Lecture 7: Period mortality

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Outline

- Measurement of mortality
- Standardization
- Period lifetables
- Stable population theory
- Another example of a lifetable

Measurement of mortality

- Quantification of mortality is central to demography
- Measurement of mortality dates back to John Graunt (1620–1674) and his analyses of the "Bills of Mortality"
- Mortality refers to the relative frequency of death in a population

Two concepts of mortality

- Life span
  - Numerical age limit of human life
  - Maximum recorded age at death
  - 122 years and 164 days, lived by the Frenchwoman Jeanne Louise Calment
- Life expectancy
  - Average expected number of years of life to be lived by a particular population at a given time

Crude death rate

- Crude death rate (*CDR*)

$$CDR = d / p * 1{,}000$$

  - *d*: deaths in the year
  - *p*: population at midyear

- Data for the United States for 2013

$$CDR = 2{,}596{,}993 / 316{,}497{,}531 * 1{,}000$$

$$CDR = 8.2$$

- World range of *CDR* in 2014
  - United Arab Emirates (UAE) and Qatar = 1
  - Lesotho = 21

CDR and age composition

- When *CDR*s are compared among countries, differences are sometimes due to differences in age composition
  - Previous examples mean that there are 8 times more deaths per 1,000 people in the US than in the UAE
  - Why is the *CDR* of the US eight times higher than of the UAE?
  - The main reason is that the UAE is much younger in average age than is the U.S.
  - Younger people have lower death rates

Young and old people

- Countries with
  - Large proportions of young people
  - Small proportions of old people
  - Usually have lower *CDR*s

- Countries with
  - Small proportions of young people
  - Large proportions of old people
  - Usually have higher *CDR*s

Changing age structure

- If age structure has changed over time
  - CDRs should not be used to compare the death experiences of the same population at different points in time
- US *CDR* did not change much in 54 years
  - CDR_{1960} = 9.5 per 1,000
  - CDR_{2014} = 8.2 per 1,000
- *CDR* is not capable of capturing the reduction in mortality when the population becomes older
  - The US became older between 1960 and 2014
  - Median age: 29 in 1960 and 37 in 2014

Crude rate

- *CDR* is a crude rate, because its denominator comprises the entire population
- However, population members are not equally at risk of experiencing death
- Risk of death varies by age, sex, race/ethnicity, socioeconomic status, and others
- Death rates vary considerably by age...

Age-specific death rates

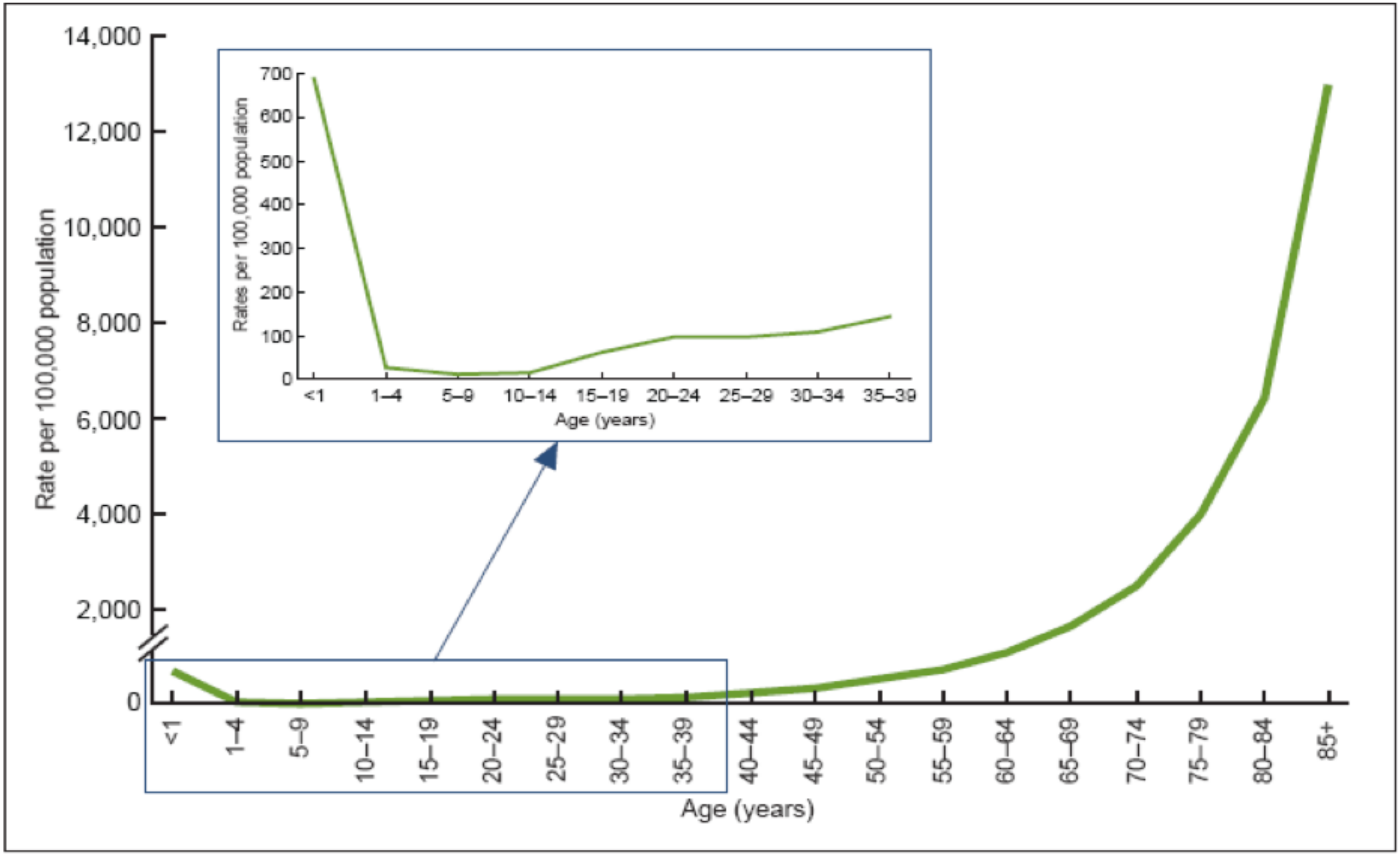
- Demographers use age-specific death rates as a more precise way to measure mortality

- Age-specific death rate (${}_nASDR_x$ or ${}_nM_x$)

$${}_nM_x = {}_nd_x \,/\, {}_np_x * 1{,}000$$

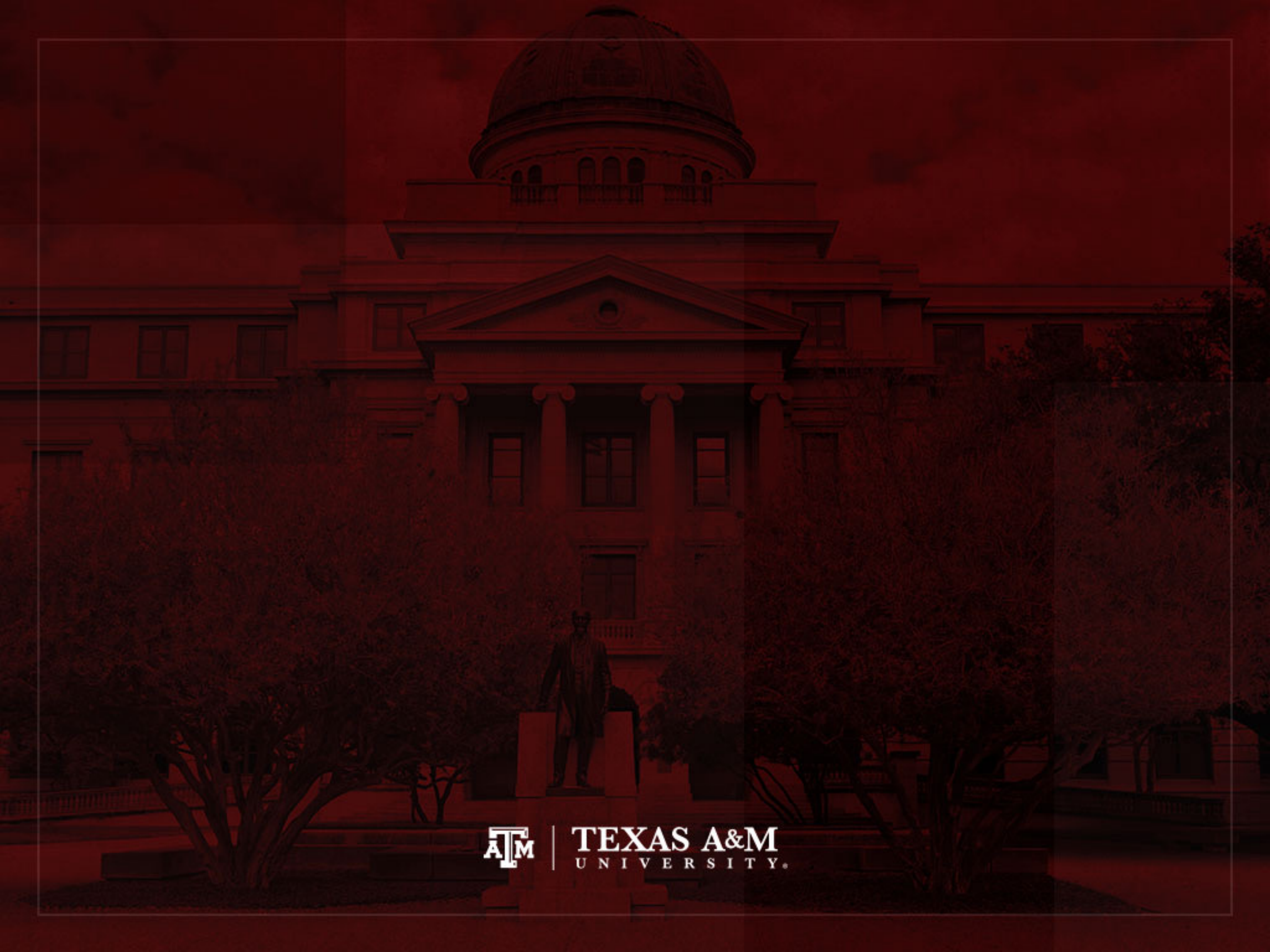
  - ${}_nd_x$: deaths to persons aged x to $x+n$
  - ${}_np_x$: persons in the population who are aged x to $x+n$
  - n: width of the age group
  - x: initial year of the age group
  - For instance, *ASDR* for age group 15–19 is ${}_5M_{15}$

Age curve of mortality, US, 2007



Source: Minino, et al., 2009: 2.

Source: Poston, Bouvier 2017, p.169.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Standardization

- ${}_nASDR_x$, and not *CDR*, should be used to compare the mortality experiences of countries with different age compositions
  - We use **standardization** to take into account age composition when we compare death rates among different countries
  - We can compare crude death rates for different countries or years
  - We need to adjust for differences in age structure
  - We estimate age-adjusted death rates and apply to a standard population

Mortality and fertility

- We cannot simply add up ${}_nASDR_x$ and multiply by the width of the age interval
  - People die just once

- This makes sense for age-specific fertility rates (${}_nASFR_x$) and total fertility rates (TFR)

$$TFR = \sum({}_nASFR_x * n)$$

  - n: width of the age group
  - Women can have more than one child

Age standardization

- Young populations tend to have low *CDR*s
- Old populations tend to have high *CDR*s
- We estimate a variation of *CDR* that allows us to account for age composition when comparing death rates among different countries

$$CDR = \Sigma\ {}_{n}ASDR_{x} * ({}_{n}P_{x} / P) * 1{,}000$$

  - P: total population
  - ${}_{n}P_{x}$: population in age group x
  - ${}_{n}ASDR_{x}$: *ASDR* for people aged x to $x+n$

Source: Formula 7.4 (Poston, Bouvier 2017, p.171).

United States, 2006

Age group	Population	Prop. population	Deaths	Age-specific death rate (ASDR)
0-1	4,147,760	0.0139	27,126	0.00654
1-4	16,352,320	0.0548	4,742	0.00029
5-9	20,142,000	0.0675	2,820	0.00014
10-14	21,454,960	0.0719	3,862	0.00018
15-19	21,604,160	0.0724	13,827	0.00064
20-24	20,947,680	0.0702	19,062	0.00091
25-29	20,022,640	0.0671	18,020	0.00090
30-34	20,261,360	0.0679	21,477	0.00106
35-39	21,067,040	0.0706	32,233	0.00153
40-44	22,857,440	0.0766	52,801	0.00231
45-49	22,588,880	0.0757	77,028	0.00341
50-54	20,142,000	0.0675	99,300	0.00493
55-59	17,158,000	0.0575	127,312	0.00742
60-64	13,040,080	0.0437	149,961	0.01150
65-69	10,115,760	0.0339	180,061	0.01780
70-74	8,474,560	0.0284	234,830	0.02771
75-79	7,400,320	0.0248	321,914	0.04350
80-84	5,580,080	0.0187	388,262	0.06958
85-89	3,192,880	0.0107	353,005	0.11056
90-94	1,342,800	0.0045	234,681	0.17477
95-99	387,920	0.0013	107,283	0.27656
100+	89,520	0.0003	39,292	0.43892
Total	**298,400,000**	**0.9999**	**2,508,899**	**1.20116**

CDR per 1,000	**8.41**

Venezuela, 2006

Age group	Population	Prop. population	Deaths	Age-specific death rate (ASDR)
0-1	487,056	0.0219	7,905	0.01623
1-4	1,779,200	0.0800	1,139	0.00064
5-9	2,308,512	0.1038	739	0.00032
10-14	2,275,152	0.1023	933	0.00041
15-19	2,250,688	0.1012	3,286	0.00146
20-24	2,057,200	0.0925	4,896	0.00238
25-29	1,872,608	0.0842	4,138	0.00221
30-34	1,614,624	0.0726	3,278	0.00203
35-39	1,530,112	0.0688	3,290	0.00215
40-44	1,401,120	0.0630	4,049	0.00289
45-49	1,145,360	0.0515	4,707	0.00411
50-54	971,888	0.0437	5,520	0.00568
55-59	769,504	0.0346	5,964	0.00775
60-64	558,224	0.0251	6,548	0.01173
65-69	400,320	0.0180	7,514	0.01877
70-74	302,464	0.0136	8,584	0.02838
75-79	215,728	0.0097	9,212	0.04270
80-84	117,872	0.0053	8,877	0.07531
85-89	53,376	0.0024	6,816	0.12769
90-94	15,568	0.0007	3,241	0.20820
95-99	2,224	0.0001	724	0.32576
100+	2,224	0.0001	1,089	0.48975
Total	**22,240,000**	**0.9951**	**102,449**	**1.37655**

CDR per 1,000	**4.61**

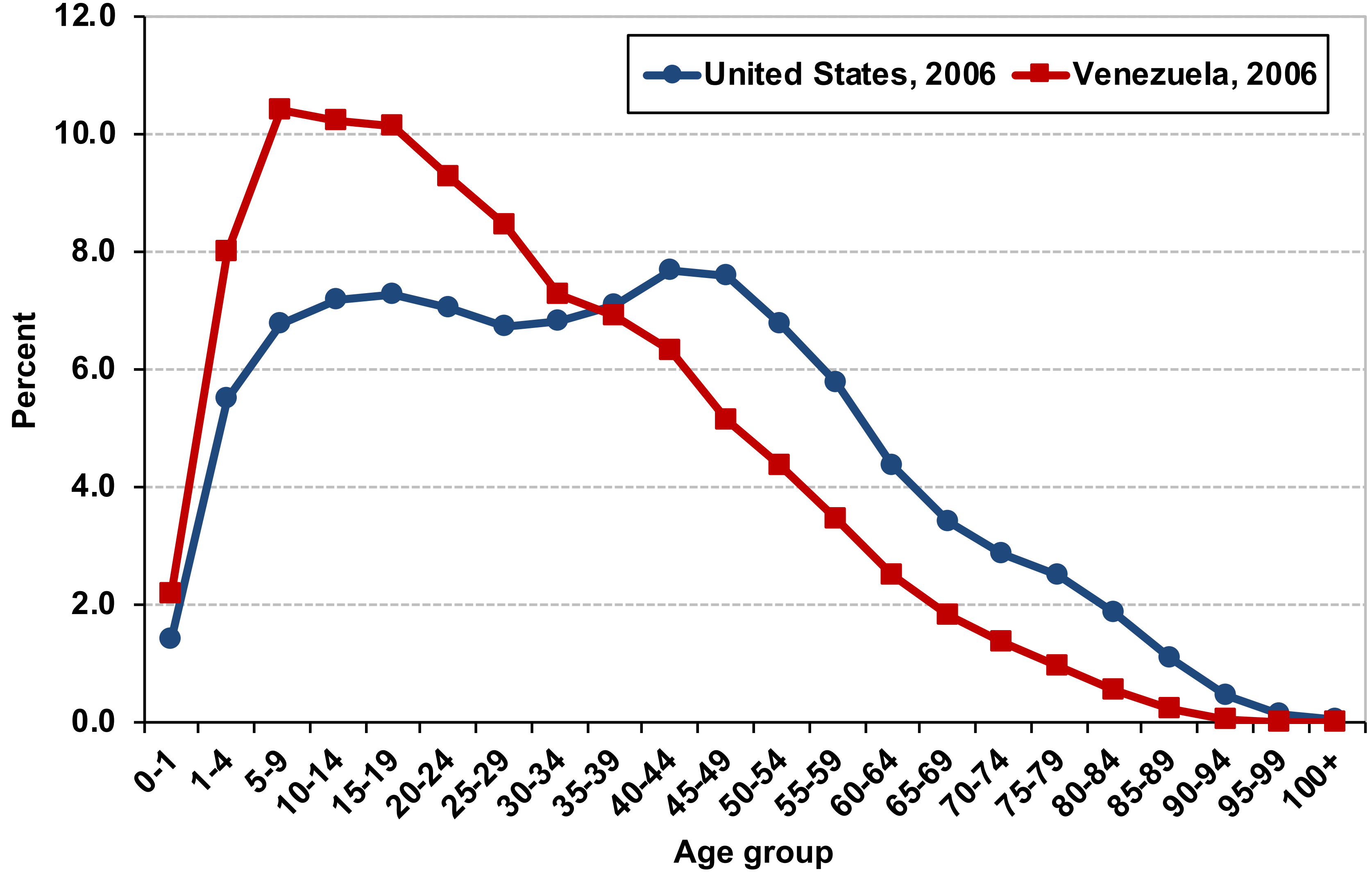
Population distribution by country

Age group	United States (%)	Venezuela (%)	Ratio United States / Venezuela
0-1	1.39	2.19	0.63
1-4	5.48	8.00	0.69
5-9	6.75	10.38	0.65
10-14	7.19	10.23	0.70
15-19	7.24	10.12	0.72
20-24	7.02	9.25	0.76
25-29	6.71	8.42	0.80
30-34	6.79	7.26	0.94
35-39	7.06	6.88	1.03
40-44	7.66	6.30	1.22
45-49	7.57	5.15	1.47
50-54	6.75	4.37	1.54
55-59	5.75	3.46	1.66
60-64	4.37	2.51	1.74
65-69	3.39	1.80	1.88
70-74	2.84	1.36	2.09
75-79	2.48	0.97	2.56
80-84	1.87	0.53	3.53
85-89	1.07	0.24	4.46
90-94	0.45	0.07	6.43
95-99	0.13	0.01	13.00
100+	0.03	0.01	3.00
Total	**99.99**	**99.51**	**1.00**

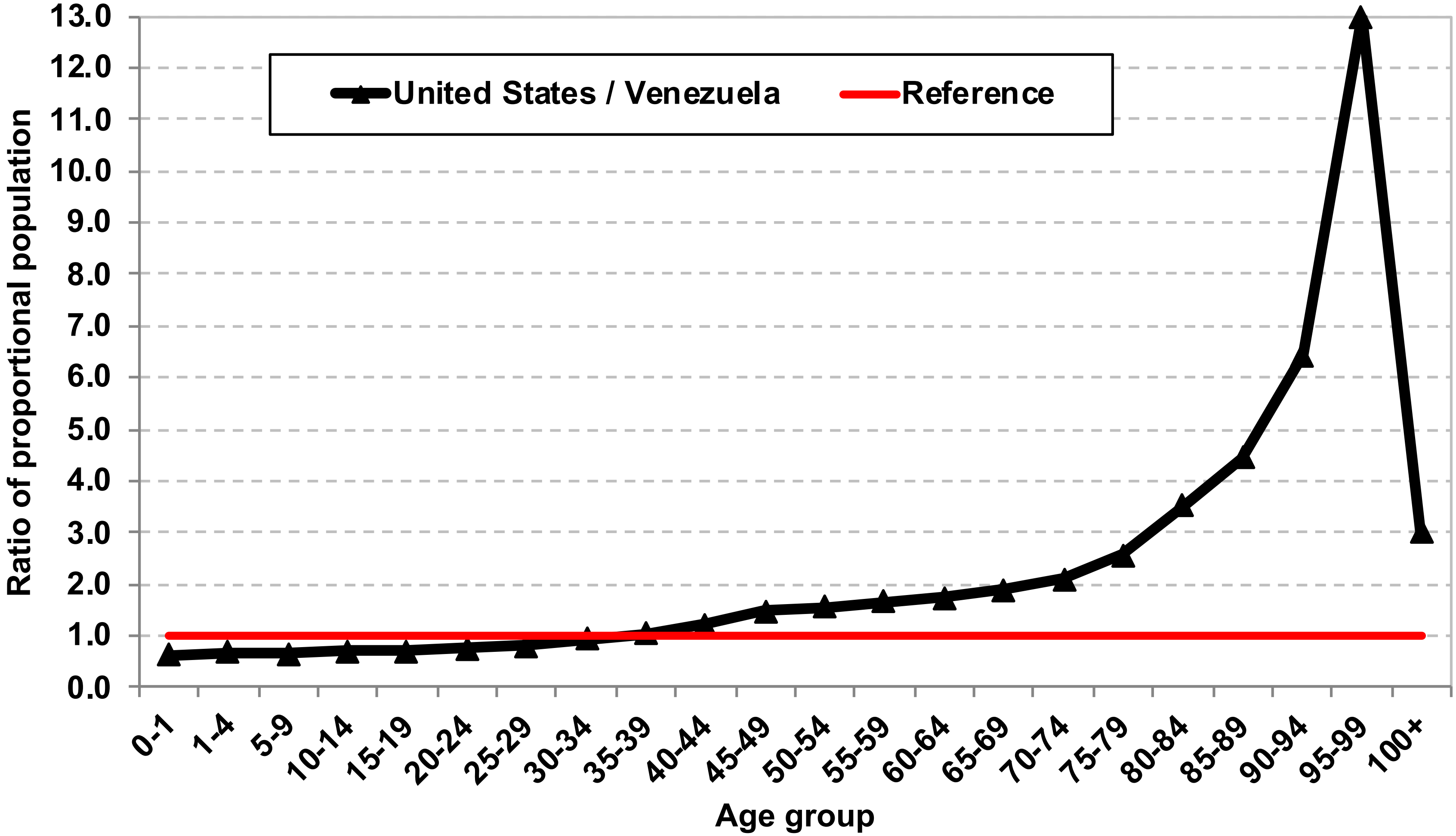
- The U.S. has an older population than Venezuela
- This is causing

$CDR_{US} > CDR_{VE}$

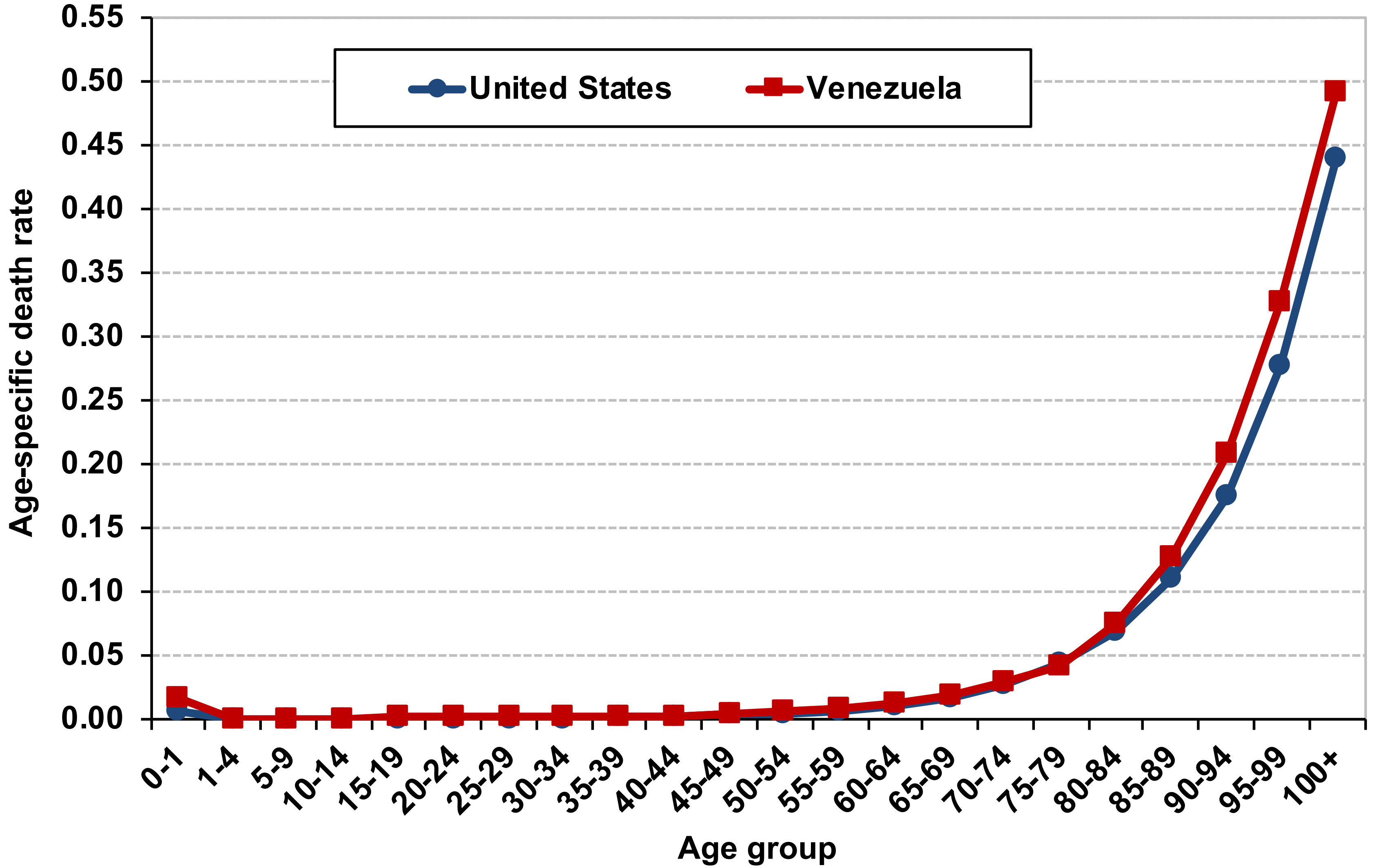
Age structure



The U.S. has an older population than Venezuela



Age-specific death rates, 2006



Standardize Venezuela's *CDR*

Standardization Age group	Venezuela (observed rates)	United States (standard prop. population)	Venezuela's rates times U.S. prop. population
0-1	0.01623	0.0139	0.0002
1-4	0.00064	0.0548	0.0000
5-9	0.00032	0.0675	0.0000
10-14	0.00041	0.0719	0.0000
15-19	0.00146	0.0724	0.0001
20-24	0.00238	0.0702	0.0002
25-29	0.00221	0.0671	0.0001
30-34	0.00203	0.0679	0.0001
35-39	0.00215	0.0706	0.0002
40-44	0.00289	0.0766	0.0002
45-49	0.00411	0.0757	0.0003
50-54	0.00568	0.0675	0.0004
55-59	0.00775	0.0575	0.0004
60-64	0.01173	0.0437	0.0005
65-69	0.01877	0.0339	0.0006
70-74	0.02838	0.0284	0.0008
75-79	0.04270	0.0248	0.0011
80-84	0.07531	0.0187	0.0014
85-89	0.12769	0.0107	0.0014
90-94	0.20820	0.0045	0.0009
95-99	0.32576	0.0013	0.0004
100+	0.48975	0.0003	0.0001
Total		**0.9999**	**0.0097**

CDR per 1,000	**9.68**

Source: Application of formula 7.4 (Poston, Bouvier 2017, p.171).

Another way... same results...

Standardization Age group	United States (standard population)	Venezuela (observed rates)	Venezuela (standardized deaths)
0-1	4,148,175	0.0162	67,325
1-4	16,353,955	0.0006	10,467
5-9	20,144,014	0.0003	6,446
10-14	21,457,106	0.0004	8,797
15-19	21,606,321	0.0015	31,545
20-24	20,949,775	0.0024	49,860
25-29	20,024,642	0.0022	44,254
30-34	20,263,386	0.0020	41,135
35-39	21,069,147	0.0022	45,299
40-44	22,859,726	0.0029	66,065
45-49	22,591,139	0.0041	92,850
50-54	20,144,014	0.0057	114,418
55-59	17,159,716	0.0078	132,988
60-64	13,041,384	0.0117	152,975
65-69	10,116,772	0.0188	189,892
70-74	8,475,408	0.0284	240,532
75-79	7,401,060	0.0427	316,025
80-84	5,580,638	0.0753	420,278
85-89	3,193,199	0.1277	407,740
90-94	1,342,934	0.2082	279,599
95-99	387,959	0.3258	126,381
100+	89,529	0.4898	43,847
Total	**298,400,000**		**2,888,718**

CDR per 1,000	**9.68**

Comparing crude death rates

- $CDR_{\text{United States original}}$

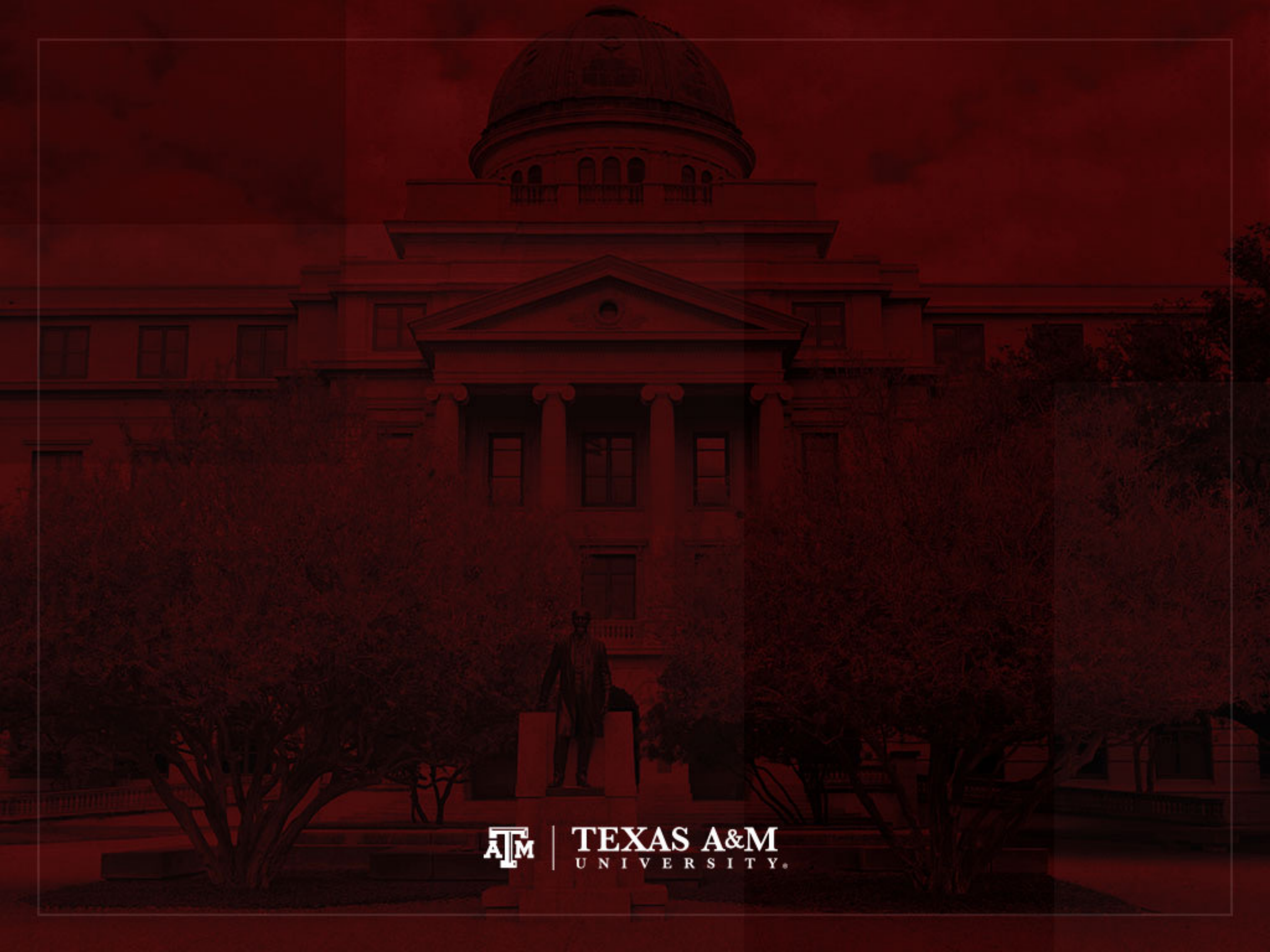
= 8.41 deaths per 1,000

- $CDR_{\text{Venezuela original}}$

= 4.61 deaths per 1,000

- $CDR_{\text{Venezuela standardized}}$

= 9.68 deaths per 1,000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Period lifetables

- One of the most important and elegant measures of the mortality experiences of a population is the life table
- It dates back to John Graunt (1620–1674) and his “Bills of Mortality”
- Demographers use the life table to determine life expectancy, not only at birth but at any age

Important information from life table

- Like the total fertility rate (*TFR*), the life table is a synthetic or hypothetical measure

- It tells us how many years of life, on average, may a person expect to live if the person during his or her lifetime is subjected to the age-specific probabilities of dying of a particular country or population at a given time

Example: United States, 2010

- Let's say that the population of the U.S. in 2010 had a life expectancy of birth of 78.7 years

- This means that if a cohort of persons, throughout the years of their life, were subjected to the *ASDR*s ($_nM_x$) of the total population in the U.S. in 2010
  - They would live, on average, 78.7 years

Examples of Life expectancy

- Life expectancy at birth is a primary indicator of quality of life

2013	Life expectancy	
	Male	Female
World	69	73
More developed countries	75	82
Less developed countries (except China)	65	69
Japan	80	86
Lesotho	42	45
Sierra Leone	45	46

Limitations of e_0

- We need to be aware of the fact that when considering life expectancy at birth (e_0), infant mortality plays a very important role

- When e_0 is low, a major reason is their very high infant mortality rate

- When comparing values of life expectancy at birth across countries, we should not think of e_0 as a modal age at death

Abridged life table

- An abridged life table is calculated for age groups

- Usually for five-year age groups

- Rather than for single-year age groups

Radix and mortality probabilities

- A life table starts with a population (a radix) of 100,000 persons born alive at age 0
  - This number is arbitrary, but conventional
  - It can also be 1,000 or 1
- From each age to the next, the population is decremented according to age-specific mortality probabilities until all members have died
  - The mortality schedule is fixed and does not change over the life of the population

Estimating period lifetables

- Estimate overall mortality of population
  - **Assumption**: age-specific rates for the period continue unchanged into the future
  - **Synthetic cohort**: imaginary cohort of new born babies would experience a life table from a specific period
  - **Life expectancy**: average age at death for a hypothetical cohort born in a particular year and being subjected to the risks of death experienced by people of all ages in that year

Mortality rates

- For cohort mortality, we did not build lifetables directly from age-specific mortality rates
  - We constructed cohort lifetables by starting with the observed survivorships l_x
  - From l_x, we computed ${}_nq_x = 1 - (l_{x+n} / l_x)$
  - From $l_x = {}_nd_x / {}_nq_x$ and ${}_nq_x = {}_nd_x / l_x$, we computed

$${}_nd_x = l_x - l_{x+n} \quad \text{or} \quad {}_nd_x = {}_nq_x * l_x$$

$${}_nL_x = (n)\,(l_{x+n}) + ({}_na_x)\,({}_nd_x)$$

  - From ${}_nd_x$ and ${}_nL_x$ come age-specific mortality rates

$${}_nm_x = {}_nd_x / {}_nL_x$$

Cohort and period data

- For a period, we do not have information on lifetime cohort survival
- We have period counts of deaths and counts of people
  - For cohort mortality (e.g. children of King Edward III), we had a death and an age at death
  - For period data, there will be many people from whom dates at death are not available, because many people will not die in the period

Mortality rates in period lifetables

- For a period lifetable, we start with age-specific mortality rates
  - Thus, mortality rates ($_{n}m_{x}$) are introduced only now

- We are assuming that age-specific mortality rates continue unchanged into the future
  - The assumption is not about mortality probabilities

Period mortality data

- Data by sex and age
  - Deaths in the period
  - Mid-period count of people
- Estimate period person-years lived (*PPYL*)
  - Multiply mid-period count by the length of the period
- *PPYL* is **not** the same as
  - Cohort person-years lived for any real cohort (*CPYL*)
  - Cohort person-years for the synthetic cohort (${}_{n}L_{x}$)

Calculating mortality rates

- Compute age-specific death rate for each age group (${}_nM_x$)

$$ {}_nM_x = \frac{{}_nD_x}{{}_nK_x \, T} $$

- ${}_nD_x$
  - Deaths between ages x and $x+n$ in the period
- ${}_nK_x$
  - Mid-period counts of people between ages x and $x+n$
- T
  - Length of the period (usually it is 1)

Rates into probabilities

- We use algebra to solve for probabilities of dying ($_nq_x$) in terms of mortality rates ($_nm_x$)
- We can substitute the period age-specific mortality rates ($_nM_x$) into the formula in place of cohort age-specific mortality rates ($_nm_x$)
- Then, we obtain the other lifetable columns with $_nq_x$

Probability of dying ($_nq_x$)

- Need to convert age-specific death rates ($_nM_x$) to probabilities of dying ($_nq_x$)
- Probability of death
  - Number of deaths during any given number of years
  - Divided by the number of people who started out being alive and at risk of dying

$$_nq_x = \frac{(n)(_nM_x)}{1 + (n - {_na_x})(_nM_x)}$$

  - $_na_x$: average years lived per person by people dying in the interval

Average years in the interval ($_na_x$)

- $_na_x$ is the average years lived per person by people dying in the interval

- For most age intervals, we can substitute the approximate value

$$_na_x = n / 2$$

- But for the first few age groups and the last one, there are better options
  - Special formulas for $_na_x$ come from empirical work by Keyfitz and Flieger (1968)...

${}_1a_0$ and ${}_4a_1$

- For the first year of life, ${}_1a_0$ depends on ${}_1M_0$

$$ {}_1a_0 = 0.07 + 1.7\ ({}_1M_0) $$

  - For very low mortality, the average is about 1 month (0.07 of a year)
  - It would require an age-specific rate almost as big as ${}_1M_0 = 0.200$ to imply an average ${}_1a_0$ near 6 months

- For a 4-year-wide age group from age 1 to age 5

$$ {}_4a_1 = 1.5 $$

${}_{\infty}a_x$

- Everybody who reaches the last open-ended interval dies at that age group

$${}_{\infty}q_x = 1$$

- Thus, for the last open-ended interval, we use the following formula that makes ${}_{\infty}q_x = 1$

$${}_{\infty}a_x = 1 \,/\, {}_{\infty}M_x$$

- We do not need ${}_{\infty}a_x$ to estimate ${}_{\infty}q_x$, but we use it to estimate person-years lived in the last open-ended interval (${}_{\infty}L_x$)

Example, U.S., 2010

- The following table gives raw counts of deaths and population for U.S. males and females for 2010

- Illustrate period lifetable calculations using males

Table 7.1 U.S. raw mortality data from 2010

		Male		Female		
Age x	n	${}_nD_x$	${}_nK_x$	${}_nD_x$	${}_nK_x$	Age x
0	1	13,703	2,029,308	10,884	1,942,651	0
1	4	2,460	8,281,720	1,856	7,932,129	1
5	5	1,325	10,372,176	1,005	9,939,790	5
10	5	1,729	10,583,108	1,220	10,104,857	10
15	35	139,372	74,583,451	76,052	73,789,207	15
50	20	365,912	34,559,040	244,524	37,013,053	50
70	5	120,193	4,277,145	97,007	5,069,352	70
75	5	143,016	3,187,811	130,346	4,135,105	75
80	5	168,836	2,306,217	183,485	3,451,358	80
85	5	157,711	1,290,434	221,744	2,357,755	85
90	5	88,592	428,765	170,943	1,033,640	90
95	5	25,975	80,683	78,713	288,017	95
100	∞	3,607	7,883	18,223	43,621	100

Male	${}_{35}a_{15} = 22.247$	${}_{20}a_{50} = 11.874$
Female	${}_{35}a_{15} = 23.996$	${}_{20}a_{50} = 12.201$

Source: Human Mortality Database (HMD) [accessed 29 June 2013].

${}_1M_0$

- Calculate observed period age-specific mortality rate for the 1-year-wide period

$${}_1M_0 = {}_1D_0 \,/\, {}_1K_0 = 13{,}703 \,/\, 2{,}029{,}308 = 0.006753$$

- It comes out to about 7 per thousand per year

${}_1a_0$

- How early in the interval do those who die in it die?

$${}_1a_0 = 0.07 + 1.7\ {}_1M_0$$

$${}_1a_0 = 0.07 + 1.7 * 0.006753$$

$${}_1a_0 = 0.081480$$

- The fraction 0.081 of a year is just about 1 month
- Infant boys who do die live about 30 days

${}_1q_0$

- The following information is a refined estimate of the probability of dying in the first year after birth

$$
{}_1q_0 = \frac{(1)({}_1M_0)}{1 + (1 - {}_1a_0)({}_1M_0)} = \frac{0.006753}{1 + (1 - 0.081480) * (0.006753)}
$$

$$
= 0.006710
$$

  - A common mistake is to use ${}_na_x$ instead of $n-{}_na_x$ in the denominator
  - The denominator itself is always close to 1

${}_4q_1$

- For the next interval, we have *x*=1 and *n*=4
- Half of *n*=4 would be 2, but the rule places deaths half a year earlier on average

$${}_4a_1 = 1.5$$

$$n - {}_4a_1 = 2.5$$

- From the data

$${}_4M_1 = 2{,}460 \,/\, 8{,}281{,}720 = 0.000297$$

- Then

$${}_4q_1 = \frac{4 * 0.000297}{1 + 2.5 * 0.000297} = 0.001187$$

Next calculations

- For period lifetables, we go on to build all the other columns from the ${}_nq_x$ column
  - Not from the l_x column as we did for cohort lifetables
- We choose a radix l_0, and calculate
  - $l_{x+n} = (1 - {}_nq_x)\, l_x$
  - $l_1 = (1 - {}_1q_0)\, l_0$
  - $l_5 = (1 - {}_4q_1)\, l_1$
  - And so on through the whole table
- Our period lifetable is the cohort lifetable for the synthetic cohort

Next columns

- Now we can calculate ${}_nd_x$, ${}_nL_x$, T_x, and e_x

- As a bonus, we have a chance to check our calculations with

$${}_nm_x = {}_nd_x \,/\, {}_nL_x$$

- This result has to come out equal to the period age-specific rates ${}_nM_x$ with which we start

Number of deaths ($_nd_x$) and alive (l_x)

- The life table assumes an initial population of 100,000 births (radix), which is subjected to the mortality schedule
  - Radix can also be 1 or 1,000
- Number of people dying during the age interval ($_nd_x$) equals probability of death during the age interval ($_nq_x$) times number alive at beginning of the age interval (l_x)

$$_nd_x = {_nq_x} * l_x = l_x - l_{x+n}$$

- Subtracting those who died in the previous age interval gives the number of people still alive at the beginning of next age interval

$$l_{x+n} = (1 - {_nq_x})\, l_x = l_x - {_nd_x}$$

Number of years lived ($_nL_x$)

- Number of years lived ($_nL_x$) considers that some people die before the end of the age interval

$$_nL_x = (n)\,(l_{x+n}) + ({}_na_x)\,({}_nd_x)$$

- We usually have $_na_x = n/2$, then formula simplifies

$$_nL_x \approx (n/2)\,(l_x + l_{x+n})$$

- $_nL_x$ for the last open-ended interval

$$_\infty L_x = ({}_\infty a_x)\,({}_\infty d_x) = (1 / {}_\infty M_x)\,({}_\infty d_x) = {}_\infty d_x / {}_\infty M_x$$

$$\text{or } {}_\infty L_x = l_x / {}_\infty M_x$$

  - l_x: number of survivors to the oldest age group
  - $_\infty M_x$: death rate at the oldest age group

Cumulative number of years lived (T_x)

- Number of years lived are added up, cumulating from the oldest to the youngest ages

- Total number of years lived in a given age interval and all older age intervals (T_x)

$$T_x = T_{x+n} + {}_nL_x$$

- T_x is easiest to compute by filling the whole ${}_nL_x$ column and cumulating sums from the bottom up

$$T_x = {}_nL_x + {}_nL_{x+n} + {}_nL_{x+2n} + \dots$$

- At the oldest age, T_x equals ${}_nL_x$

Life expectancy (e_x)

- Expectation of life is the average remaining lifetime
  - It is the total years remaining to be lived at exact age x
- Division of total number of years lived (T_x) by number of people alive at that exact age (l_x)

$$e_x = T_x / l_x$$

- This index summarizes the level of mortality prevailing in a given population at a particular time

Average age at death ($x + e_x$)

- e_x is the expectation of future life beyond age x
  – It is not an average age at death
- We add x and e_x to obtain the average age at death for cohort members who survive to age x
  – Not all lifetables include $x + e_x$
  – The $x + e_x$ column always go up
- e_x does not always go down
  – It often goes up after the first few years of life, because babies who survive infancy are no longer subject to the high risks of infancy

Probability of surviving (p_x)

- Probability of surviving from birth to age x is designated p_x

$$p_x = l_x / l_0$$

- We can also estimate the probability of surviving from one particular age group to the subsequent age group

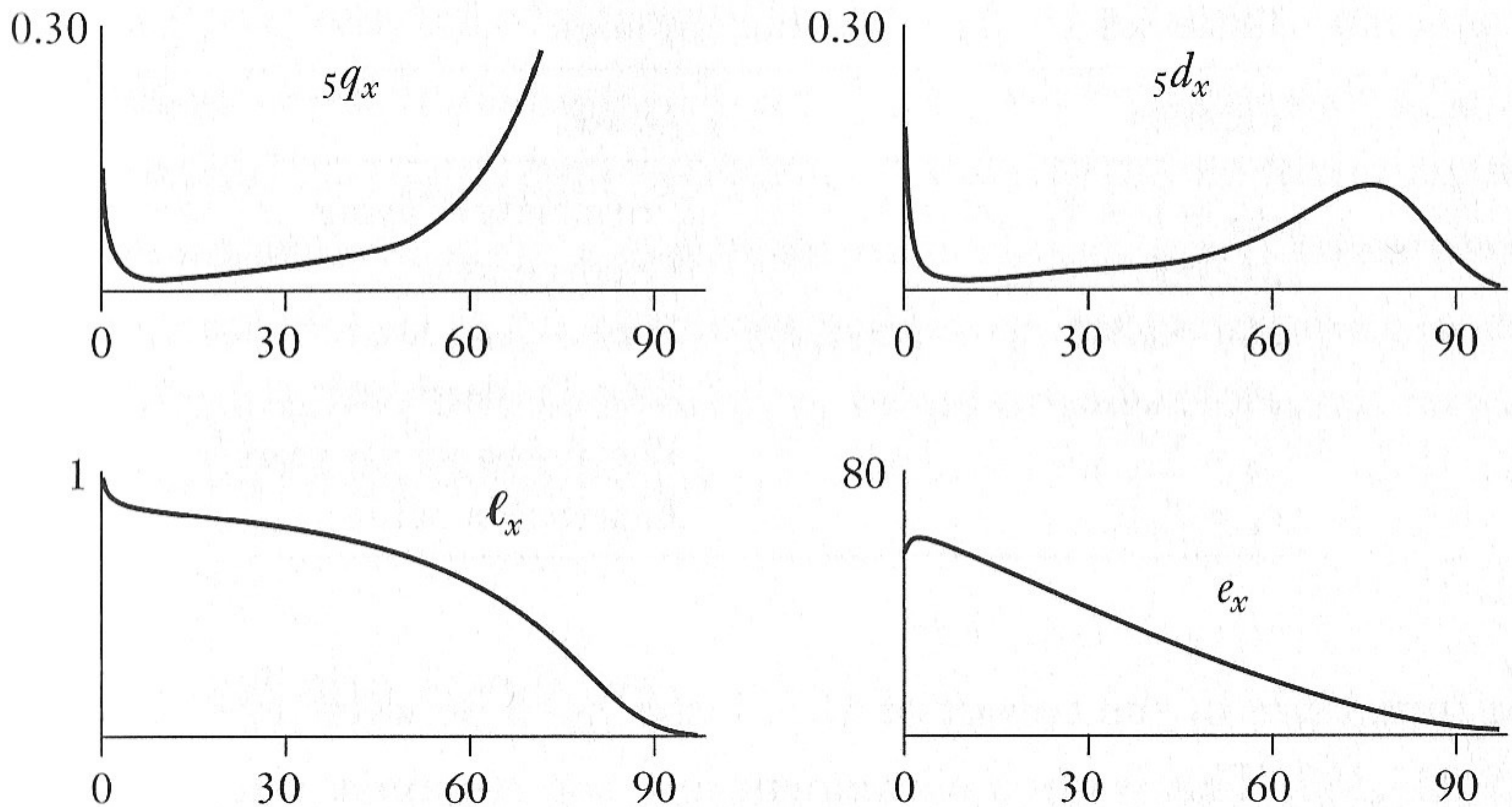
Crude death and birth rates

- Crude death rate (*CDR*) equals total number of deaths (l_0) divided by total population (T_0)

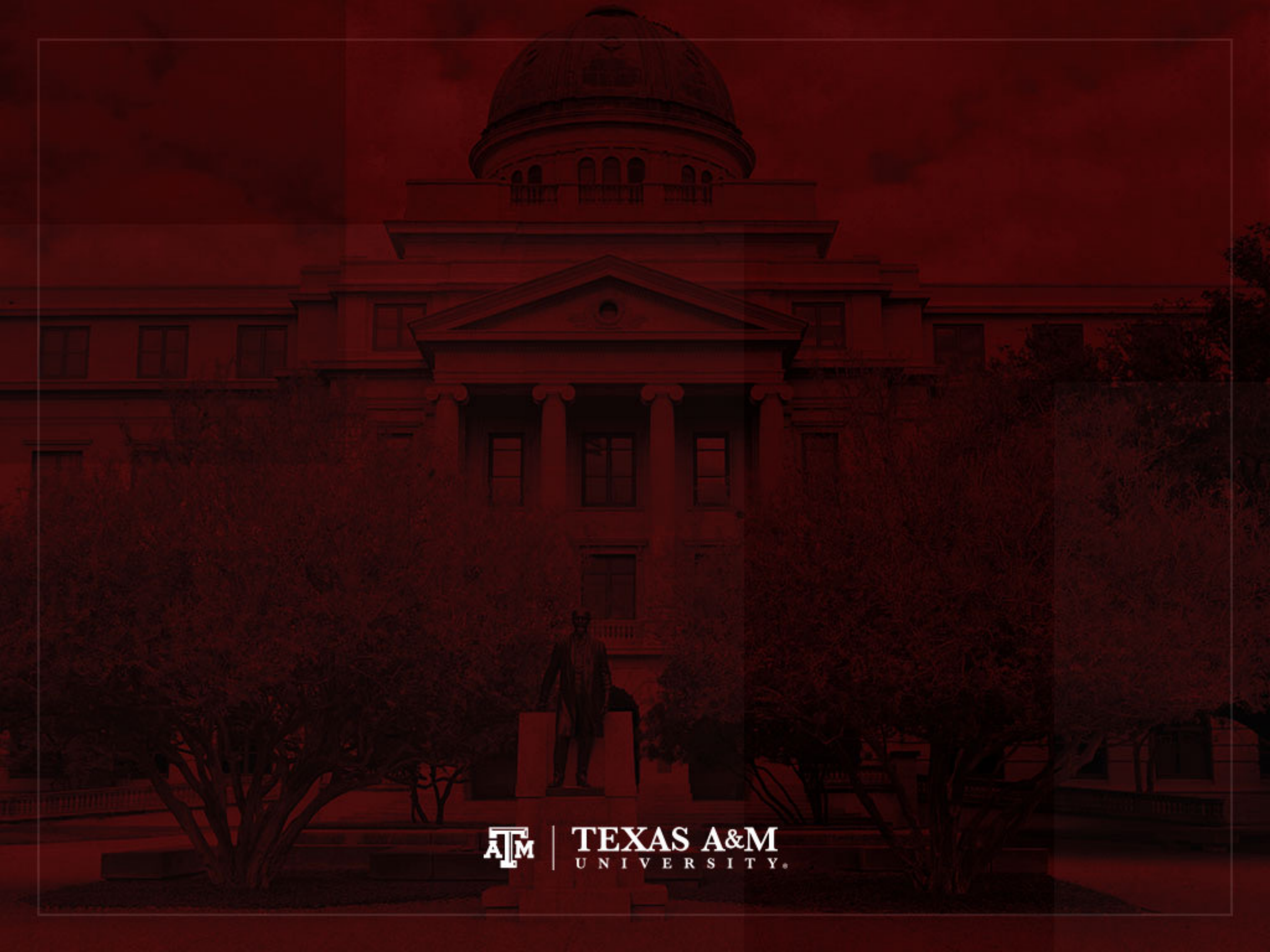
- Crude birth rate (*CBR*) equals total number of births (l_0) divided by total population (T_0)

$$CDR = CBR = l_0 \,/\, T_0 = 1 \,/\, (T_0 \,/\, l_0) = 1 \,/\, e_0$$

Typical shapes of lifetable functions



Source: Wachter, 2014, p. 61.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Stable population theory

- Alfred Lotka (1880–1949) used life tables in the development of his stable population theory

- If a population that is closed to migration experiences constant schedules of age-specific fertility and mortality rates
  - It will develop a constant age distribution
  - It will grow at a constant rate, irrespective of its initial age distribution

Alternative interpretations

- **Synthetic cohort** (history of a hypothetical cohort)
  - Lifetime mortality experience of a single cohort of newborn babies, who are subject to specific age-specific mortality rates
  - Used in public health/mortality studies, calculation of survival rates for estimating population, fertility, net migration...

- **Stationary population**
  - Results from unchanging schedule of age-specific mortality rates and a constant annual number of births/deaths (radix)
  - Used in the comparative measurement of mortality and in studies of population structure

Same interpretation

- ***x* to *x+n***
  - Period of life between two exact ages
  - For instance, 20–25 means the 5-year interval between the 20th and 25th birthdays

- $_{n}q_{x}$
  - Proportion of persons in the cohort alive at the beginning of an indicated age interval (*x*) who will die before reaching the end of that age interval (*x*+*n*)
  - Probability that a person at his/her *x*th birthday will die before reaching his/her *x*+*n*th birthday

- **e_x (life expectancy)**
  - Average remaining lifetime (in years) for a person who survives to the beginning of the indicated age interval

l_x

- **Synthetic cohort**
  - Number of persons living at the beginning of the indicated age interval (*x*) out of the total number of births assumed as the radix of the table

- **Stationary population**
  - Number of persons who reach the beginning of the age interval each year

$_nd_x$

- **Synthetic cohort**
  - Number of persons who would die within the indicated age interval (*x* to *x*+*n*) out of the total number of births assumed in the table

- **Stationary population**
  - Number of persons that die each year within the indicated age interval

${}_{n}L_{x}$

- **Synthetic cohort**
  – Number of person-years that would be lived within the indicated age interval (*x* to *x*+*n*) by the assumed birth cohort (e.g., l_0 = 100,000)

- **Stationary population**
  – Number of persons in the population who at any moment are living within the indicated age interval

T_x

- **Synthetic cohort**
  - Total number of person-years that would be lived after the beginning of the indicated age interval by the assumed birth cohort (e.g., l_0 = 100,000)

- **Stationary population**
  - Number of persons in the population who at any moment are living within the indicated age interval and all higher age intervals

Female and male life tables, U.S., 2007

ABRIDGED LIFE TABLE FOR THE FEMALE POPULATION OF THE UNITED STATES: 2007

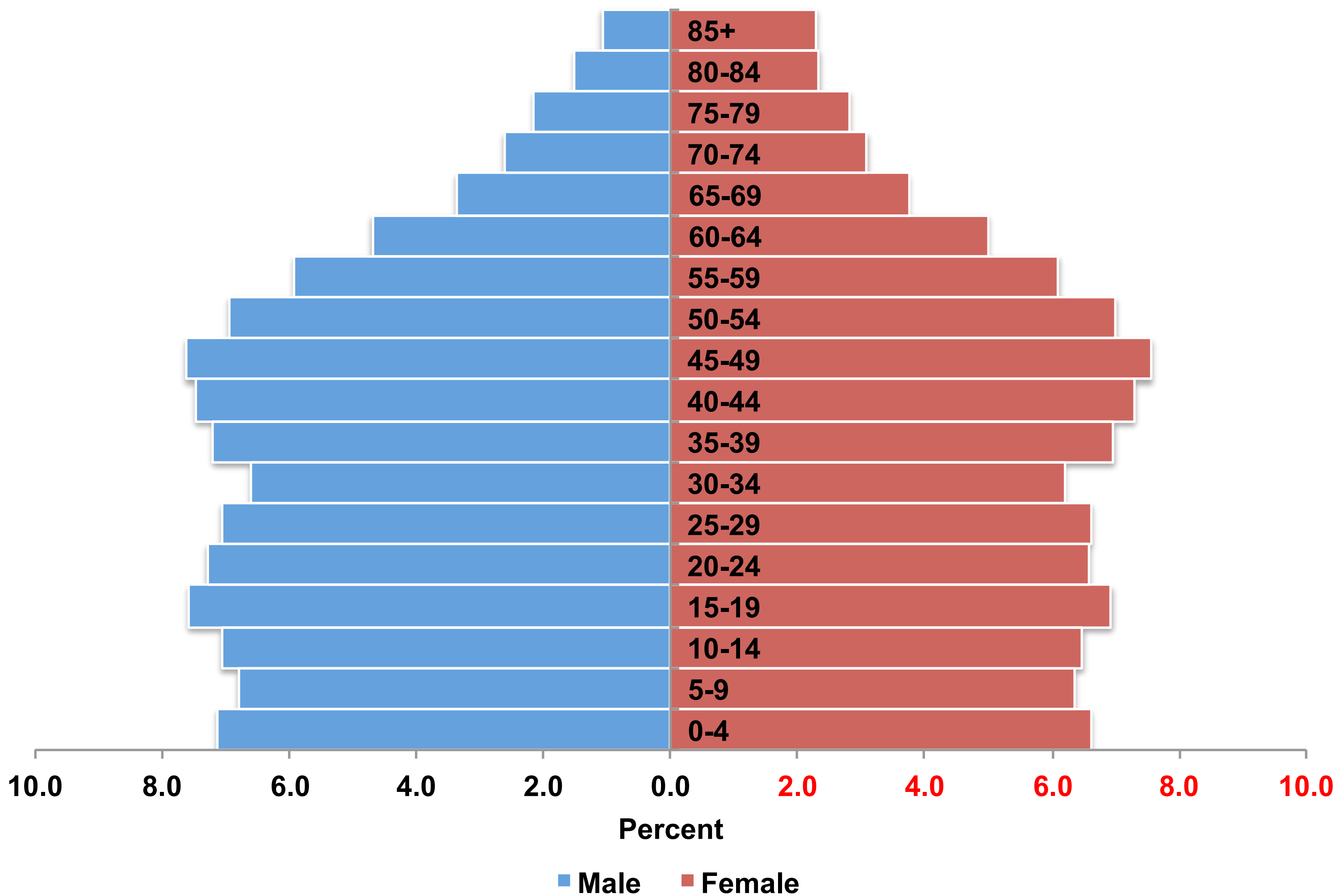
						Of 100,000 born alive		Stationary population		
Age group	Width n	Population nPx	Deaths nDx	Age-specific death rates nMx	Proportion dying nqx	# living at beginning of interval lx	# dying during interval ndx	In the age interval nLx	In this and following ages Tx	Average remaining lifetime ex
0	1	1,998,761	12,845	0.0064	0.0064	100,000	641	99,684	8,103,588	81.0
1-4	4	8,109,371	2,069	0.0003	0.0010	99,359	101	397,248	8,003,904	80.6
5-9	5	9,720,587	1,192	0.0001	0.0006	99,258	61	496,150	7,606,656	76.6
10-14	5	9,918,543	1,370	0.0001	0.0007	99,197	68	495,828	7,110,506	71.7
15-19	5	10,617,178	3,741	0.0004	0.0018	99,129	175	495,242	6,614,678	66.7
20-24	5	10,073,754	4,925	0.0005	0.0024	98,954	242	494,215	6,119,436	61.8
25-29	5	10,122,681	5,824	0.0006	0.0029	98,713	284	492,910	5,625,222	57.0
30-34	5	9,469,789	6,956	0.0007	0.0037	98,429	361	491,314	5,132,312	52.1
35-39	5	10,666,827	11,126	0.0010	0.0052	98,068	510	489,165	4,640,998	47.3
40-44	5	11,155,652	18,375	0.0016	0.0082	97,558	800	485,944	4,151,834	42.6
45-49	5	11,572,428	29,834	0.0026	0.0128	96,757	1,240	480,926	3,665,890	37.9
50-54	5	10,709,011	40,396	0.0038	0.0187	95,518	1,786	473,463	3,184,963	33.3
55-59	5	9,339,919	50,868	0.0054	0.0269	93,732	2,521	462,827	2,711,501	28.9
60-64	5	7,636,068	62,624	0.0082	0.0402	91,211	3,670	447,543	2,248,674	24.7
65-69	5	5,725,079	74,499	0.0130	0.0631	87,541	5,528	424,827	1,801,131	20.6
70-74	5	4,738,379	96,395	0.0203	0.0971	82,012	7,962	391,395	1,376,304	16.8
75-79	5	4,314,403	139,360	0.0323	0.1500	74,050	11,109	343,929	984,910	13.3
80-84	5	3,582,388	192,519	0.0537	0.2378	62,941	14,970	278,566	640,981	10.2
85+	---	3,511,395	464,781	0.1324	1.0000	47,971	47,971	362,415	362,415	7.6

ABRIDGED LIFE TABLE FOR THE MALE POPULATION OF THE UNITED STATES: 2007

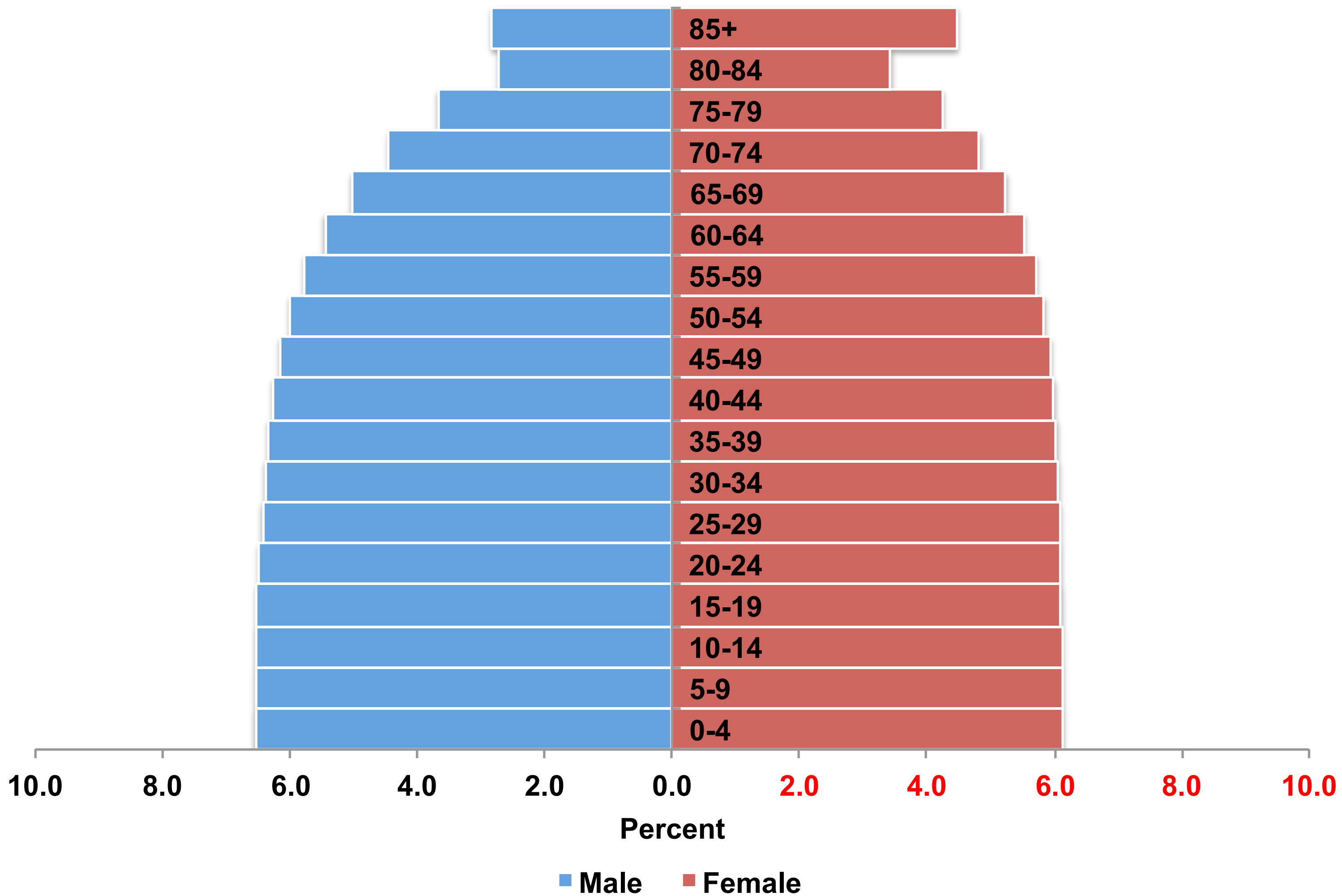
						Of 100,000 born alive		Stationary population		
Age group	Width n	Population nPx	Deaths nDx	Age-specific death rates nMx	Proportion dying nqx	# living at beginning of interval lx	# dying during interval ndx	In the age interval nLx	In this and following ages Tx	Average remaining lifetime ex
0	1	2,079,846	16,293	0.0078	0.0078	100,000	780	99,615	7,582,342	75.8
1-4	4	8,507,893	2,634	0.0003	0.0012	99,220	123	396,648	7,482,726	75.4
5-9	5	10,095,353	1,519	0.0002	0.0008	99,097	75	495,313	7,086,078	71.5
10-14	5	10,484,813	2,066	0.0002	0.0010	99,022	98	494,887	6,590,765	66.6
15-19	5	11,252,863	9,558	0.0008	0.0042	98,925	419	493,658	6,095,878	61.6
20-24	5	10,828,130	15,758	0.0015	0.0073	98,505	714	490,881	5,602,220	56.9
25-29	5	10,489,470	15,107	0.0014	0.0072	97,791	702	487,338	5,111,340	52.3
30-34	5	9,802,132	14,685	0.0015	0.0075	97,089	725	483,776	4,624,002	47.6
35-39	5	10,684,227	19,755	0.0018	0.0092	96,364	887	479,777	4,140,226	43.0
40-44	5	11,085,591	30,350	0.0027	0.0136	95,477	1,299	474,390	3,660,450	38.3
45-49	5	11,318,167	47,904	0.0042	0.0210	94,179	1,974	466,332	3,186,060	33.8
50-54	5	10,313,298	66,552	0.0065	0.0318	92,205	2,931	454,237	2,719,728	29.5
55-59	5	8,790,943	81,590	0.0093	0.0454	89,274	4,055	436,954	2,265,491	25.4
60-64	5	6,979,426	92,028	0.0132	0.0640	85,218	5,451	413,393	1,828,537	21.5
65-69	5	5,003,042	100,492	0.0201	0.0959	79,767	7,651	380,904	1,415,144	17.7
70-74	5	3,889,104	117,852	0.0303	0.1414	72,116	10,196	336,467	1,034,240	14.3
75-79	5	3,192,676	149,669	0.0469	0.2107	61,920	13,046	278,295	697,773	11.3
80-84	5	2,235,826	171,134	0.0765	0.3220	48,874	15,739	205,629	419,478	8.6
85+	---	1,606,146	248,866	0.1549	1.0000	33,135	33,135	213,850	213,850	6.5

Source: Formulas from Kintner (2003); Population data from 2007 ACS; Death data from CDC ((http://www.cdc.gov/nchs/data/dvs/mortfinal2007_worktable310.pdf).

Population, U.S., 2007



${}_nL_x$ from previous life tables, U.S., 2007

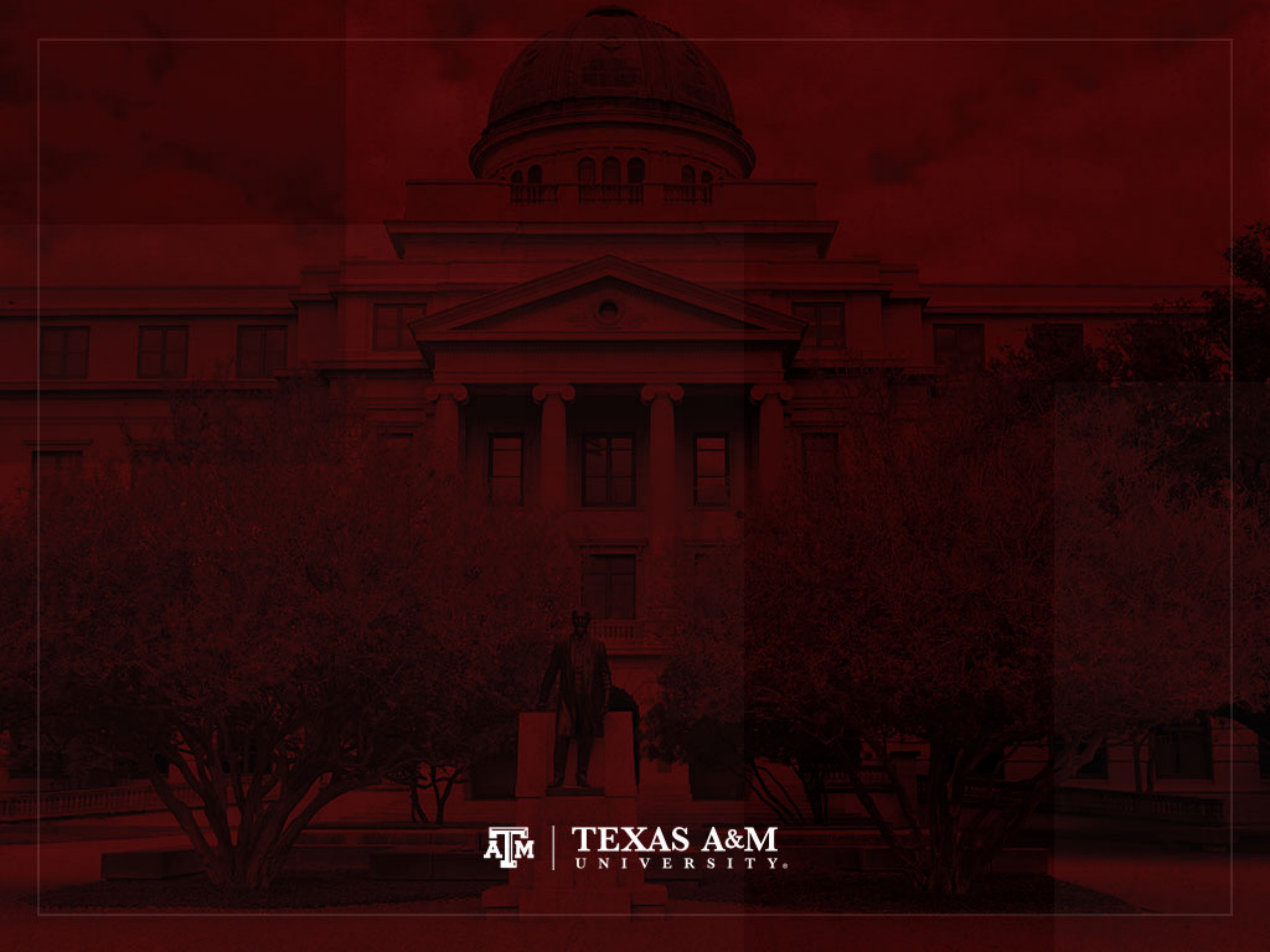


Problems with life tables

- We saw life tables based on complete empirical data
- We might experience some issues
  - Have partial information to build our life table
  - Have data for only some age groups
  - Information for some ages may be more reliable than for other ages
  - Have ideas about mortality level, but not a full life table to make projections
- We can use model life tables to solve these issues

Model life tables

- A life table constructed from mathematical formulas is called a model life table
  - Use mathematical formulas to fill in missing parts
  - Have a whole life table from partial information
  - Identify suspicious and poor quality data with model expectations
  - Supply standard assumptions for projections
  - Find regularities for the invention of indirect measures
  - Reconstruct rates from historical counts of births and deaths (inverse projection)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Another example of a lifetable

Life Table for the Total Population, United States, 2010

(1) Age range	(2) $_nq_x$	(3) l_x	(4) $_nd_x$	(5) $_nL_x$	(6) T_x	(7) e_x
<1	0.006123	100,000	612	99,465	7,866,027	78.7
1–4	0.001071	99,388	106	397,294	7,766,561	78.1
5–9	0.000573	99,281	57	496,250	7,369,267	74.2
10–14	0.000708	99,224	70	495,989	6,873,017	69.3
15–19	0.002463	99,154	244	495,240	6,377,028	64.3
20–24	0.004317	98,910	427	493,529	5,881,789	59.5
25–29	0.004791	98,483	472	491,249	5,388,260	54.7
30–34	0.005497	98,011	539	488,744	4,897,011	50.0
35–39	0.006913	97,472	674	485,753	4,408,267	45.2
40–44	0.009979	96,798	966	481,758	3,922,514	40.5
45–49	0.016044	95,833	1,538	475,584	3,440,756	35.9
50–54	0.024343	94,295	2,295	466,066	2,965,173	31.4
55–59	0.035106	92,000	3,230	452,347	2,499,106	27.2
60–64	0.049847	88,770	4,425	433,348	2,046,759	23.1
65–69	0.074406	84,345	6,276	406,912	1,613,411	19.1
70–74	0.112315	78,069	8,768	369,612	1,206,499	15.5
75–79	0.174782	69,301	12,113	317,694	836,886	12.1
80–84	0.274384	57,188	15,692	248,038	519,193	9.1
85–89	0.430820	41,497	17,878	162,723	271,155	6.5
90–94	0.615282	23,619	14,532	79,720	108,432	4.6
95–99	0.783397	9,087	7,119	24,670	29,212	3.2
100+	1.00000	1,968	1,968	4,542	4,542	2.3

Source: Arias (2014: 62).

Source: Poston, Bouvier 2017.

Basic life table columns

1. Age intervals of each group
2. ${}_nq_x$: probability of dying between age *x* and age *x*+*n*
3. l_x: number of survivors at each age x
4. ${}_nd_x$: number of deaths between age *x* and age *x*+*n*
5. ${}_nL_x$: number of years lived by all persons who enter the age interval while in the age interval
6. T_x: number of years lived by the population in the age interval and in all subsequent intervals
7. e_x: remaining life expectancy at each age

1. Age intervals of each group

- Age groups refer to the range of years between two birthdays

- The age group 5–9 refers to the five-year interval between the fifth and the tenth birthdays

2. Probabilities of dying ($_nq_x$)

- The most basic column of the life time shows probabilities of dying for each age group ($_nq_x$)
  - These are probabilities that persons alive at the beginning of an age interval will die during the interval, before they reach the start of the next age interval

$$_nq_x = {}_nd_x / l_x$$

  - For last age group, $_nq_x$=1.0 because everybody dies

Rates and probabilities

- Difference between mortality rates (${}_nM_x$) and mortality probabilities (${}_nq_x$) is the denominator

- ${}_nM_x$: denominator is midyear population

- ${}_nq_x$: denominator is population alive at the beginning of the age interval

3. Number of survivors (l_x)

- Number of people alive at the beginning of the age interval (l_x)
  - Known as "the little l column"
- It is calculated by subtracting the number of people dying (${}_nd_x$) from the l_x value in the age interval immediately preceding the one being calculated
- Example of U.S. life table in 2010
  - Of the 99,224 people alive at the beginning of the age interval 10–14 (l_{10})
  - 70 of them die during the age interval (${}_5d_{10}$)
  - Thus, the value of l_{15} is 99,154 = 99,224 – 70

4. Number of deaths ($_nd_x$)

- Number of people who die during a particular age interval ($_nd_x$)

$$_nd_x = l_x * {_nq_x}$$

- For the number of people who die during the age interval of 40–44

$$_5d_{40} = {_5q_{40}} * l_{40}$$

$$_5d_{40} = 0.009979 * 96{,}798$$

$$_5d_{40} = 966$$

5. Years lived in age interval ($_nL_x$)

- Total number of years lived by all persons who enter that age interval while in the age interval ($_nL_x$)
  - Known as "the big L column"
- Example of U.S. life table in 2010
  - 98,011 persons are alive at the beginning of age interval 30–34 (l_{30})
  - If none of them died during the age interval, they would have lived 490,055 years (98,011 times 5)
  - But 539 of them died ($_5d_{30}$)

Different formulas for ${}_nL_x$

- Demographers assume that deaths are roughly distributed during the five-year period for many of the age intervals

- This assumption does not apply to the first few age intervals
  - There are several formulas to produce the nLx value for the first few age groups

- At the other age extreme, 100+ in the life table, another formula is used

6. Years lived in current and subsequent age intervals (T_x)

- Total number of years lived by the population in the age interval and in all subsequent age intervals (T_x)
  - We sum ${}_nL_x$ from the oldest age backwards to get T_x

$$T_x = \sum_{i=x}^{w} L_i$$

  - L_i: entry i in the ${}_nL_x$ column
  - $\sum_{i=x}^{w}$: sum of the ${}_nL_x$ column starting at entry x through the last ${}_nL_x$ entry (w)

Example of T_x

- Example of U.S. life table in 2010

$$T_{95} = {}_5L_{95} + {}_5L_{100}$$

$$T_{95} = 24{,}670 + 4{,}542$$

$$T_{95} = 29{,}212$$

7. Remaining life expectancy (e_x)

- Average number of years of life remaining at the beginning of the age interval (e_x)
- It provides life expectancy at any age

$$e_x = T_x / l_x$$

- Example of U.S. life table in 2010

$$e_0 = T_0 / l_0 = 7{,}866{,}027 / 100{,}000 = 78.7$$

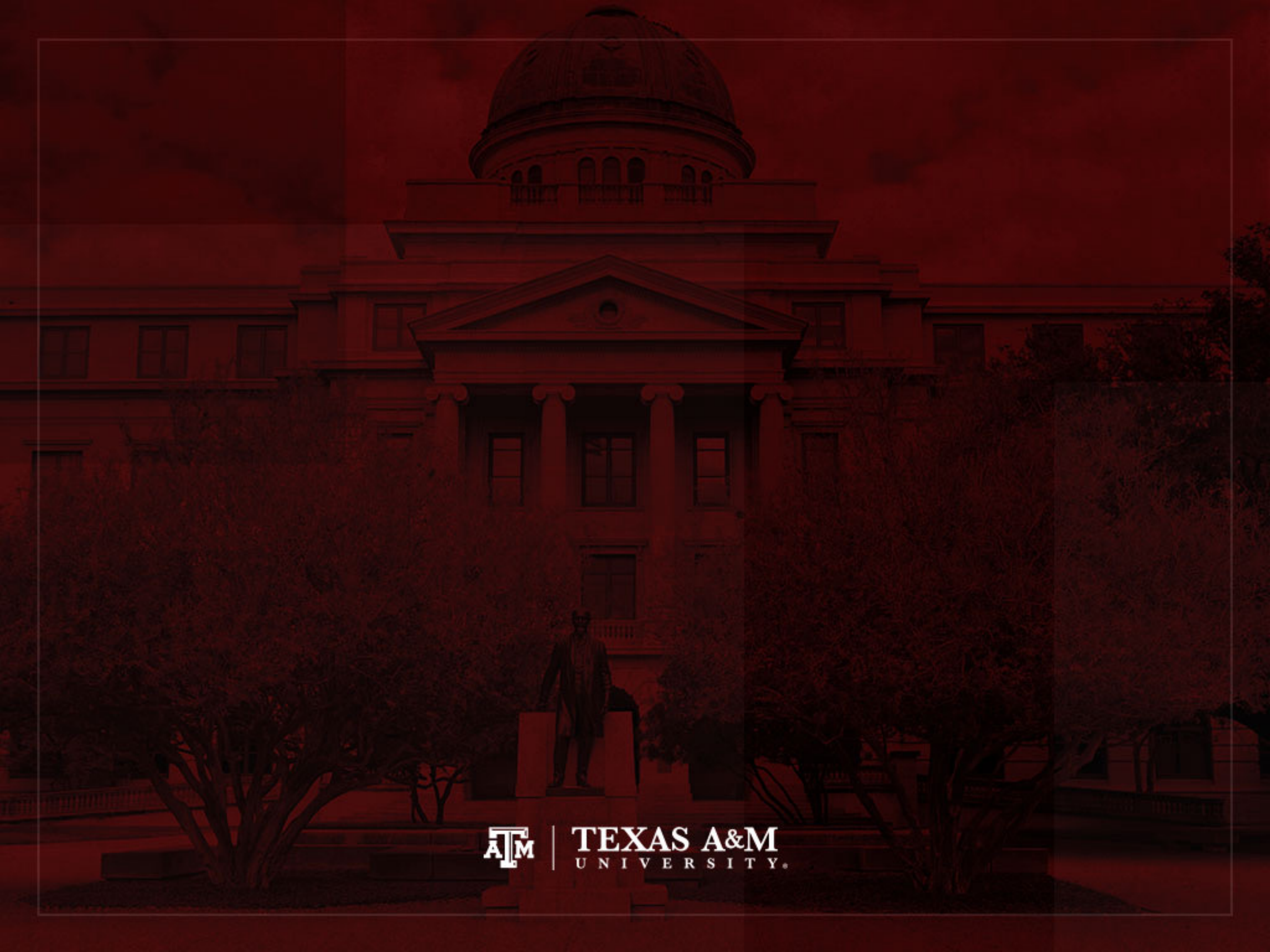
$$e_{25} = T_{25} / l_{25} = 5{,}388{,}260 / 98{,}483 = 54.7$$

  – Persons aged 25–29 can expect to live an additional 54.7 years

References

Poston DL, Bouvier LF. 2017. Population and Society: An Introduction to Demography. New York: Cambridge University Press. 2nd edition. Chapter 7 (pp. 163–214).

Wachter KW. 2014. Essential Demographic Methods. Cambridge: Harvard University Press. Chapter 7 (pp. 153–173).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