Lecture 10: Stable age structures

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Demographic Methods (SOCI 633/320)

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Outline

- Age pyramids
- Stable and stationary populations
- Births * Life expectancy at birth

Age pyramids

- There is theory to deal with age structure
  - It accounts for the relative numbers of young and old men and women in a population
- Basic idea is to obtain formulas for how a population will be theoretically distributed by age
  - If population has been closed to migration
  - If its birth and death rates have been unchanging for a long time

Actual ≠ Theoretical

- The actual age distribution of the population naturally differs from the theoretical age distribution

- Deviations are explained by
  - Events of migration
  - Changes in rates in the prior history of the population

General and special features

- The age distribution of each population has
- General features
  - Which it shares with populations with the same vital rates
- Special features
  - Which are derived from its own particular history

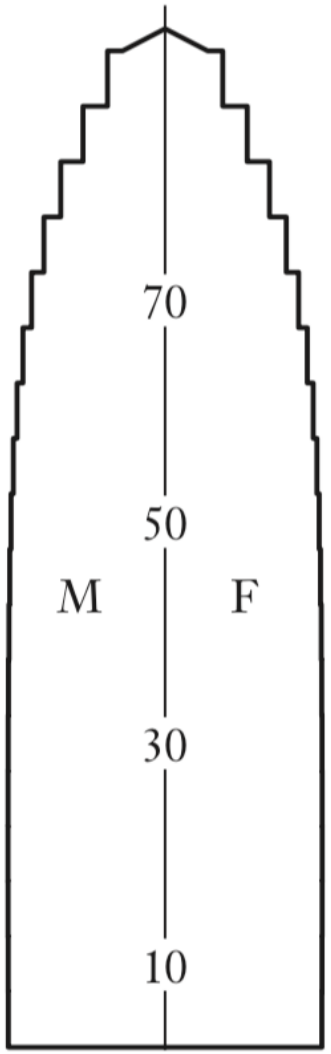
Graphical diagrams

- Age pyramid, age distribution, age structure
  - They represent the distribution of the population by age and sex
  - They are made up of a pair of bar graphs, one for men and one for women, turned on their sides and joined
- The vertical axis corresponds to age
  - The young are toward the bottom, the elderly toward the top
  - The open-ended age group at the very top is sometimes drawn with a triangle instead of bars
- For each age group
  - The bar coming off the axis to the right represents the number of women in that age group
  - The bar to the left the number of men

Idealized age pyramids

- Examples of idealized stable pyramids that occur in closed populations with unchanging vital rates
- Tall and slender
  - It is a case for a long-lived population with near zero growth
- Quite pyramidal in shape
  - It is a case for a population with heavy mortality and rapid growth

Tall and slender



Quite pyramidal in shape

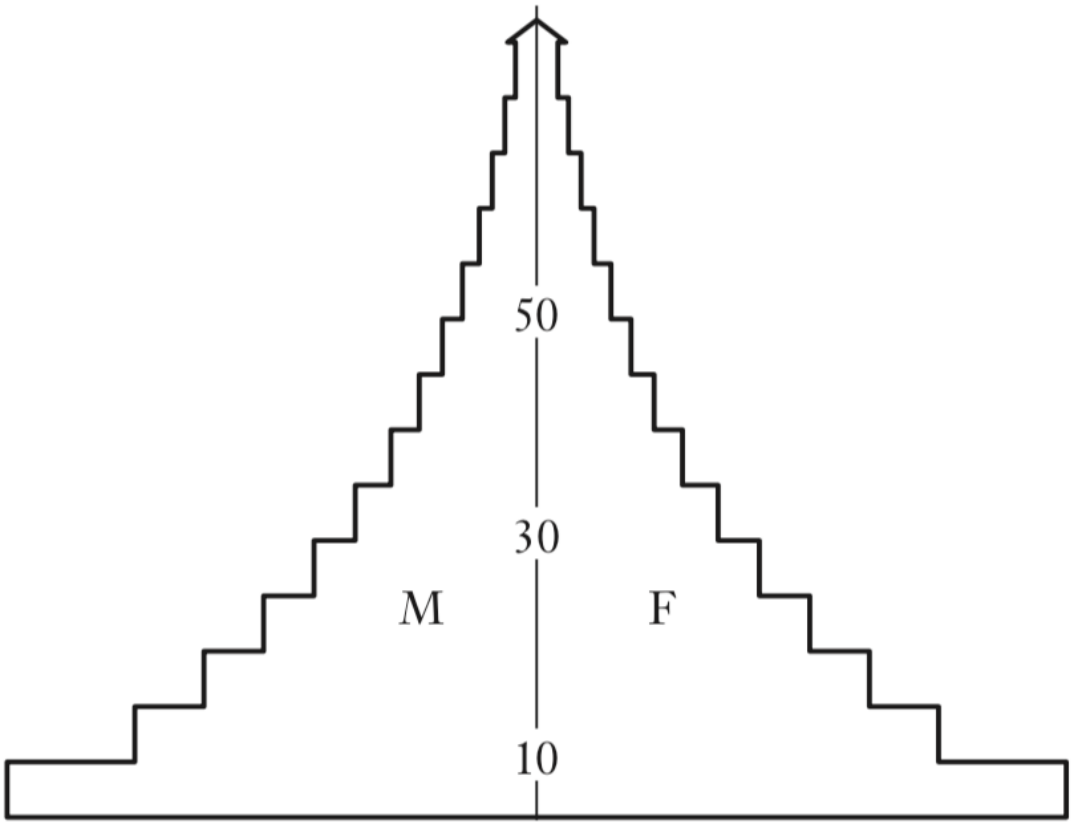
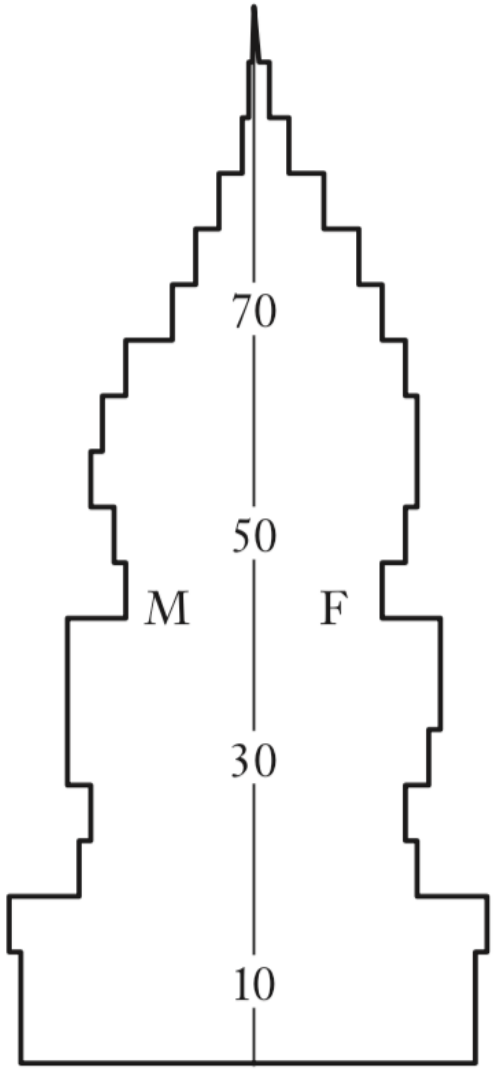


Figure 10.1 Examples of stable age pyramids

Observed age pyramids

- Examples of observed age pyramids
- France in 1960
  - It shares overall shape with the low-growth sable case
  - But notches among 20 and 40 years of age due to low births during World Wars I and II
- Mauritius in 1963
  - It shares overall shape with high-growth stable case
  - But indentations at working ages hint at changes around 1945 from increasing growth
    - Gains against infant mortality

France, 1960



Mauritius, 1963

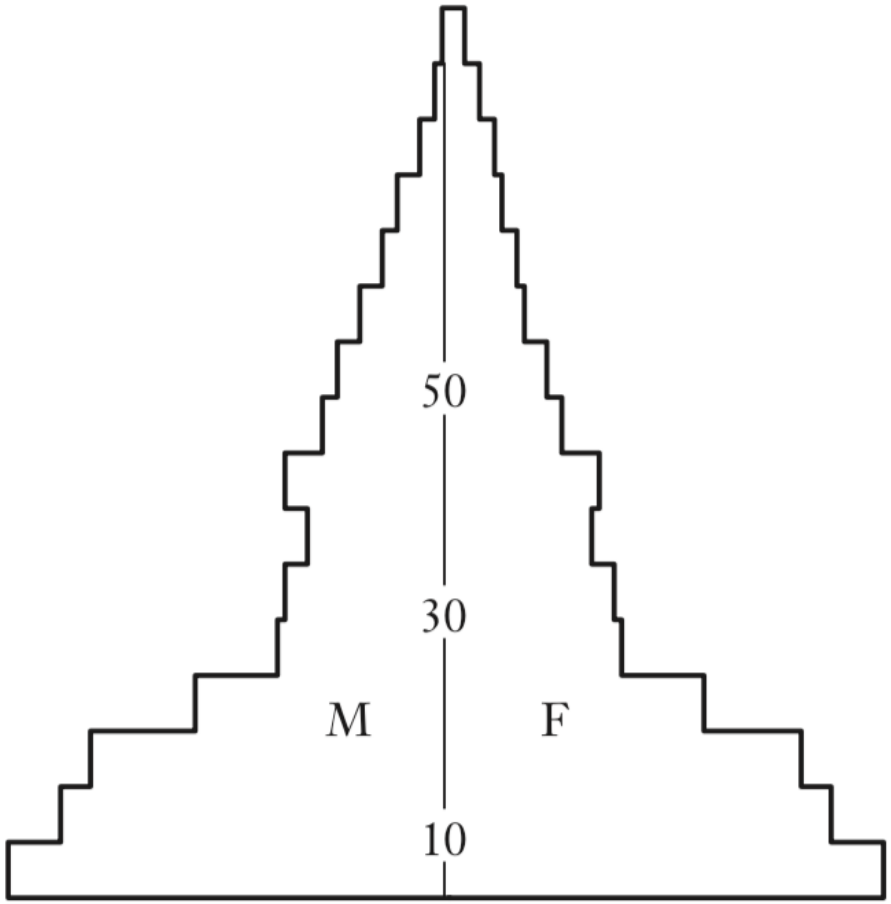
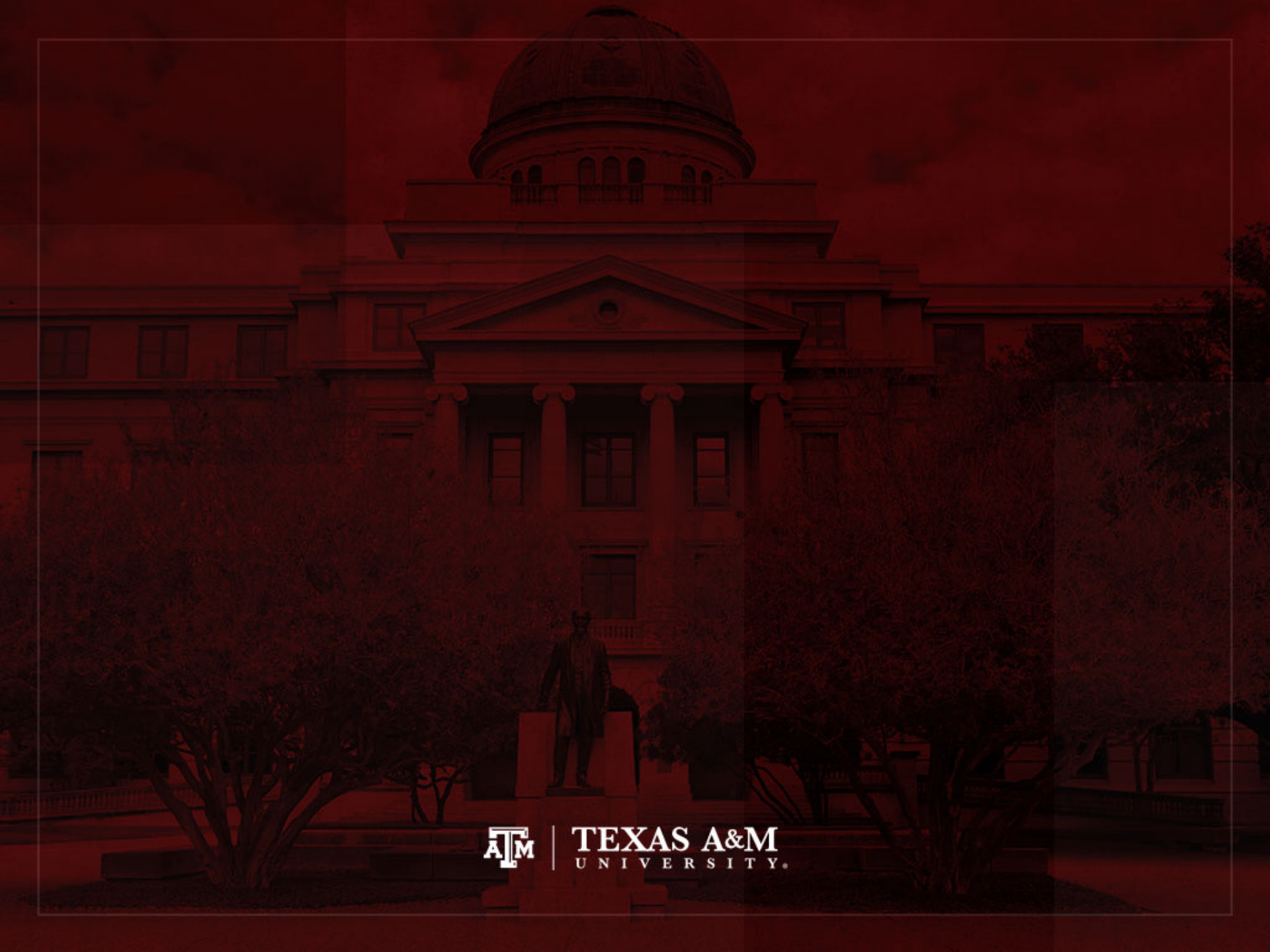


Figure 10.2 Examples of observed age pyramids

Idealized ≠ Observed

- Stable theory captures general features well
- Observable differences from stable shapes due to each nation's own history
  - Changing rates
  - Movements across borders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Stable population

- Stable population is any population produced by age-specific rates of fertility and mortality constant over a long period of time
  - Its age pyramid is determined uniquely by its lifetable and its long-term growth rate
  - Proportions in each age group in a stable population do not change over time
  - Numbers in each age group may change over time
    - Population may be growing or declining in size
    - It depends on what the growth rate happens to be

Stable population theory

- Stable population theory is the mathematical analysis of stable age pyramids
- It is a theory that goes back to the work of Leonhard Euler in 1760
- It was extensively developed by
  - Alfred Lotka in the early 1900s
  - Nathan Keyfitz and Ansley Coale in the last half-century

Stable population

- Alfred Lotka (1880–1949) used life tables in the development of his stable population theory

- If a population that is closed to migration experiences constant schedules of age-specific fertility and mortality rates
  - It will develop a constant age distribution
  - It will grow at a constant rate, irrespective of its initial age distribution

Stable population theory

- It considers a closed population
  - A population in which migration does not occur
- If a population experiences constant age-specific fertility and mortality rates for a long time
  - It develops a constant age distribution and grows at a constant rate, irrespective of its initial age distribution
  - Demographers sometimes indicate that stable populations forget their past
- Age distribution of a stable population depends on
  - The underlying age-specific mortality rates
  - The rate of growth

Stationary population

- Stationary population is a stable population in which the birth rate equals the death rate
- This results in no change in the size of the population
- It is also considered in the absence of migration

Stationary population identity

- Cohort members born each year

$$B = \text{Population} * \text{CBR} = Kb$$

- Cohort members dying each year

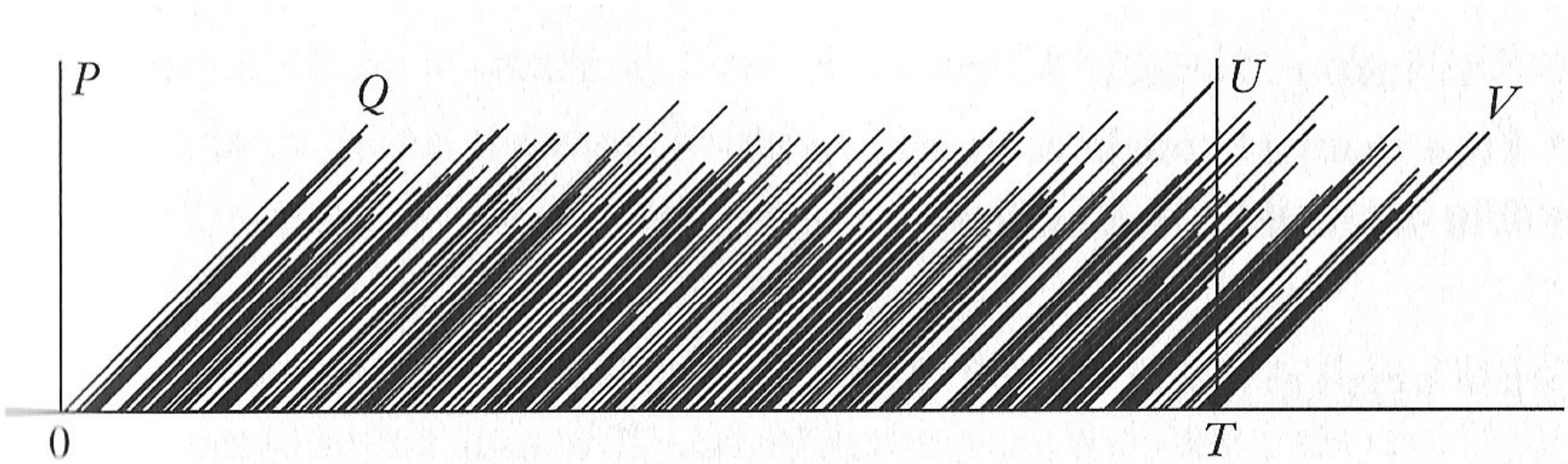
$$D = \text{Population} * \text{CDR} = Kd$$

- Years lived on average in each lifetime: e_0
- Number of cohorts: T
- Count of cohort person-years: $B\ e_0\ T = K\ b\ e_0\ T$
- Count of period person-years: KT
- Stationary population identity (R=0)
  - Period count equals cohort count

$$KT = K\ b\ e_0\ T$$

$$1 = b\ e_0$$

Lexis diagram for a stationary population



Source: Wachter 2014, p. 45.

Stable ≠ Stationary

- Stable population
  - Rates stay the same
  - Population size may change
- Stationary population
  - Rates and population size remain the same
  - Growth rate is zero
  - It is a special case of a stable population
  - It satisfies the extra condition of having zero population growth (ZPG)

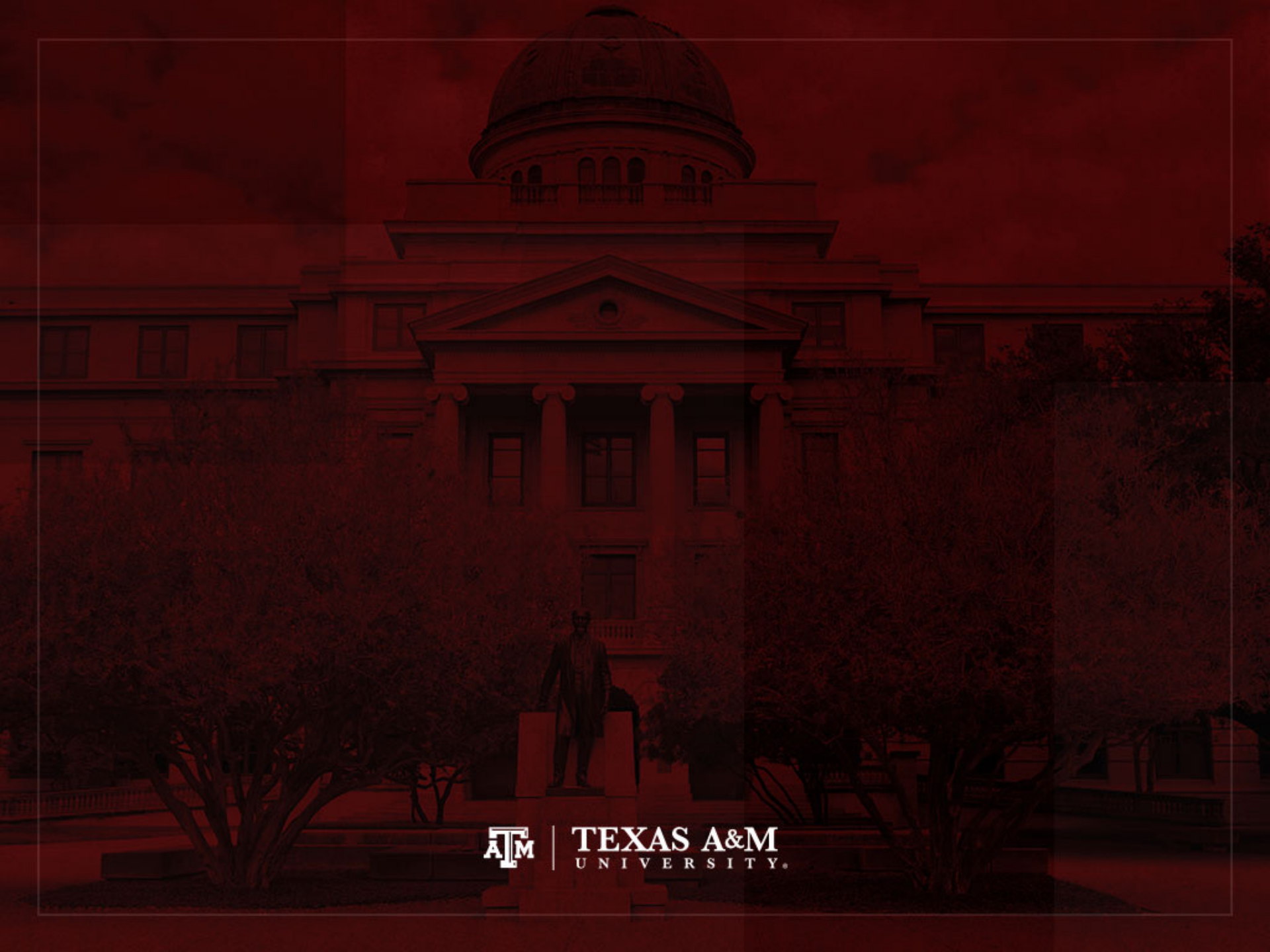
Stable and stationary populations

- Stable population
  - Demographic rates are unchanging
  - Birth and death rates are constant
  - Population size might be growing, constant or declining

- Stationary population
  - Numbers are unchanging
  - Numbers of births and deaths are constant
  - Number of births equals number of deaths (B=D)
  - Total population is the same from year to year

Little more on stationary

- We can imagine complicated cases in which age-specific rates are changing in ways that cancel each other out
  - So that population size remains the same
  - Sometimes such a population is called stationary
- But we reserve the word stationary for cases with
  - Unchanging rates
  - Unchanging size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Births * Life expectancy at birth

Elon Musk @elonmusk · Jan 18

UN projections are utter nonsense. Just multiply last year's births by life expectancy. Given downward trend in birth rate, that is best case unless reversed.

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Source: Ilya Kashnitsky, Assistant Professor at the University of Southern Denmark (@ikashnitsky).

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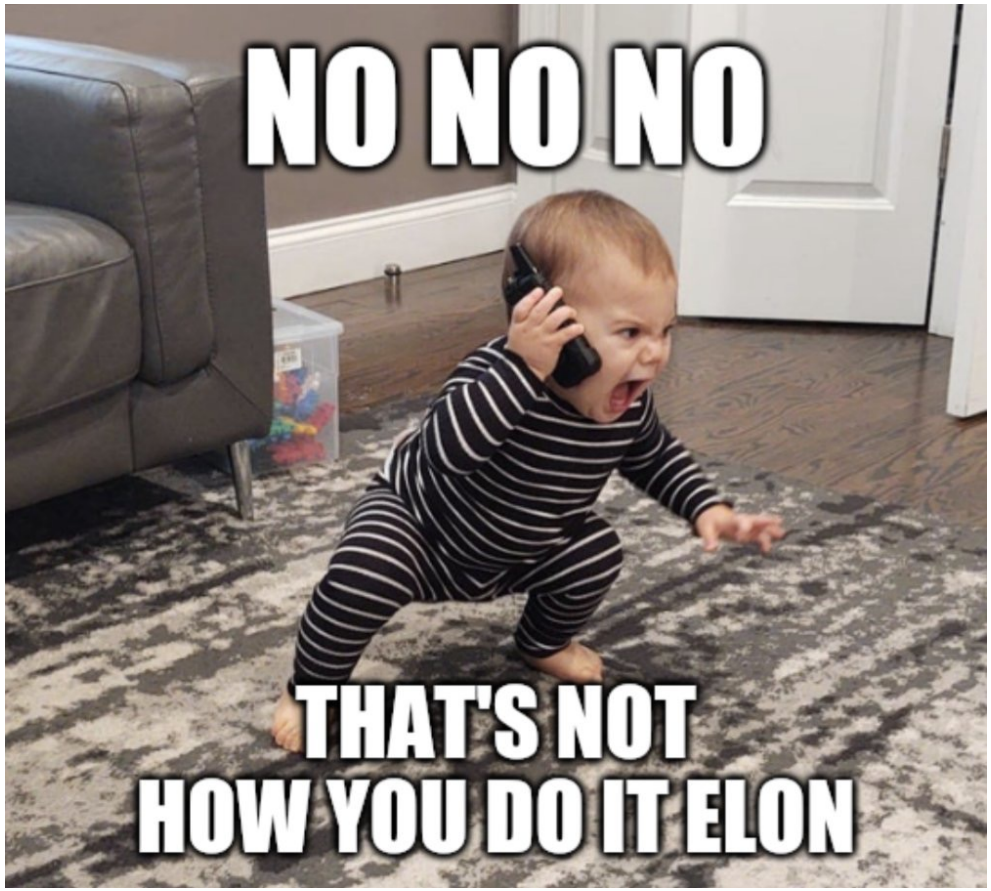

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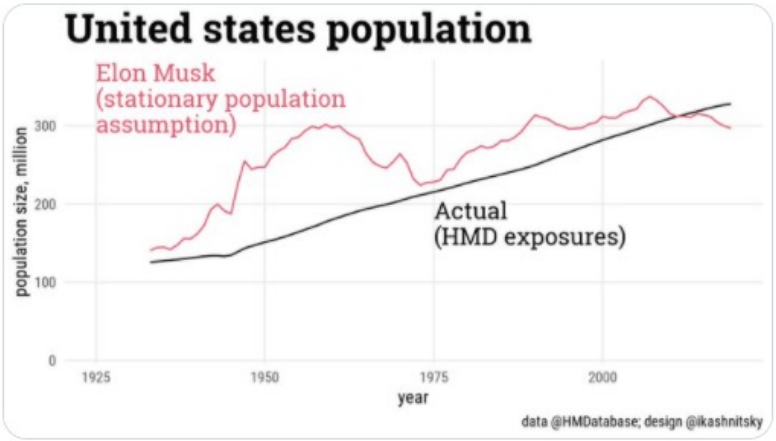
Ilya Kashnitsky @ikashnitsky · Jan 19

Okay. Maybe this is a bit overkill but let's illustrate how this projection method of @elonmusk (in fact, a stationary population assumption) worked with past data using @HMDatabase

Here is just the US 🇺🇸



Ilya Kashnitsky
@ikashnitsky

If there is just one take-home message from this thread let it be

✅ Life expectancy is a snapshot of the *current* mortality

❌ It's not a projection/forecast of the actual experience of the newborn cohorts

11/

9:14 AM · Mar 5, 2021 · Typefully

Jonas Schöley
@jschoeley

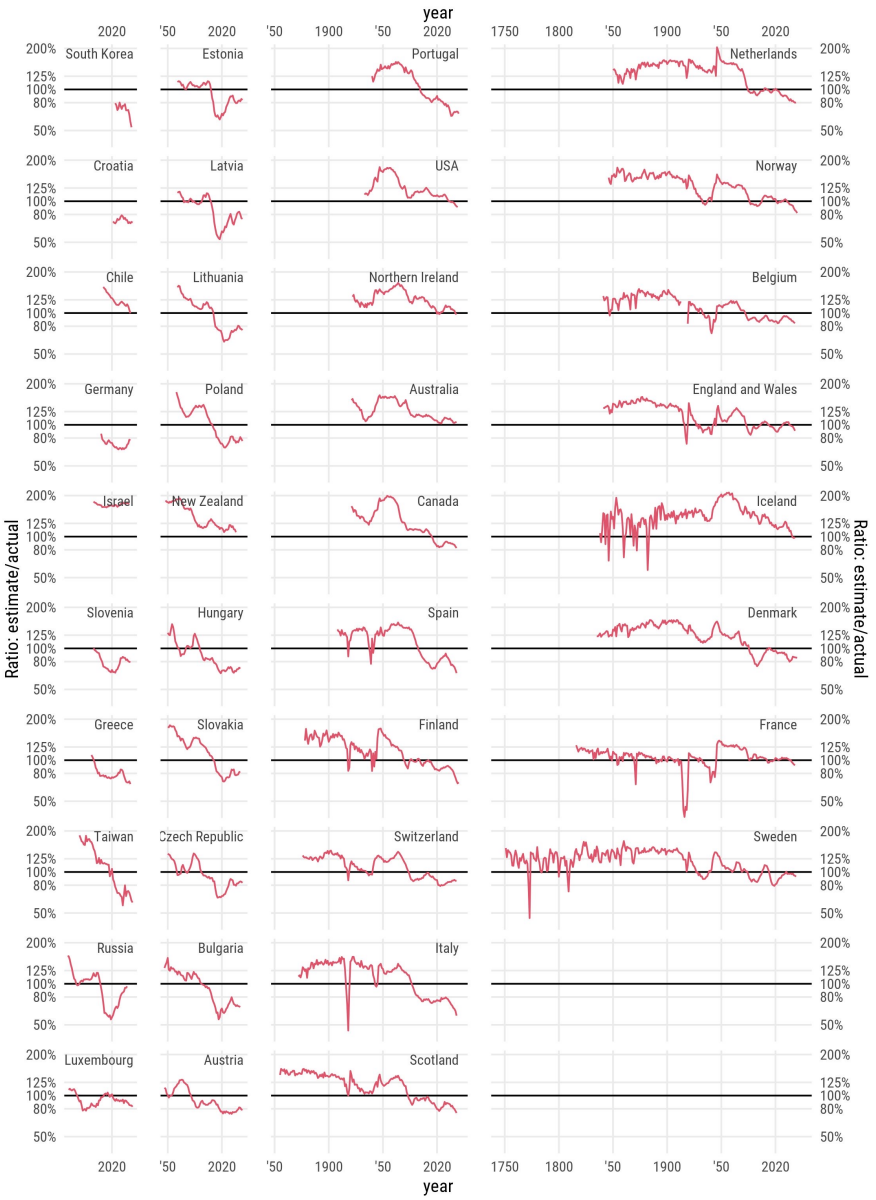
Replying to @ikashnitsky @elonmusk and @HMDatabase

It's true IF we look at world population (0 migration) AND it's stationary (birth rates = death rates since generations) AND mortality remains constant whereas fertility is allowed to decline. Then we reached peak population which can be estimated by B*e0. Bullshit assumptions.

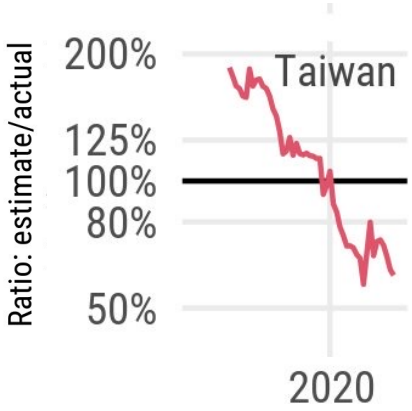
5:41 AM · Jan 19, 2022 · Twitter Web App

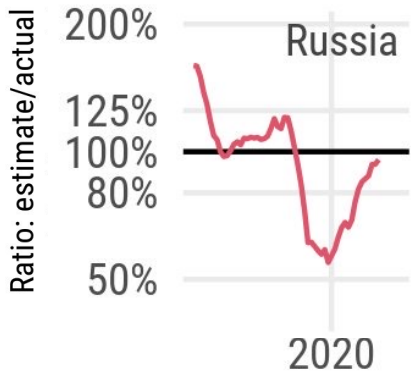
Source: Ilya Kashnitsky, Assistant Professor at the University of Southern Denmark (@ikashnitsky).

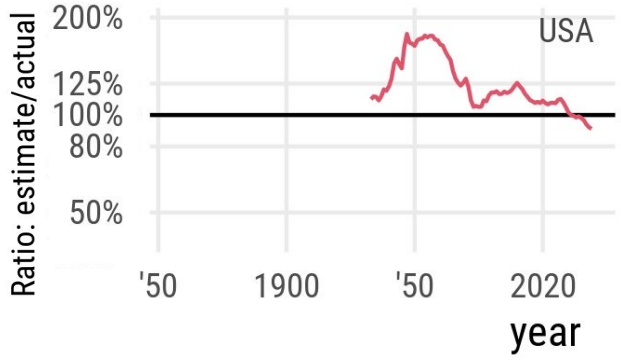
Population size under the stationary assumption relative to the actual dynamics



data @HMDatabase; design @ikashnitsky







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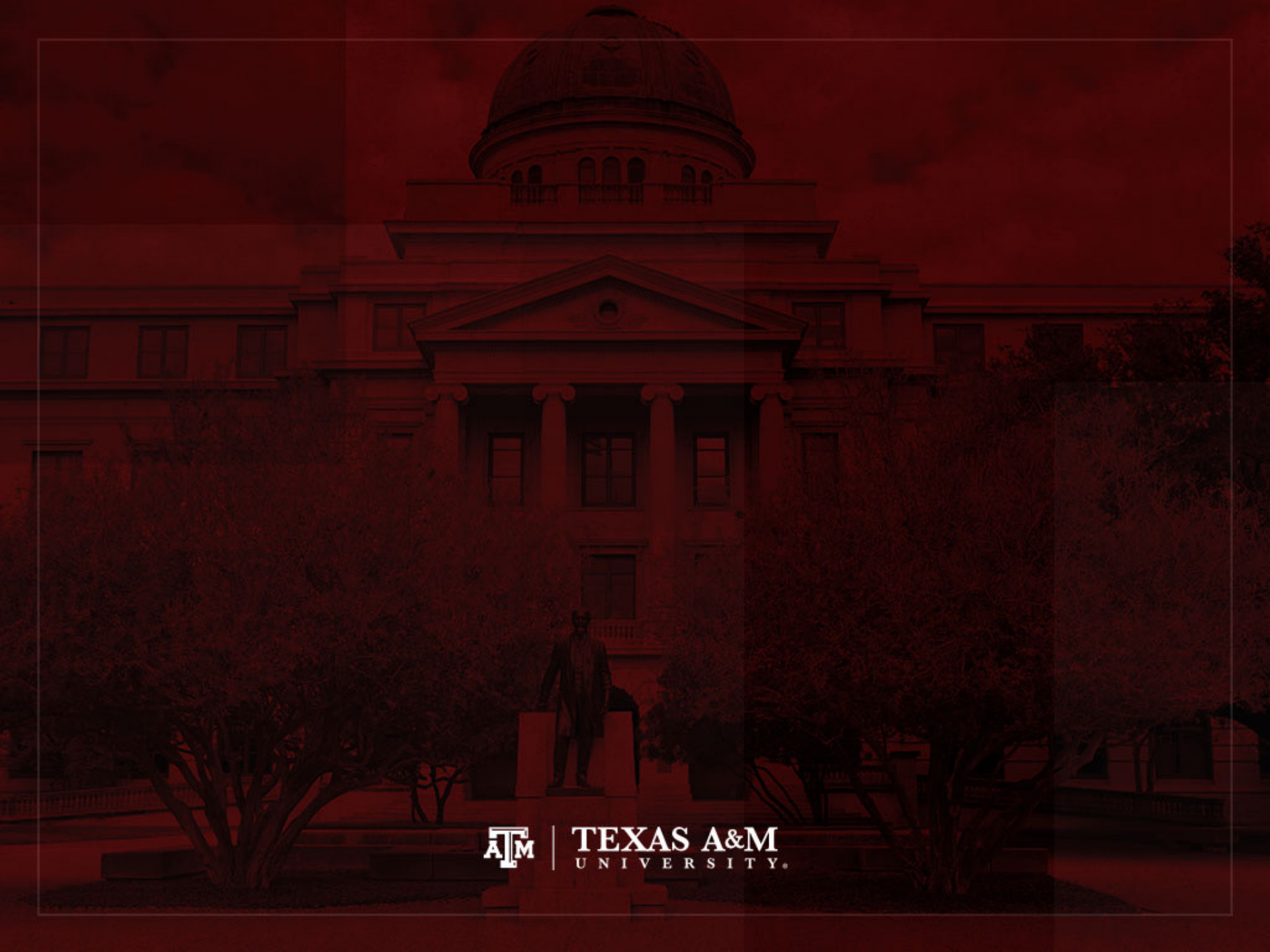
References

Wachter KW. 2014. Essential Demographic Methods. Cambridge: Harvard University Press. Chapter 10 (pp. 218–249).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