Lecture 10:
Stable age structures

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April 14, 2025
Demographic Methods (SOCI 633/320)

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Outline

- Age pyramids
- Stationary equivalent populations
- Unchanging rates
- Stable age pyramids
- Lotka's *r*
- Population momentum

Age pyramids

- There is theory to deal with age structure
  - It accounts for the relative numbers of young and old men and women in a population

- Basic idea is to obtain formulas for how a population will be theoretically distributed by age
  - If population has been closed to migration
  - If its birth and death rates have been unchanging for a long time

Actual ≠ Theoretical

- The actual age distribution of the population naturally differs from the theoretical age distribution

- Deviations are explained by
  - Events of migration
  - Changes in rates in the prior history of the population

General and special features

- The age distribution of each population has

- General features
  - Which it shares with populations with the same vital rates

- Special features
  - Which are derived from its own particular history

Graphical diagrams

- Age pyramid, age distribution, age structure
  - They represent the distribution of the population by age and sex
  - They are made up of a pair of bar graphs, one for men and one for women, turned on their sides and joined
- The vertical axis corresponds to age
  - The young are toward the bottom, the elderly toward the top
  - The open-ended age group at the very top is sometimes drawn with a triangle instead of bars
- For each age group
  - The bar coming off the axis to the right represents the number of women in that age group
  - The bar to the left the number of men

Idealized age pyramids

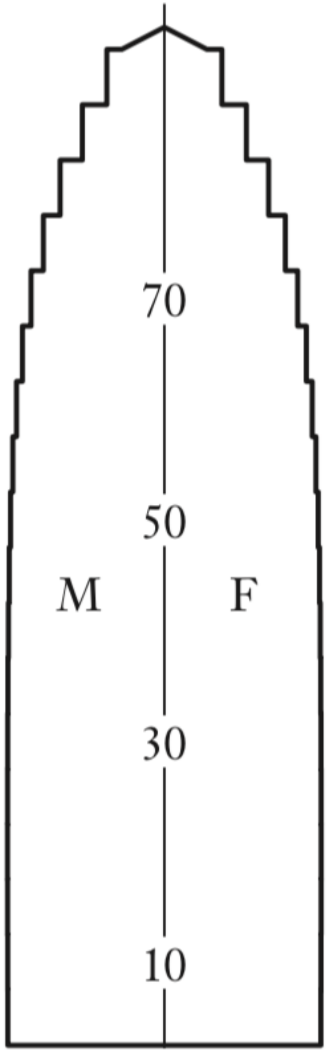
- Examples of idealized stable pyramids that occur in closed populations with unchanging vital rates

- Tall and slender
  - It is a case for a long-lived population with near zero growth

- Quite pyramidal in shape
  - It is a case for a population with heavy mortality and rapid growth

Idealized pyramids

Tall and slender



Quite pyramidal in shape

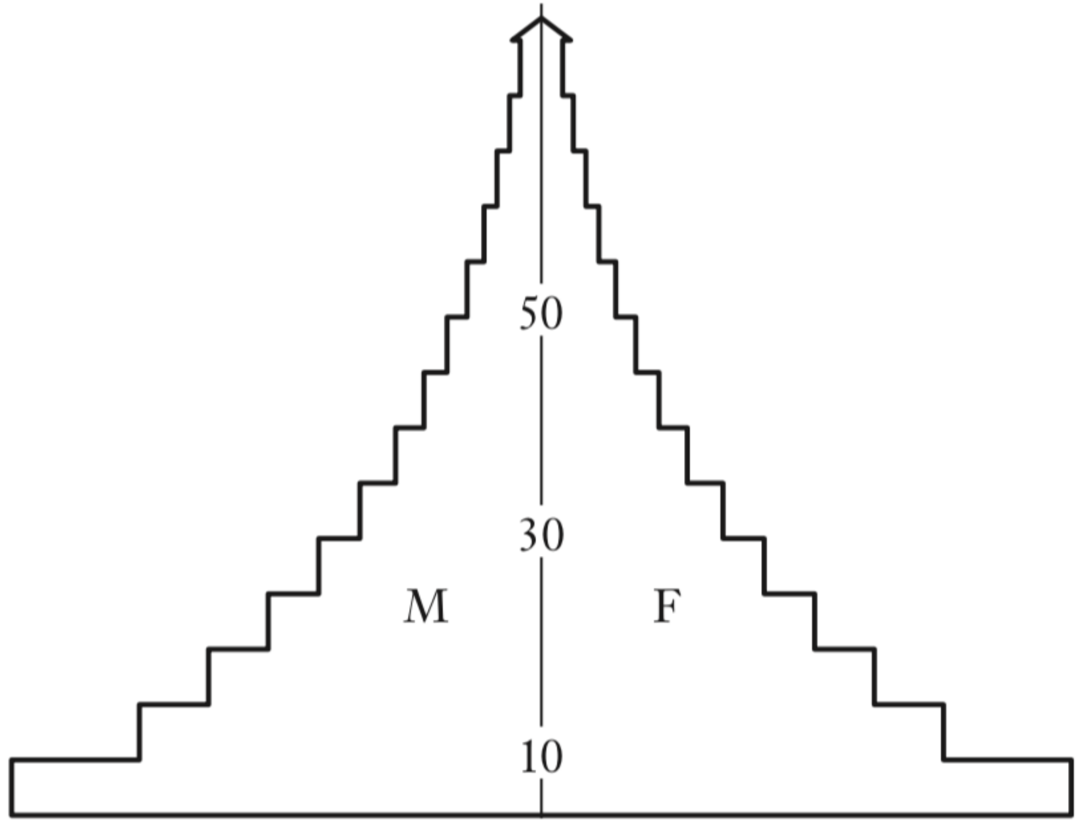


Figure 10.1 Examples of stable age pyramids

Stable population

- Stable population is any population produced by age-specific rates of fertility and mortality constant over a long period of time
  - Its age pyramid is determined uniquely by its lifetable and its long-term growth rate
  - Proportions in each age group in a stable population do not change over time
  - Numbers in each age group may change over time
    - Population may be growing or declining in size
    - It depends on what the growth rate happens to be

Observed age pyramids

- Examples of observed age pyramids
- France in 1960
  - It shares overall shape with the low-growth stable case
  - But notches among 20 and 40 years of age due to low births during World Wars I and II
- Mauritius in 1963
  - It shares overall shape with high-growth stable case
  - But indentations at working ages hint at
    - Changes around 1945 from increasing growth probably due to gains against infant mortality
    - Out-migration

Observed pyramids

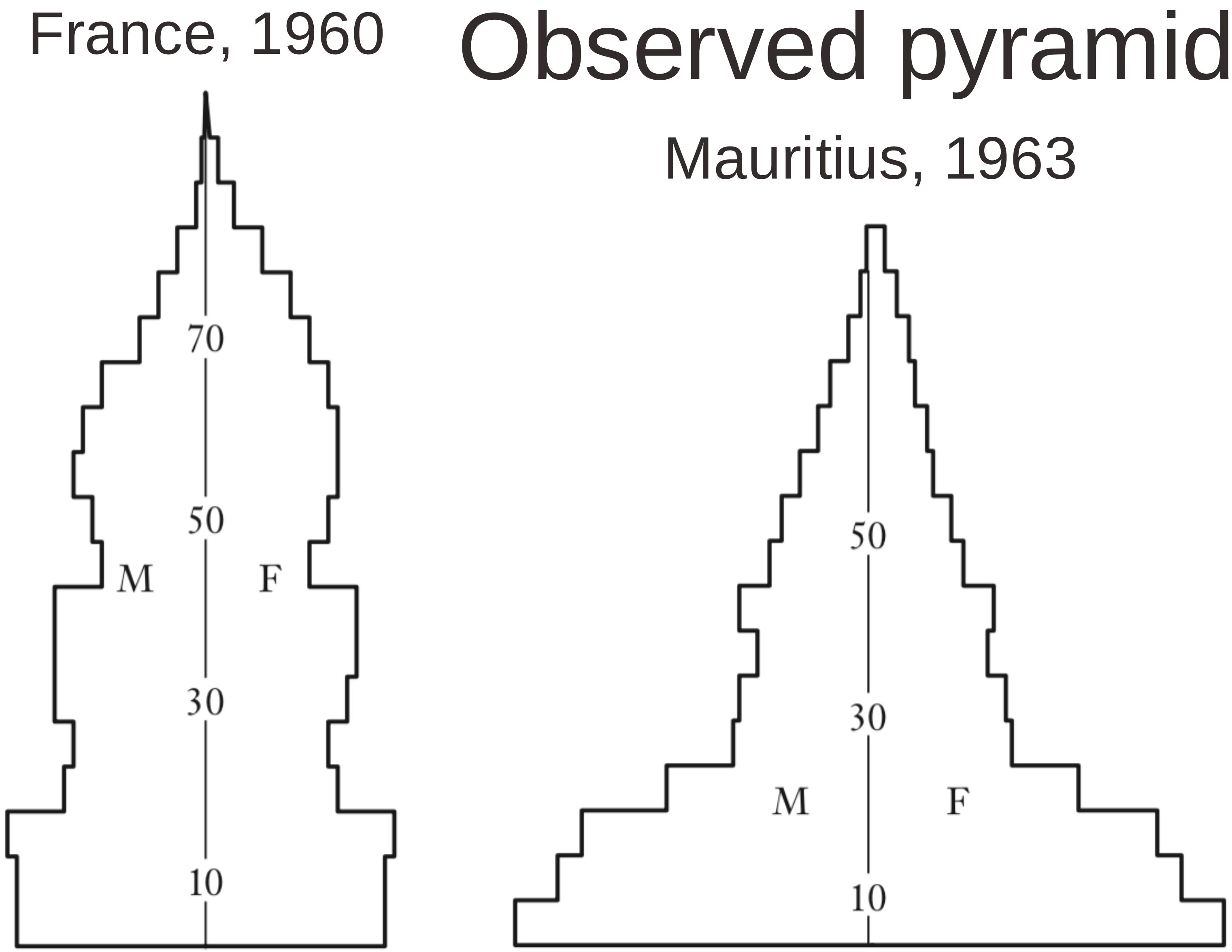


Figure 10.2 Examples of observed age pyramids

Idealized ≠ Observed

- Stable theory captures general features well

- Observable differences from stable population shapes are due to each nation's own history
  - Changing mortality and fertility rates
  - Movements across borders

Stable population conditions

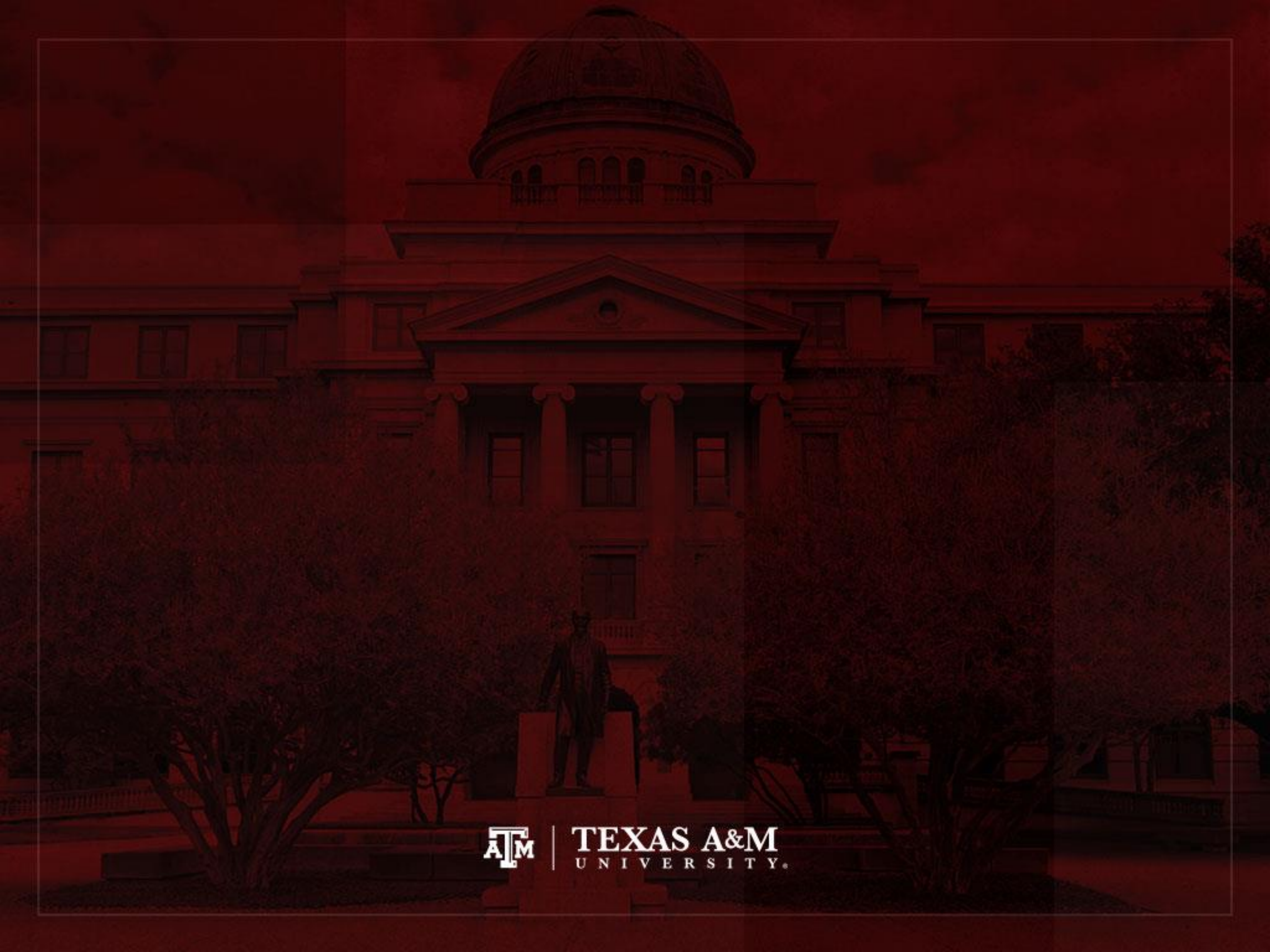
- It considers a closed population
  - A population in which migration does not occur
- If a population experiences constant age-specific fertility and mortality rates for a long time
  - It develops a constant age distribution and grows at a constant rate, irrespective of its initial age distribution
  - Demographers sometimes indicate that stable populations forget their past
- Age distribution of a stable population depends on
  - The underlying age-specific mortality rates
  - The rate of growth

Stable & stationary populations

- Stable population
  - Rates stay the same, but the population size may change
- Stationary population
  - Rates and population size remain the same
  - The growth rate is zero
  - It is a special case of a stable population which satisfies the extra condition of having zero population growth (ZPG)

Stable population theory

- Stable population theory is the mathematical analysis of stable age pyramids
- It is a theory that goes back to the work of Leonhard Euler in 1760
- It was extensively developed by
  - Alfred Lotka (1880–1949) used life tables to develop his stable population theory in the early 1900s
  - Nathan Keyfitz and Ansley Coale in the last half-century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Stationary equivalent populations

- Unchanging age-specific rates along with exponentially growing births imply the equation

$$k(x,t) = B(t)(\ell_x/\ell_0)e^{-Rx}$$

- $k(x,t)$ is the density of individuals age x at time t, which is the height of the Lexis surface

Unchanging ℓ_x and R

- When cohort survivorship ℓ_x is unchanging, those individuals age x at time t are the survivors

$$B(t-x)\ell_x/\ell_0$$

  - Out of the $B(t-x)$ babies born at time $t-x$

- When it is also true that births are growing exponentially at rate R, then

$$B(t-x) = B(0)\exp(R(t-x)) = B(t)\exp(-Rx)$$

Age groups of width *n*

- The formula for a stable population with age groups of width *n* followed across *n*-year-long steps of time turns out to be

$$ {}_nK_x(t) = B(t)({}_nL_x/\ell_0)\ \exp(-Rx) $$

- Similar to the expression in continuous time (below) with big "*L*" in place of little "ℓ"

$$ k(x,t) = B(t)(\ell_x/\ell_0)\ \exp(-Rx) $$

Growth rate *R* as *r*

- The growth rate "*R*" turns out to be a special growth rate which will be written as "*r*"

- This can be understood with the special case of a stationary population
  - $R=0$
  - Births $B(t)$ equal to an unchanging count B

Leslie matrix subdiagonal

- With *R*=0, the same reasoning leading from Figure 5.2 to the formula for the subdiagonal elements of a Leslie matrix tells us that

$$ {}_nK_x = B\,{}_nL_x/\ell_0 $$

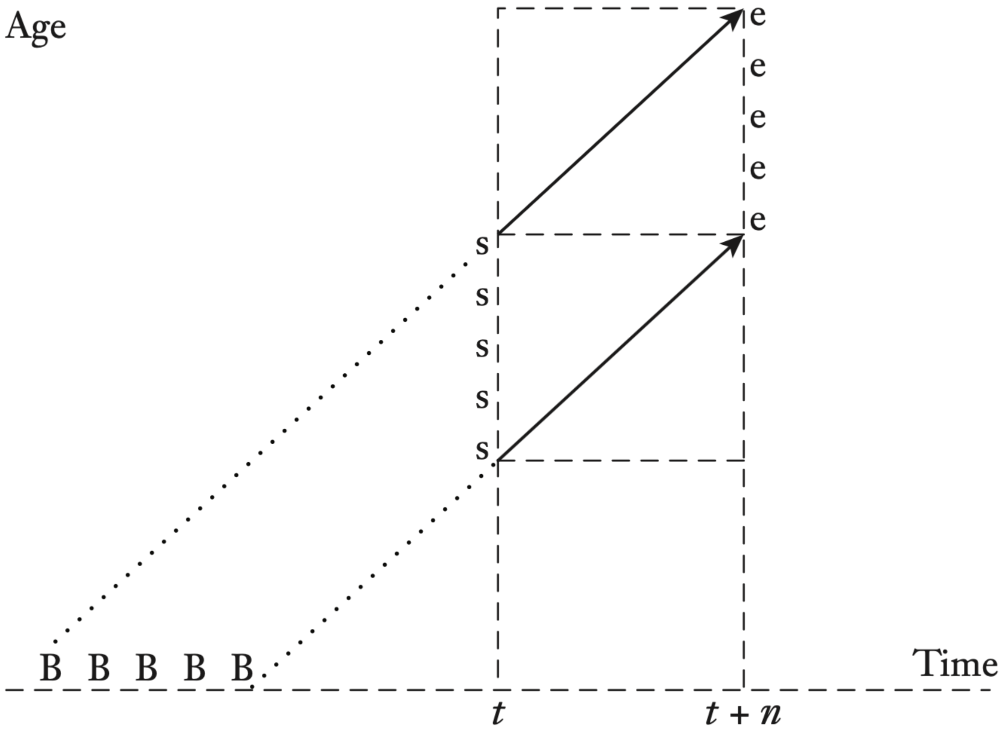


Figure 5.2 Contributions to the Leslie matrix subdiagonal

Total stationary population size

- Total population size for a stationary population is obtained by adding the sizes of the age groups
- Adding up the *L* values and dividing by the radix (ℓ_0) gives us the life expectancy at birth (e_0)

$$e_0 = \Sigma({}_nL_x) / \ell_0$$

- The total population size is

$$B\, e_0 = B\, \Sigma({}_nL_x) / \ell_0$$

CBR and age group proportion

- We can divide the size of each age group by the total size to find the proportion in the age group
  - We avoid having too many letters by writing ${}_{\infty}K_0$ for total size (whole population from ages zero to infinity)
- The Crude Birth Rate is

$$b = B \,/\, ({}_{\infty}K_0)$$

- The proportion aged x to $x+n$ in a stationary population is

$$\frac{{}_{n}K_x}{{}_{\infty}K_0} = b \, \frac{{}_{n}L_x}{\ell_0}$$

Disagreement in units

- Going back to the formula for counts (${}_nK_x$)

$$ {}_nK_x = B \; {}_nL_x / \ell_0 $$

  – Units on the left-hand side are people

  – Units on the right-hand side are people (B) times person-years (${}_nL_x$) divided by people (ℓ_0), which ends with person-years

- So, the units disagree

People = Person-years

- This is a clue...

${}_nL_x$ as stationary population

- A correspondence is being established between people and person-years

$$ {}_nK_x = B\ {}_nL_x / \ell_0 $$

- ${}_nL_x$ sometimes has the label “stationary population” instead of “person-years lived”
- We can choose the radix (ℓ_0) equal to B

$$ {}_nK_x = B\ {}_nL_x/\ell_0 = {}_nL_x $$

- This is related to the synthetic cohort concept

Going in the opposite direction

- Previously, we started with a period population and ended up with a synthetic cohort
- We can try going in the opposite direction
  - Starting with a synthetic cohort and trying to end up with a population in a period
  - But we do not get the original period population
- We end up with a stationary population
  - It has the same lifetable as the period population
  - But it usually has very different age-group counts

Stationary equivalent population

- The population derived from a synthetic cohort is called a “stationary equivalent population”
- It is a population equivalent to a cohort

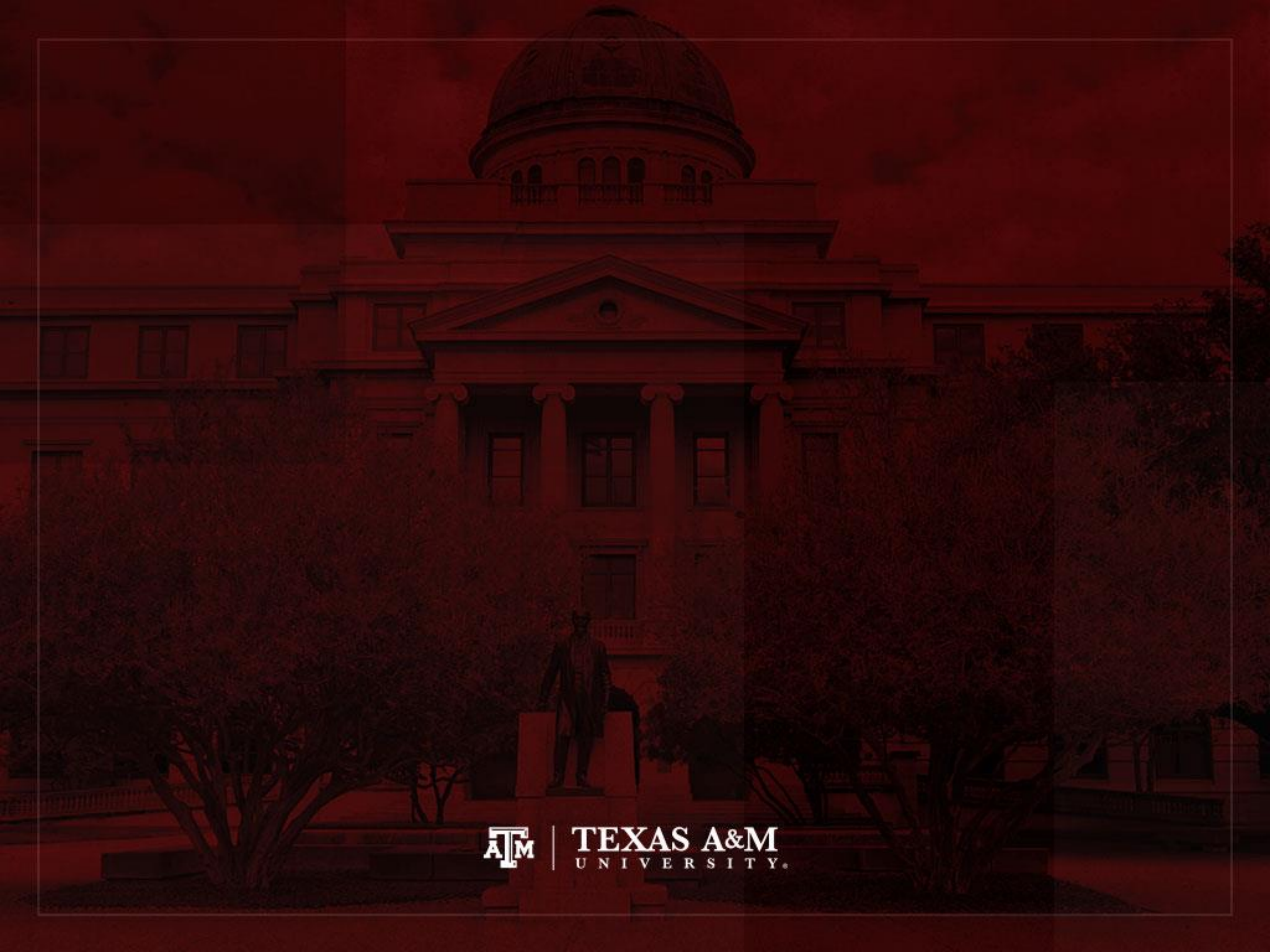
Construction

- Constructing the stationary equivalent population
- Make each year in the life of each cohort member match up with a unique population member
- One cohort person-year is associated with one population person, forcing units to agree
- Interpret the count of births B in the stationary formula as a density of births per year

Population correspondence

- The cohort size is much smaller than the population size
- On average, each cohort member lives for e_0 years
- So a cohort of size ℓ_0 corresponds to a stationary equivalent population of size

$$\ell_0 \, e_0$$



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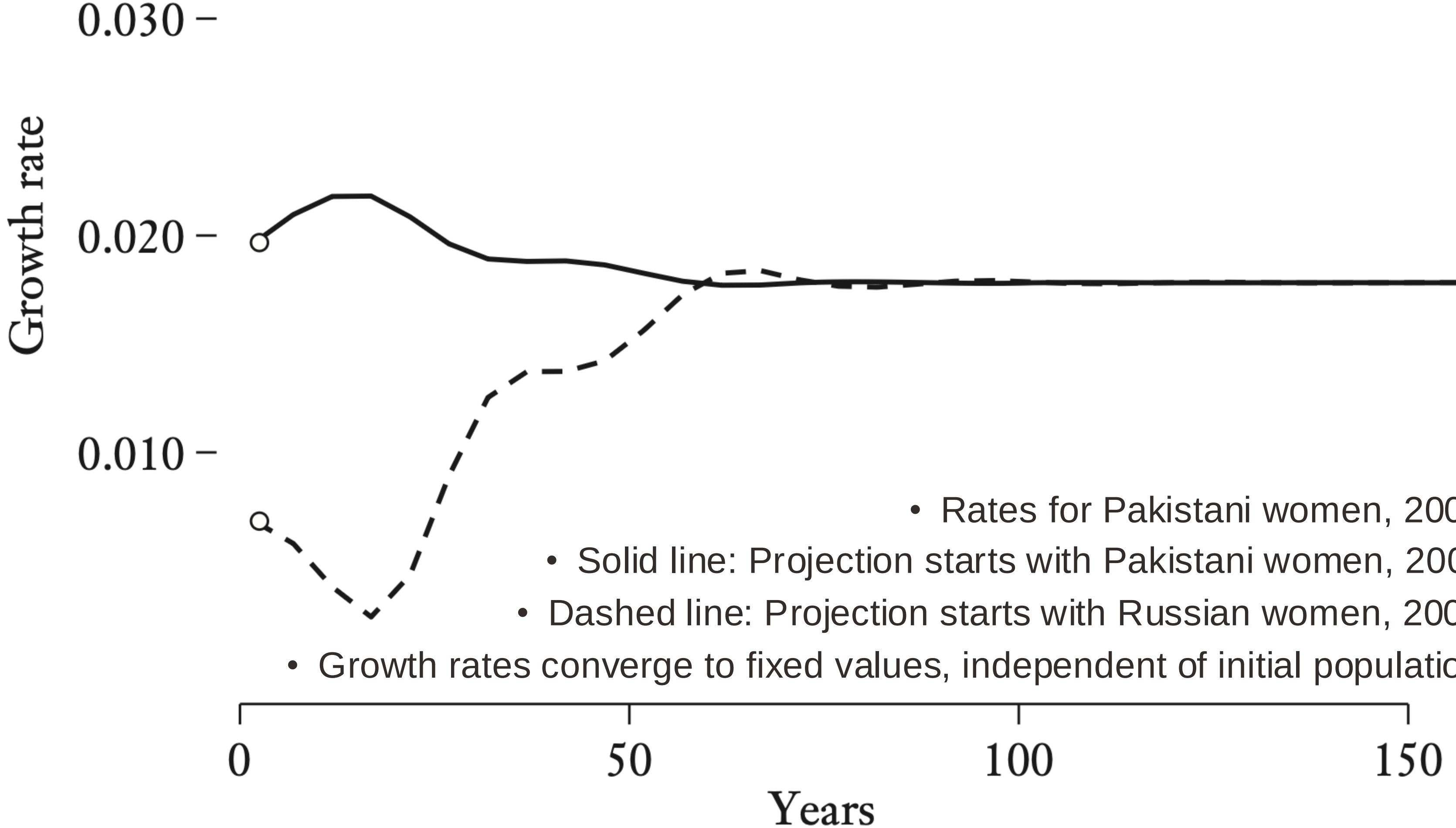
Unchanging rates

- Stable populations are more general than stationary populations
  - Stable populations may have any growth rate
  - Stationary populations have zero growth
- Study the effects of constant age-specific fertility and mortality rates
  - But births do not necessarily balance deaths
- Consequences can be seen by projecting a population over many steps using Leslie matrices

- Behavior of growth rate *R* with unchanging age-specific rates



- Rates for Pakistani women, 2000
- Solid line: Projection starts with Pakistani women, 2000
- Dashed line: Projection starts with Russian women, 2000
- Growth rates converge to fixed values, independent of initial population

Figure 10.3 Projected growth rates over time

Long-term growth rate "r"

- Populations projected forward with the same constant age-specific rates may differ in their short-term growth rates
  - They share the same constant long-term growth rate
  - This rate has its own symbol, the letter "r"
- Capital *R* stands for any growth rate
- Little *r* stands for the special long-term growth rate produced by a long stretch of unchanging demographic rates

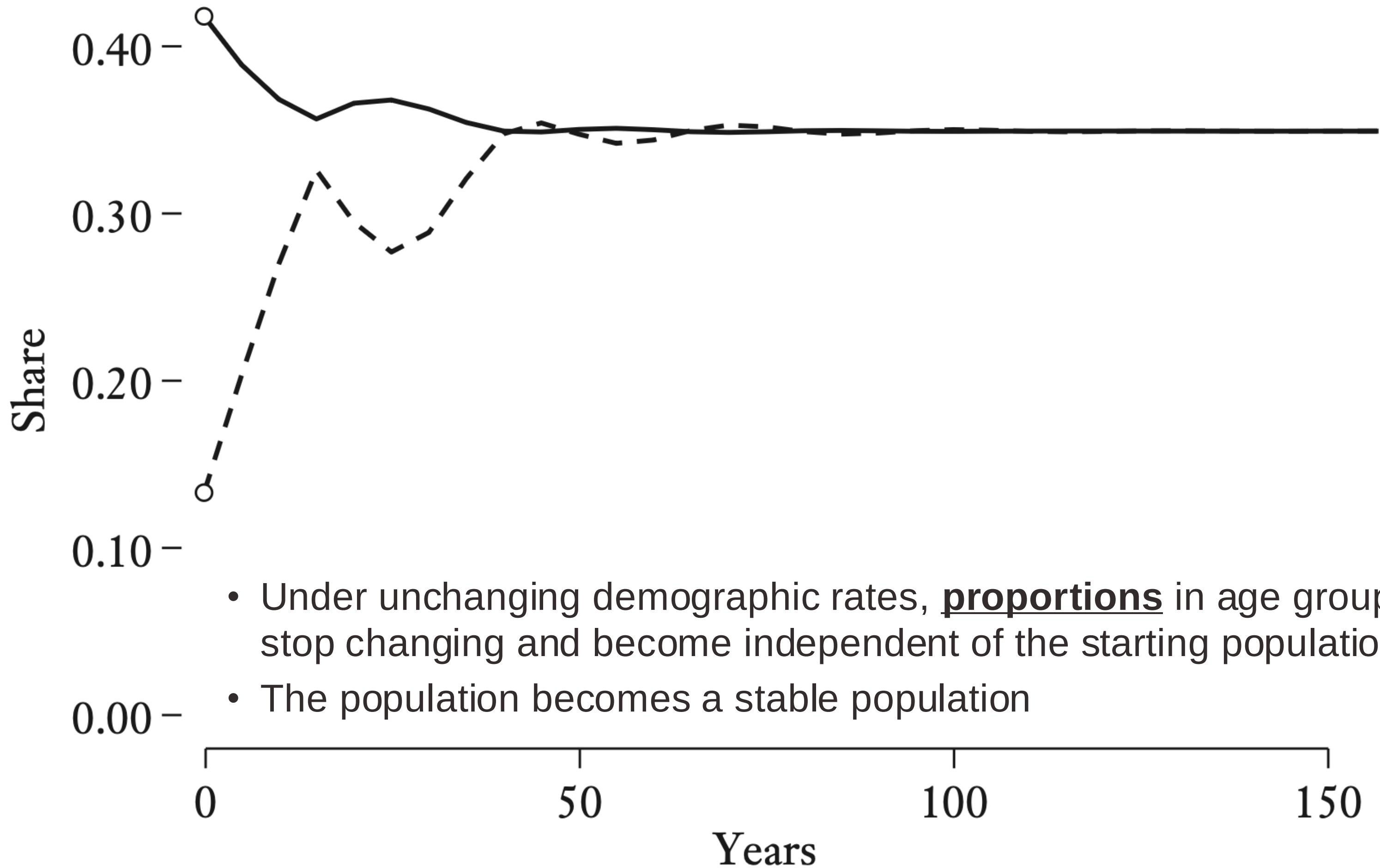
Intrinsic rate of natural increase

- Little *r* is often called "Lotka's r" after Alfred Lotka
- Its official name is the "intrinsic rate of natural increase"
  - "**Intrinsic**" because it is built into the fertility and mortality schedules, without regard to population numbers
  - "**Natural increase**" because it is defined for closed populations without regard to migration

Lotka's *r* definition

- Lotka's intrinsic rate of natural increase (*r*) is the long-term growth rate of any closed population subject to unchanging age-specific rates of fertility and mortality

- Share of children under age 15 in the population for the same projections as Figure 10.3



- Under unchanging demographic rates, **<u>proportions</u>** in age groups stop changing and become independent of the starting population
- The population becomes a stable population

Figure 10.4 An age group share over time

Proportions not sizes

- A stable population is a population in which the **proportions** in age groups remain exactly constant over time
- **Proportions** in age groups are constant over time in a stable population, not the **sizes** of the groups
- The sizes of age groups at any time in the future do continue to depend on the starting population
- They depend both on initial size and on initial age distribution

- Sizes of the group of children under age 15 in the population for the same projections as Figure 10.3

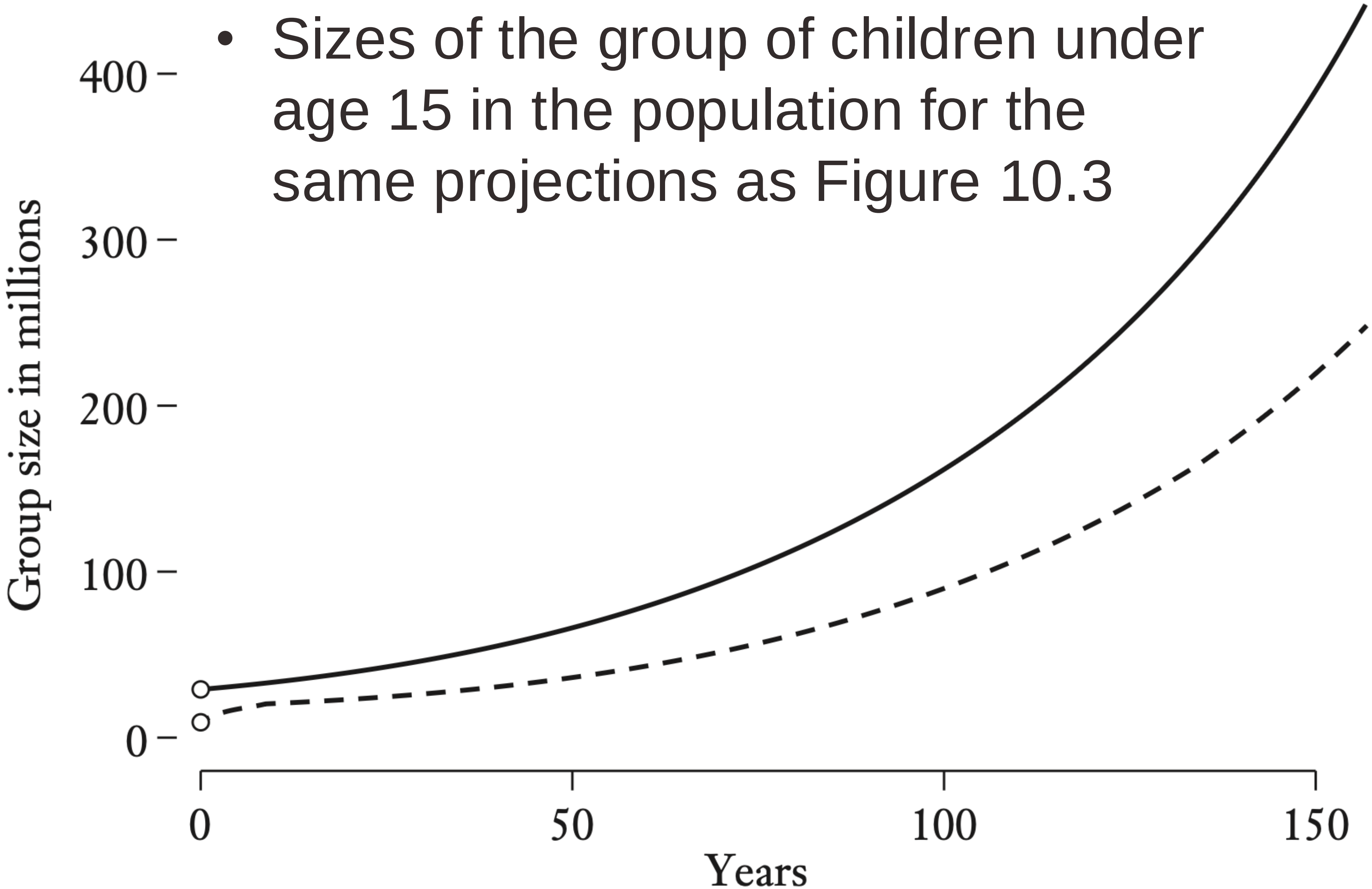


Figure 10.5 Age group size over time

Exponential growth

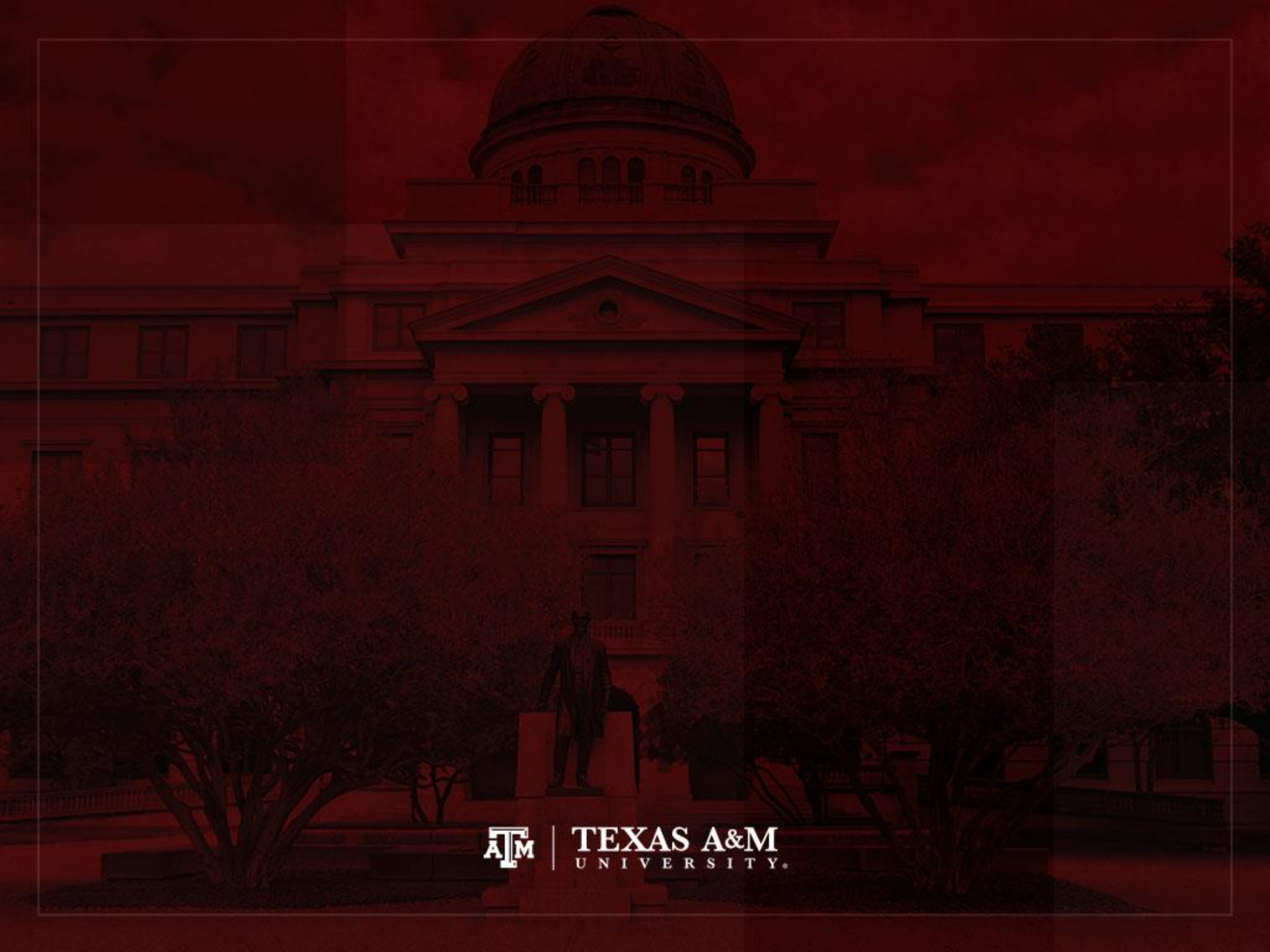
- In a stable population, the size of every age group obeys the formula for exponential growth
- The growth rate is the slope of the logarithm of population size over time
  - When it settles down to r, total population is growing exponentially like e^{rt}
  - When the proportions in age groups have also settled down to fixed values, each age group is also growing exponentially like e^{rt}

4 main features

1. Growth rates become fixed over time
2. Growth rates become independent of starting populations
3. Age pyramid proportions become fixed over time
4. Age pyramid proportions become independent of starting populations

5 assumptions

1. Age-specific rates of giving birth and dying are unchanging over time
2. The population is closed to migration
3. The female population can be projected independent of the males
4. Randomness can be ignored, letting population estimates represent mean or expected values
5. Children are estimated in continuous fractions
   - Projected numbers of children from any mother are typically fractions rather than whole numbers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Stable age pyramids

- A stable population grows exactly exponentially and its growth rate is Lotka's intrinsic rate of natural increase (*r*)
- These are the generalizations of the stationary population formulas

$$\text{Stable Counts: } {}_nK_x = B\,\frac{{}_nL_x}{\ell_0}\,e^{-rx}$$

$$\text{Stable Proportions: } \frac{{}_nK_x}{{}_\infty K_0} = b\,\frac{{}_nL_x}{\ell_0}\,e^{-rx}$$

Exponential factor

- Exponential growth as a function of time has a plus sign: e^{+rt}
- The exponential factor in the stationary population has a minus sign: e^{-rx}
- In a growing population (r > 0), the age pyramid has to shirking as we go up in age, because birth cohorts shrink as we go back in time
- In declining populations (r < 0), older people come from large cohorts from the past

Possibilities of *r*

- The first step to study an age pyramid is to assign one of the three classes of population growth
- $r > 0$
  - With positive growth, the shape is broad at the bottom
  - There are lots of young people and fewer old people
- $r = 0$
  - With zero growth, ${}_nL_x$ values determine the slope
- $r < 0$
  - With negative growth, the base is narrow and the pyramid bulges out at the top

Age group sizes

- Total size of a stationary population: Be_0
- No simple counterpart for stable populations
  - Have to add up all the age group sizes
- The constant B is used for all stable populations

$$_nK_x = B\ _nL_x/\ell_0\ e^{-rx}$$

- For the youngest age group (x=0)

$$e^{-rx} = e^{-r0} = 1$$

- For very small n (e.g., day, hour)

$$_nL_0/\ell_0 \cong n$$

$$_nK_0 \cong Bn$$

Example with dependency ratios

- We can calculate dependency ratios with
  - Observed data
  - Stable population with intrinsic growth rate (r) and lifetable (${}_nL_x$)
- Observed ratios are influenced by migration, prior history, and other special features
- The systematic effect of growth is easier to see when we calculate ratios from the stable population formula

Data from India, 2000

- Numerator: Number of children below age 15 (0–4, 5–9, 10–14)
- Intrinsic growth rate (r = 0.006); radix (ℓ_0 = 1)

Table 10.1 Stable population data for India, youth, 2000

x	Observed Count (millions)	Lifetable $_5L_x$	Factor e^{-rx}	Product $_5L_x/\ell_0\ e^{-rx}$
0	120.878	4.649	1.000	4.649
5	116.296	4.549	0.970	4.414
10	109.984	4.497	0.942	4.235
	347.158			13.298

Data from India, 2000

- Denominator: Working-age population (15–64)
- Intrinsic growth rate (r = 0.006); radix (ℓ_0 = 1)

Table 10.2 Stable population data for India, adults, 2000

x	Observed Count (millions)	Lifetable ${}_5L_x$	x	Observed Count (millions)	Lifetable ${}_5L_x$
15	100.852	4.466	60	26.743	3.386
20	88.504	4.426	65	20.861	2.948
25	83.604	4.371	70	14.426	2.381
30	75.671	4.301	75	8.617	1.706
35	66.900	4.228	80	4.250	1.023
40	58.114	4.145	85	1.468	0.477
45	48.176	4.042	90	0.340	0.158
50	39.033	3.900	95	0.047	0.034
55	32.257	3.694	100	0.005	0.005

- **Child dependency ratio**
  - For observed population: 347.158 / 619.854 = 0.560
  - For stable population: 13.298 / 33.042 = 0.402

x	Count (millions)	Lifetable: 5Lx	Factor: exp(-rx)	Product
15	100.852	4.466	0.914	4.082
20	88.504	4.426	0.887	3.926
25	83.604	4.371	0.861	3.762
30	75.671	4.301	0.835	3.592
35	66.900	4.228	0.811	3.427
40	58.114	4.145	0.787	3.261
45	48.176	4.042	0.763	3.086
50	39.033	3.900	0.741	2.889
55	32.257	3.694	0.719	2.656
60	26.743	3.386	0.698	2.362
Total	**619.854**	**40.959**		**33.042**

r = 0.006

2000 observed & stable population

Data from India	2000 observed population			Stable population 2000 intrinsic rate of natural increase (r) 0.006		
x	Count (millions)	Population counts	Lifetable: 5Lx	Factor: exp(-rx)	Product	Population counts
0	120.878		4.649	1.000	4.649	
5	116.296		4.549	0.970	4.415	
10	109.984	**347.158**	4.497	0.942	4.235	**13.299**
15	100.852		4.466	0.914	4.082	
20	88.504		4.426	0.887	3.926	
25	83.604		4.371	0.861	3.762	
30	75.671		4.301	0.835	3.592	
35	66.900		4.228	0.811	3.427	
40	58.114		4.145	0.787	3.261	
45	48.176		4.042	0.763	3.086	
50	39.033		3.900	0.741	2.889	
55	32.257		3.694	0.719	2.656	
60	26.743	**619.854**	3.386	0.698	2.362	**33.042**
65	20.861		2.948	0.677	1.996	
70	14.426		2.381	0.657	1.564	
75	8.617		1.706	0.638	1.088	
80	4.250		1.023	0.619	0.633	
85	1.468		0.477	0.600	0.286	
90	0.340		0.158	0.583	0.092	
95	0.047		0.034	0.566	0.019	
100	0.005	**50.014**	0.005	0.549	0.003	**5.682**
Chid dependency ratio		**0.560**				**0.402**
Old-age dependency ratio		**0.081**				**0.172**
Total dependency ratio		**0.641**				**0.574**

2013 observed & ZPG

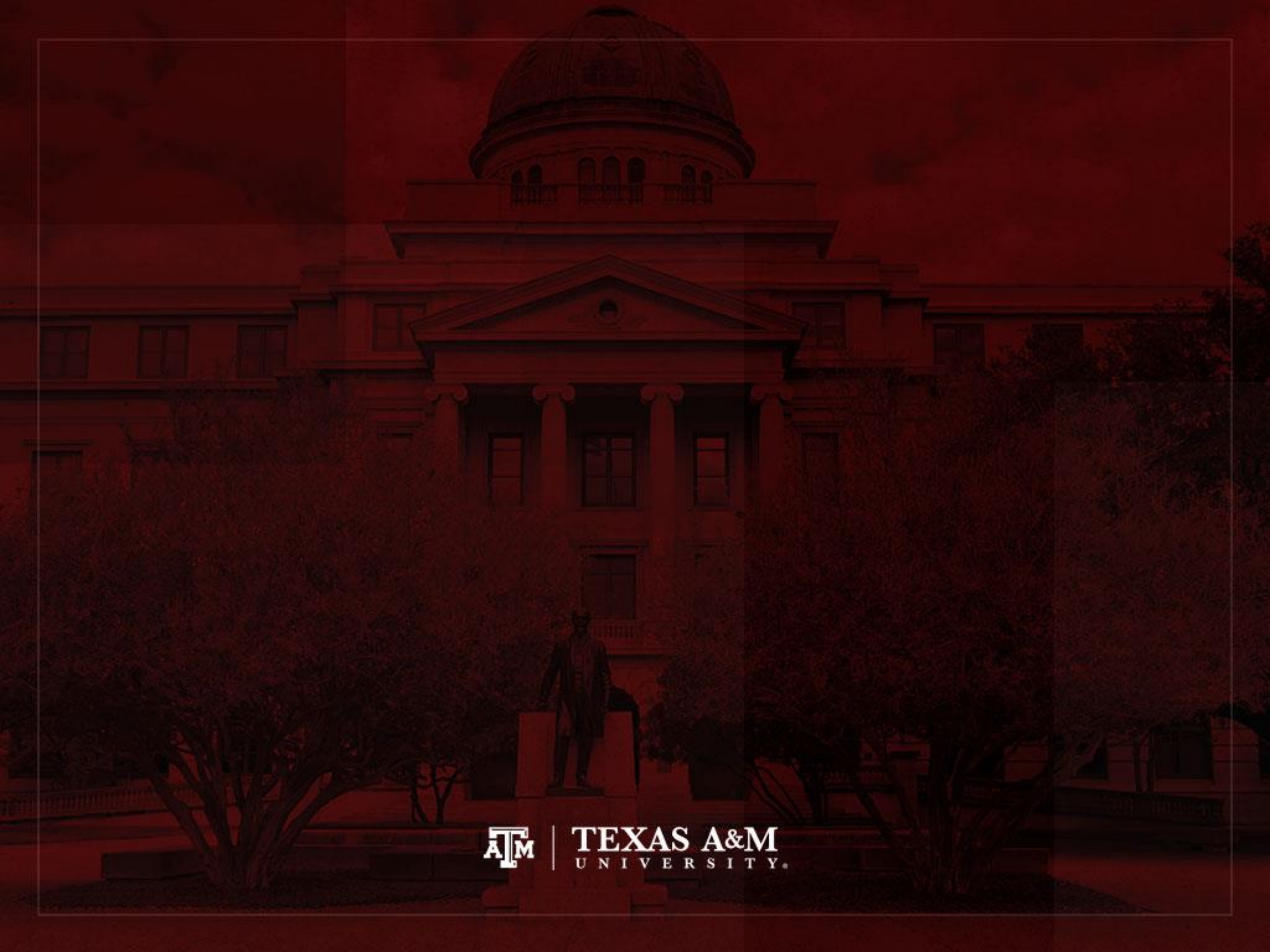
- Dependency ratios for stable populations are revealing because we can compare what we would see with different growth rates
- If India had an intrinsic growth rate (*r*) equal to its *R* of 0.015 in 2013, child dependency ratio would be close to observed ratio: 12.740/24.263=0.525
- If India reached sustained zero population growth (*r*=0), child dependency ratio would be 13.695/40.959=0.334

2013 observed & ZPG

Data from India	2000 observed population	2013 growth rate (R) 0.015			Zero population growth 0.000		
x	Lifetable: 5Lx	Factor: exp(-rx)	Product	Population counts	Factor: exp(-rx)	Product	Population counts
0	4.649	1.000	4.649		1.000	4.649	
5	4.549	0.928	4.220		1.000	4.549	
10	4.497	0.861	3.871	**12.740**	1.000	4.497	**13.695**
15	4.466	0.799	3.566		1.000	4.466	
20	4.426	0.741	3.279		1.000	4.426	
25	4.371	0.687	3.004		1.000	4.371	
30	4.301	0.638	2.742		1.000	4.301	
35	4.228	0.592	2.501		1.000	4.228	
40	4.145	0.549	2.275		1.000	4.145	
45	4.042	0.509	2.058		1.000	4.042	
50	3.900	0.472	1.842		1.000	3.900	
55	3.694	0.438	1.619		1.000	3.694	
60	3.386	0.407	1.377	**24.263**	1.000	3.386	**40.959**
65	2.948	0.377	1.112		1.000	2.948	
70	2.381	0.350	0.833		1.000	2.381	
75	1.706	0.325	0.554		1.000	1.706	
80	1.023	0.301	0.308		1.000	1.023	
85	0.477	0.279	0.133		1.000	0.477	
90	0.158	0.259	0.041		1.000	0.158	
95	0.034	0.241	0.008		1.000	0.034	
100	0.005	0.223	0.001	**2.991**	1.000	0.005	**8.732**
Chid dependency ratio				**0.525**			**0.334**
Old-age dependency ratio				**0.123**			**0.213**
Total dependency ratio				**0.648**			**0.548**

Comparative statics

- These calculations with stable age pyramids can be referred as “comparative statics”
  - “Statics”: Study of things that stay fixed
  - “Dynamics”: Study of things in the process of movement or change
- In stable population calculations
  - We are comparing what we would see in a country whose growth rate stayed fixed at one value
  - To what we would see in a country whose growth rate stayed fixed at some other value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Lotka's *r*

- Lotka's *r* is the growth rate of the stable population (intrinsic rate of natural increase)
- To estimate it, start with the growth rate formula

$$R = (1/T) \log(K(T)/K(0))$$

  - The net reproduction ratio (*NRR*) is the factor by which the population grows in one generation ($K(T)/K(0)$)
  - Cohort mean age of childbearing (μ) is the length of time it takes for the daughter generation to come along
- Thus, an approximation is

$$r \approx \frac{\log(NRR)}{\mu}$$

Example

- Lotka's *r*, *NRR* and *μ* depend on lifetable ($_nL_x$,), fertility rates ($_nF_x$) and the fraction female at birth
- At the peak of the Baby Boom around 1960, the *NRR* for the United States was about 1.7
- The mean age *μ* was around 27

$$r \cong \log(NRR) / \mu$$

$$r \cong \log(1.7) / 27$$

$$r \cong 0.019653$$

- For 2012, $r \cong \log(0.95) / 29 \cong -0.001769$

Interpretation

- Does the negative value for *r* mean that the population of the United States is declining?
  - No
- Lotka's *r* is the long-term growth rate that would occur if the rates for 2012 remained in effect for a very long time
- In the U.S., the value of r has been negative while the period growth rate has been strongly positive
- The U.S. does not have a stable population

Lotka's *r* and *NRR*

- Relationship between Lotka's r and the net reproduction ratio (*NRR*)

$$r \begin{cases} > 0 & \text{if and only if } NRR > 1 \\ = 0 & \text{if and only if } NRR = 1 \\ < 0 & \text{if and only if } NRR < 1 \end{cases}$$

Little *r* goes by several names

- Lotka's parameter
  - Although it goes back to Euler, little *r* became familiar through the work of Alfred Lotka
- Intrinsic rate of natural increase
- Malthusian parameter

Intrinsic rate of natural increase

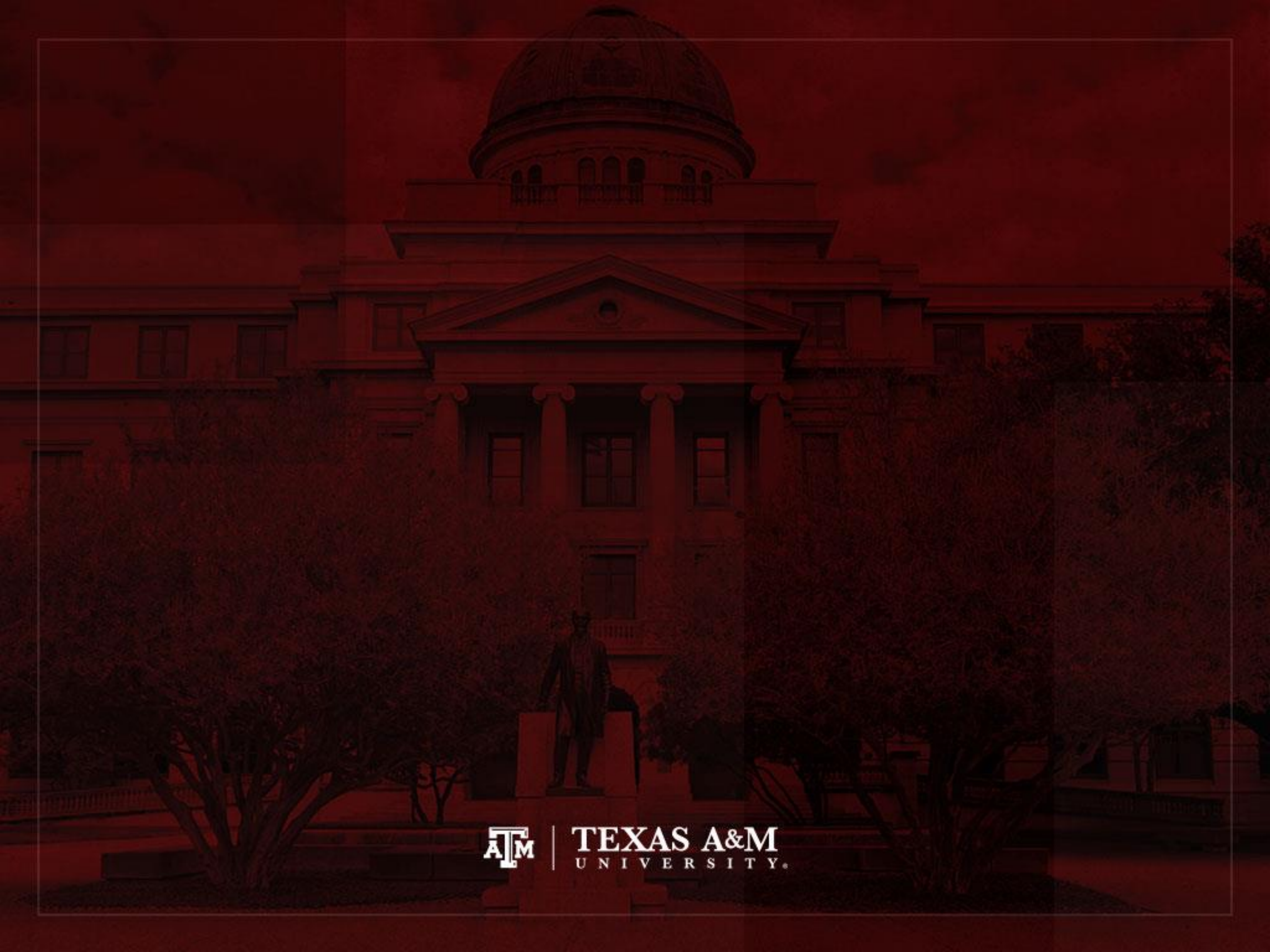
- It is “intrinsic” because it is built into the rates of age specific fertility and mortality without reference to population numbers
- It refers only to “natural” increase from an excess of births over deaths, taking no account of migration

Malthusian parameter

- This name can be confusing
- Malthus emphasized limits to population growth, whereas *r* is a rate assuming no Malthusian limits
- The name comes from passages in which Malthus also discussed exponential growth without limits

3 roles for little *r*

- *r* is the exact growth rate of the stable population
- *r* is the long-term growth rate when any initial population is projected forward in time with permanently unchanging rates
- *r* is the parameter that appears in the e^{-rx} factor in the formula for the stable age pyramid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Population momentum

- Stable population theory gives a complete account of the age structure and growth implied by a long period of unchanging vital rates
  - Cases of great interest are one-time shifts from one set of previously unchanging rates to a new set of unchanging rates
- Population momentum
  - The tendency for populations which have been growing to keep growing for many years, even when births rates and intrinsic growth rates drop

Keyfitz scenario

- The sudden shift from a growing stable population to a stationary one was studied by Nathan Keyfitz (Keyfitz scenario)
  - At time t=0, a previously stable population with some nonzero value of Lotka's r experiences a sudden change in fertility with no change in lifetable
  - Fertility rates for all age groups rapidly change by the same factor and make the new *NRR* equal to 1
  - The new rates persist for a long time, ultimately creating a stationary age distribution
  - The factor by which all the age-specific fertility rates are multiplied in order to achieve a new *NRR* of 1 is just 1 over the old *NRR*

new *NRR* = 1 / old *NRR*

Time before and after drop

- We write $t=-\epsilon$ for a time just before the intrinsic growth rates drop and $t=+\epsilon$ for a time just afterward
  - The Greek letter epsilon (ϵ) stands for some tiny positive number
- Nearly the same women are at risk of childbearing just after the drop as before
  - But their post-drop rates of childbearing are only $1/NRR$ as high, so births $B(+\epsilon)=B(-\epsilon)/NRR$
  - Logarithmic scale: $\log(B(+\epsilon))=\log(B(-\epsilon))-\log(NRR)$

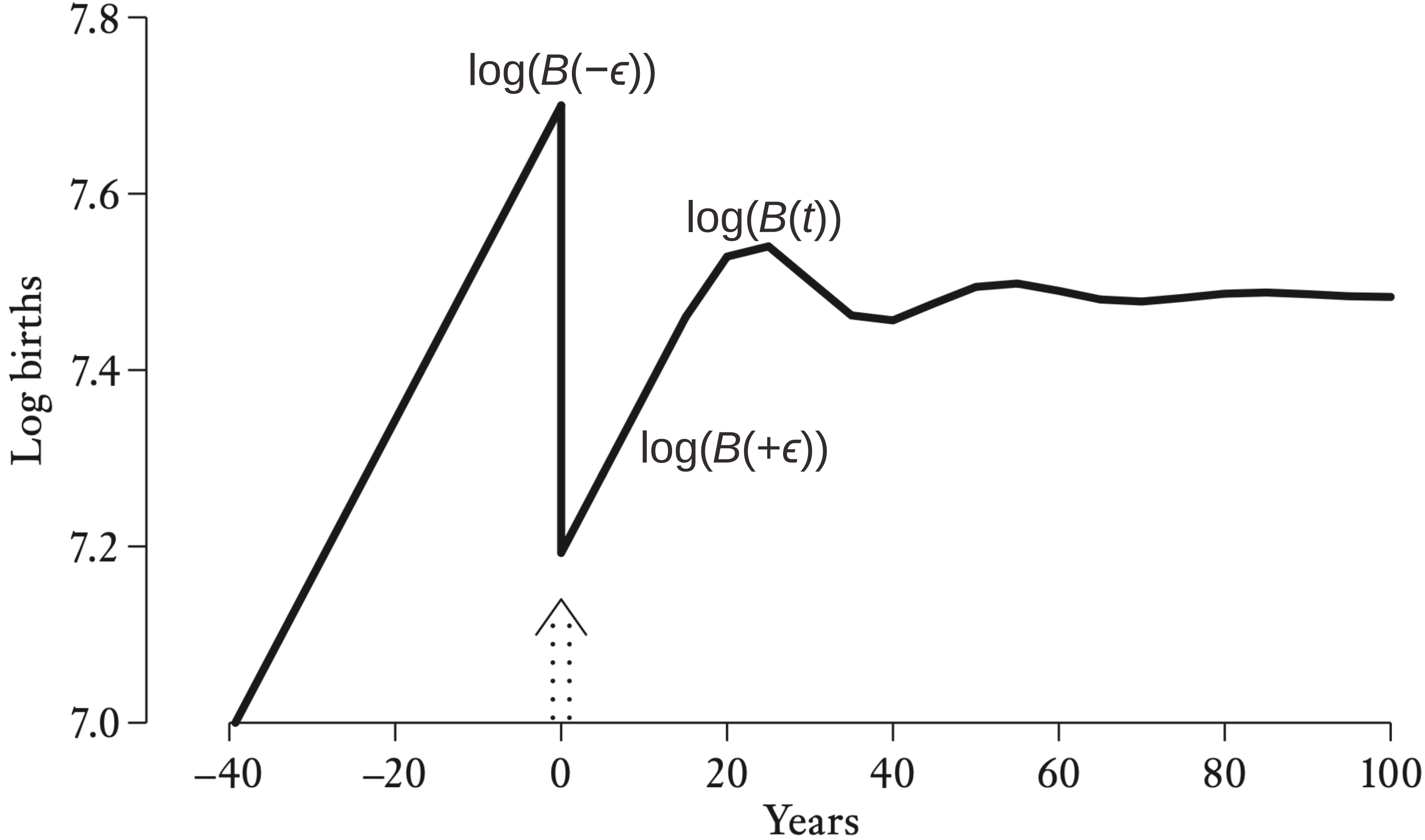


Figure 10.7 Logarithm of births in the Keyfitz scenario

Population momentum

- An approximation recommended to Keyfitz by James Frauenthal supposes that $\log(B(t))$ ultimately settles down about halfway between $\log(B(+\epsilon))$ and $\log(B(-\epsilon))$
- Writing U for a future time at which an ultimate stationary population is been achieved, we have

$$B(U) \approx B(-\epsilon)/\sqrt{NRR}$$

Convert births to population sizes

- We divide births before the drop by the Crude Birth Rate before the drop $b(-\epsilon)$ to find

$$K(-\epsilon) = B(-\epsilon) \,/\, b(-\epsilon)$$

- Using the stationary population identity ($1/b(U)= e_0$), we multiply births in the ultimate stationary population by e_0

$$K(U)= B(U)e_0$$

- This converts the approximation for births into an approximation for the momentum effect on population size

$$\frac{K(U)}{K(-\epsilon)} \approx \frac{b(-\epsilon)e_0}{\sqrt{NRR}}$$

Exact formula

- This is the approximate effect of momentum on population size

$$\frac{K(U)}{K(-\epsilon)} \approx \frac{b(-\epsilon)e_0}{\sqrt{NRR}}$$

- Exact formula with age groups of width *n*

$$\frac{K(U)}{K(-\epsilon)} = \frac{NRR - 1}{NRR} \frac{b(-\epsilon)e_0}{\mu(1 - e^{-rn})/n}$$

Example for China, 1980

- Population: 985 million
- $CBR = b(-\epsilon) = 0.024$
- $NRR = 1.5$
- $e_0 = 70$
- If fertility rates dropped instantly to replacement levels by the same factor at all ages, the ultimate stationary population would have been

$$K(U) / K(-\epsilon) \approx b(-\epsilon)e_0 / \sqrt{NRR}$$

$$K(U) / 985 \approx 0.024 * 70 / \sqrt{1.5}$$

$$K(U) \approx 985 * 0.024 * 70 / \sqrt{1.5}$$

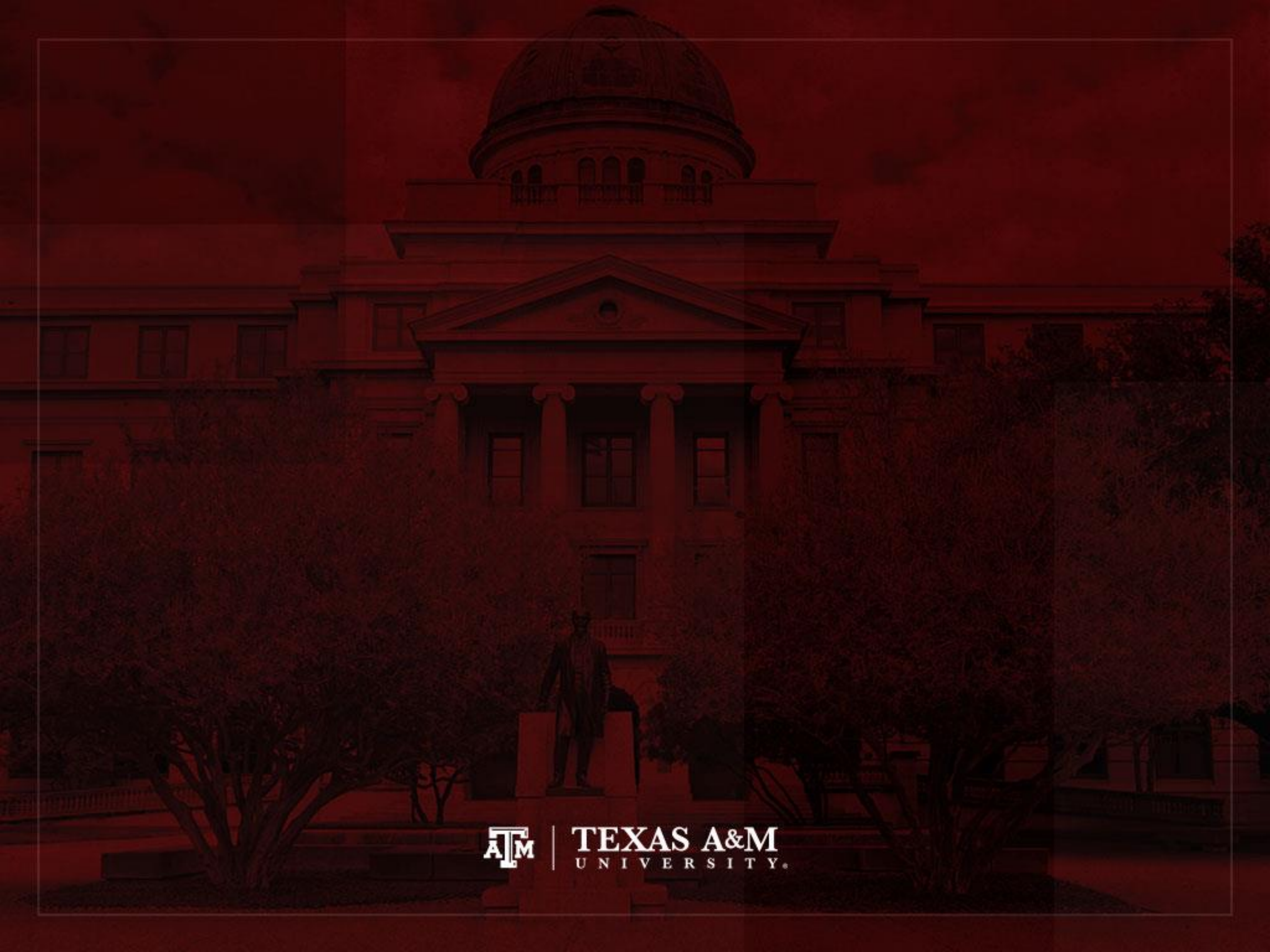
$$K(U) \approx 1.350 \text{ billion}$$

Challenges

- Population momentum is one of the most important concepts that demographers bring to public policy
- In government and among the press, it is not widely appreciated that very substantial future increases in human numbers are already built into the present structure of world population
- Momentum amplifies the challenges faced by any program for sustainable development

References

Wachter KW. 2014. Essential Demographic Methods. Cambridge: Harvard University Press. Chapter 10 (pp. 218–249).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