality for the age interval from x to $x + n$:

$$ {}_nh_x = \int_x^{x+n}\mu(a)da$$

Manton and Stallard (1984) have suggested adding a column for the cumulative force of mortality to the life table by using

$$ {}_nh_x = -\ln(1-{}_nq_x) = -\ln{}_np_x$$

This column can also be calculated for multiple decrement life tables.

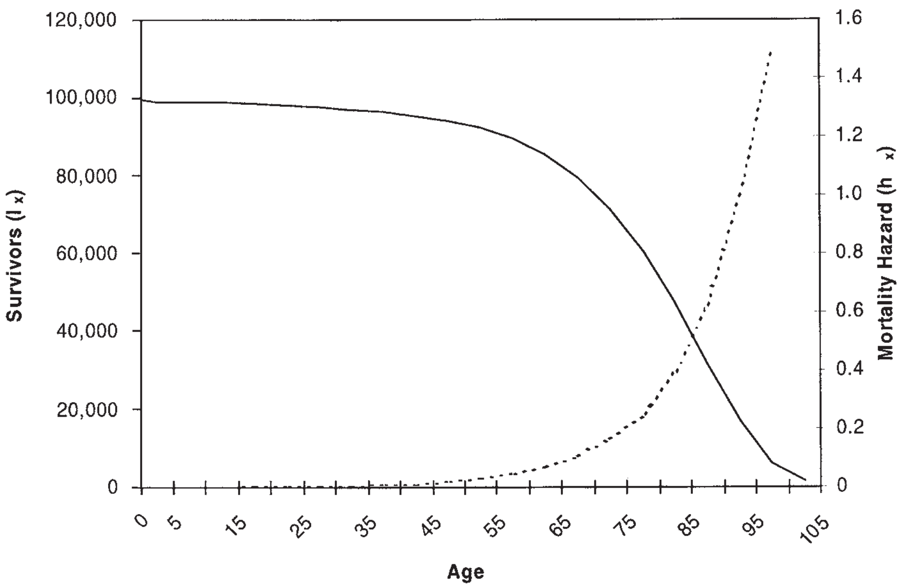

FIGURE 13.4 Survivors of 100,000 births and force of mortality by age in life table for total United States, 1989–91

Figure 13.4 compares the survival function l_x to the hazard function h_x for the U.S. abridged life table shown in Table 13.2. The number of survivors falls with increasing age, while the mortality hazard increases with age. Note that this graph uses the approximation $\mu\left(x+\frac{1}{2}\right)=h_x$.

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