Probabilistic population projections with migration uncertainty

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We produce probabilistic projections of population for all countries based on probabilistic projections of fertility, mortality, and migration. We compare our projections to those from the United Nations' Probabilistic Population Projections, which uses similar methods for fertility and mortality but deterministic migration projections. We find that uncertainty in migration projection is a substantial contributor to uncertainty in population projections for many countries. Prediction intervals for the populations of Northern America and Europe are over 70% wider, whereas prediction intervals for the populations of Africa, Asia, and the world as a whole are nearly unchanged. Out-of-sample validation shows that the model is reasonably well calibrated.

Bayesian hierarchical model | international migration | predictive distribution | United Nations | World Population Prospects

In this paper we describe a method for probabilistic projection of population for all countries, with a focus on accounting for uncertainty in projections of international migration. In particular, we are motivated by the needs of the United Nations (UN) Population Division in producing population projections for all countries until 2100 based on projections of fertility, mortality, and migration.

A variety of forces contribute to the ebb and flow of international migration. Economic theories at varying levels of granularity indicate that migration flows can arise from individual attempts to maximize income (1, 2), household-level mitigation of risk (3, 4), or differences in global supply and demand for labor (5, 6). Individuals decide to migrate based on an assessment of push and pull factors (7), which may include migration policy (8), geopolitical conflict (9), and quality of the natural environment (10, 11). Networks of migrants provide a feedback mechanism such that migration flows tend to perpetuate themselves over time (12, 13). Bijak (14) gives a thorough overview of theories and models of international migration. Despite their acknowledged role in driving migration, our model does not make use of push and pull factors, economic or otherwise, as covariates. Such factors are largely too difficult to predict in the long term to be of use. Instead, we appeal to the inertia of self-perpetuating migration patterns by modeling migration as an autoregressive process.

Historically, most methods for projecting population have been deterministic. If the current population is known, broken down by age and sex, and future age- and sex-specific rates are projected for fertility, mortality, and migration, then the cohort-component method produces population projections (15). However, the UN Population Division now produces probabilistic projections of population, fertility, and mortality for all countries, but these projections still condition on deterministic migration projections (16, 17). The current methodology in the UN's World Population Prospects (WPP) differs from country to country but typically projects that net migration counts will remain constant until 2050 and drop deterministically to zero by 2150 (17). A deterministic gravity model that assumes migration is proportional to population size raised to some power (18, 19) is more flexible than the simpler WPP migration projections but still lacks quantification of uncertainty. Probabilistic population projection models that account for migration uncertainty have been developed for a small number of countries, typically only those with good data (20). Our method produces projections for all countries.

In many countries, migration is a substantial driver of population change. By failing to account for uncertainty about future migration, the current projection methodology understates the uncertainty in population projections.

We have developed a probabilistic methodology for projecting net migration for all countries (21). Based on this, we have developed a method for projecting populations for all countries by age and sex that takes account of uncertainty about migration. The method ensures that migration balances across the globe in all sex and age groups and that the large labor migration flows to and from the countries of the Arabian Peninsula are projected appropriately so that projected migration to the Gulf States does not substantially drive down projected migration in all other countries.

Results

We have produced probabilistic projections of migration and population for all countries until 2100. These are included in *SI Appendix*. They compare the medians and 80% prediction intervals from our model to a model that uses the UN's deterministic migration projections (17) along with probabilistic projections of fertility and mortality.

Case Studies. The countries in this section illustrate a range of common patterns seen as a result of including migration uncertainty in population projections.

Significance

Projected populations to the end of this century are an important factor in many policy decisions. Population forecasts become less reliable as we look farther into the future, suggesting a probabilistic approach to convey uncertainty. Migration projections have been largely deterministic until now, even in probabilistic population projections. Deterministic migration projections neglect a substantial source of population uncertainty. We incorporate a probabilistic migration model with probabilistic models of fertility and mortality to produce probabilistic population projections for all countries until 2100. The result is a substantial increase in uncertainty about the populations of Europe and Northern America, with very little change to uncertainty about the population of Africa, Asia, and the world as a whole.

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Over the 2010–2015 period, the United States experienced a net inflow of 5 million migrants, the largest net migration of any country (17). Brown and Bean (22) argue that the United States needs substantial continued in-migration as a source of labor force participants to counterbalance an aging population and to fund social safety-net programs. The WPP's deterministic projections for the United States have net migration counts remaining constant at their most recent levels until 2050, then declining linearly to zero in 2150 (blue line in the top row of Fig. 1). These deterministic projections were intended to reflect a belief that migration will be stable in the short to medium term and that in the far future we simply do not know what net migration numbers will look like. However, historical trends in the United States data show that although it is often realistic to project that net migration will not change much from one time period to the next, there are occasionally large deviations from this trend. The deterministic projections for the United States therefore understate uncertainty substantially. When carried through to population projections for the United States, we see that incorporating uncertainty due to migration roughly doubles the width of predictive intervals for population.

Many European countries are similar to the United States in that migration contributes substantially to population change. Germany (second row of Fig. 1) is one such country. In a model with a deterministic migration component, an 80% prediction interval for Germany's population in 2050 ranges from a low of 71 million to a high of 78 million. (By design, we expect the true population to fall outside an 80% prediction interval 20% of the time—that is, in one 5-year period every 25 years on average.) Compared with a current population of 81 million, the projections with a deterministic migration component indicate population decline to be very likely. Including uncertainty from migration more than doubles the width of this interval, which in our results goes from a low of 65 million to a high of 82 million. This indicates population decline to be far less likely.

Germany's 80% prediction interval for the 5-year period 2015–2020 has an upper bound on net migration of 2.2 million, substantially higher than the 1.25 million observed in the previous 5-year period, 2010–2015. Although this bound seems high relative to recent history, surpassing it in 2015–2020 remains a possibility, in large part due to an influx of Syrian and other

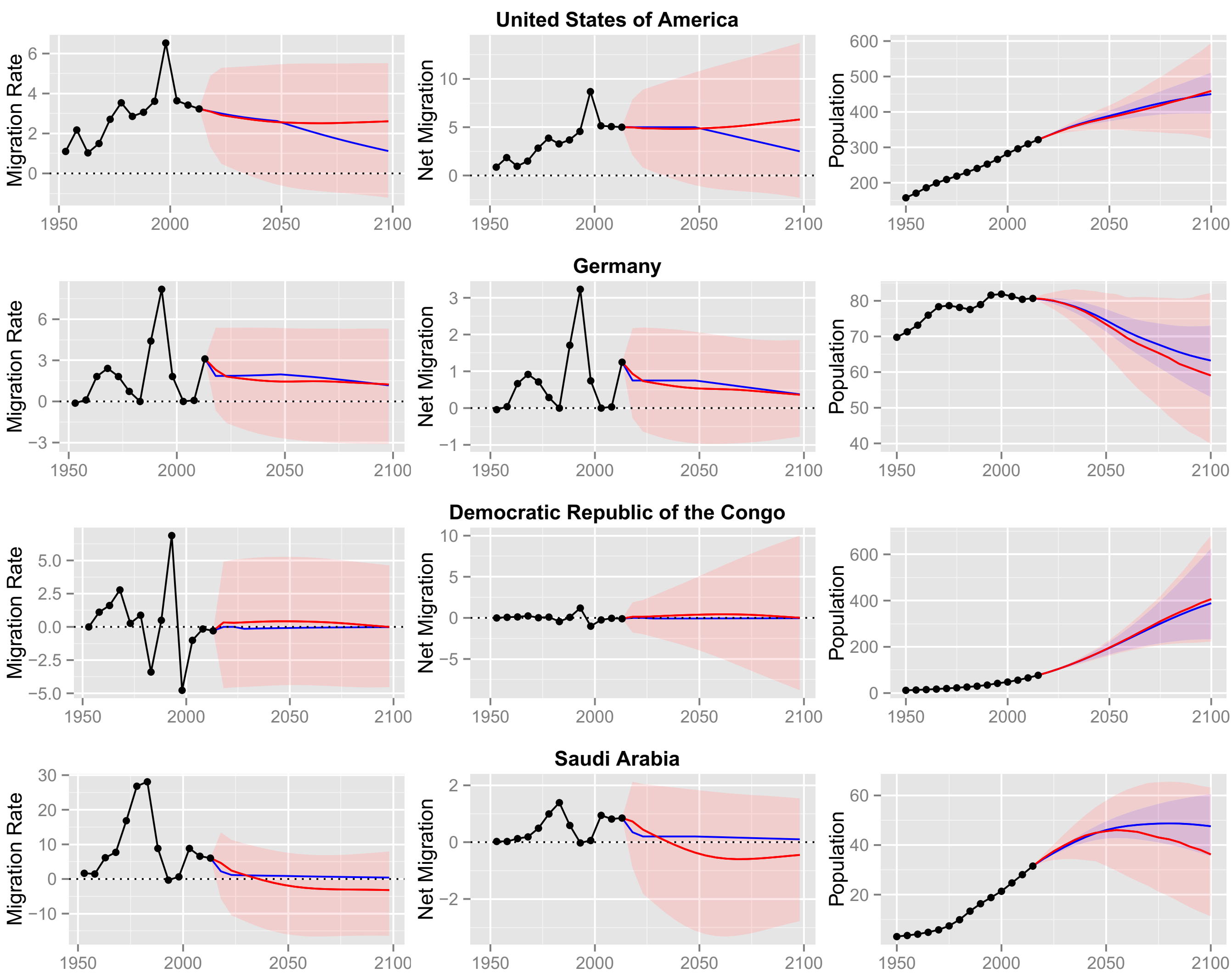

Fig. 1. Projected net migration rate (net annual migrants per thousand individuals), net migration count (5-year count, in millions of individuals), and population (in millions of individuals) for the United States, Germany, the Democratic Republic of the Congo, and Saudi Arabia. Probabilistic projections (in red) show the median and 80% prediction interval. Projections from WPP 2015 (in blue) are deterministic for the migration quantities (*Left* and *Center*).

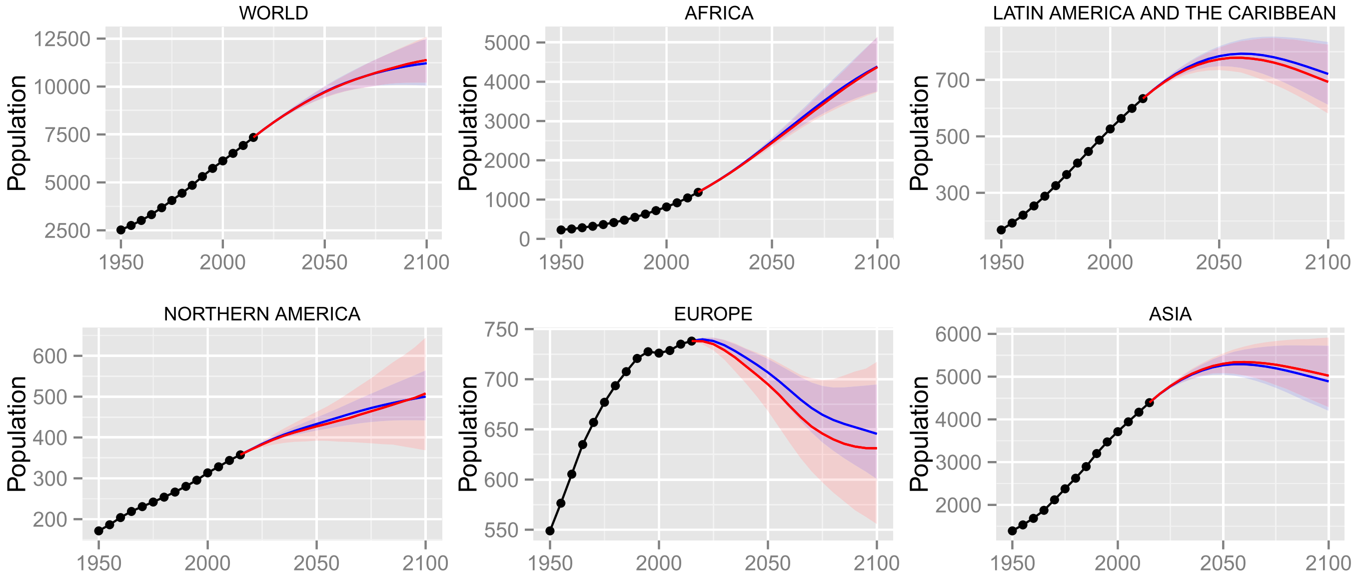

Fig. 2. Regional population projections (in millions) with median projections and 80% prediction intervals. Projections including probabilistic migration projections shown in red, and projections with deterministic migration projections in blue.

refugees. From January through November of 2015, Germany registered 965,000 migrants, with 484,000 from Syria alone (23).

In contrast to what we see in the United States and Europe, in many developing countries net migration makes up only a small fraction of total population change. When this is the case, the contribution of uncertainty in migration to uncertainty in population change is dwarfed by the contribution of uncertainty in fertility and mortality. The Democratic Republic of the Congo (DRC) is one such country (third row of Fig. 1). We project substantial uncertainty in net migration for the DRC, with an 80% prediction interval encompassing ±5 million migrants in the 2050–2055 time period. Although this is a large amount of uncertainty in terms of absolute numbers of individuals, it is small relative to the uncertainty about the rate of fertility and mortality decline in the DRC. When carried through to population projections, the result is that probabilistic migration projections have very little impact on uncertainty in population. This pattern holds across much of Africa and Asia, where population momentum dominates long-term population dynamics.

Finally, Saudi Arabia and the other Gulf Cooperation Council (GCC) countries receive special treatment in our projection method to ensure that growth from in-migration remains within reasonable bounds. (Full details of this special treatment for the GCC countries are given in *SI Appendix*.) In Saudi Arabia our migration projections introduce two distinct changes to the population projections (bottom row of Fig. 1). First, as is the typical pattern in countries where migration is a major contributor to population change, the probabilistic treatment of migration results in wider prediction intervals for population. Second, a threshold on net migration lowers the median population projection. The asymmetry of the migration thresholds in our method tends to result in larger out-migrations than in-migrations. On average this produces lower projected populations in Saudi Arabia than we saw with deterministic migration projections.

Regional Aggregates. The impact of introducing probabilistic migration projections is especially pronounced in the context of population projections for regional aggregates. Fig. 2 shows projected populations for a selection of multicountry regions both with probabilistic migration projection (red) and without (blue). In Europe and Northern America, uncertainty in migration contributes a large proportion of the total uncertainty in population projections. In Africa and Asia, introducing uncertainty in migration leaves population projections mostly unchanged. Global population projections are also nearly unchanged. In Latin America and the Caribbean, our migration model tends to project lower net migration than the deterministic migration model. This brings median population projections down, but it induces little change in the amount of uncertainty.

By comparing predictive distributions for models with and without a probabilistic migration component, we obtain an estimate of the proportion of variance in population projections that is attributable to migration. Table 1 contains these proportions for global and regional projections. In Europe, Northern America, and Oceania, more than half of the uncertainty about population projections comes from migration uncertainty. In Northern America, at least 80% of the uncertainty in population is attributable to migration. In these regions, treating migration deterministically leads to substantial understatement of uncertainty in population projections.

Which European Union Country Will Be Largest? Our projections take account of the major sources of uncertainty about future population change, and this allows us to provide answers to questions about any population quantity involving multiple countries. As an example, we consider the question of which country of the current European Union will be the largest in the 21st century. Germany is currently the most populous European Union member state, with a 2015 population of 81 million.

Table 1. Proportion of variance in population projection explained by variance in migration projections

	Proportion of variability (%)		
Region	2025	2050	2100
World	0.4	1.0	2.2
Africa	48	25	11
Latin America and the Caribbean	28	30	25
Northern America	82	84	82
Europe	75	75	64
Asia	7.5	6.9	18
Oceania	90	91	90

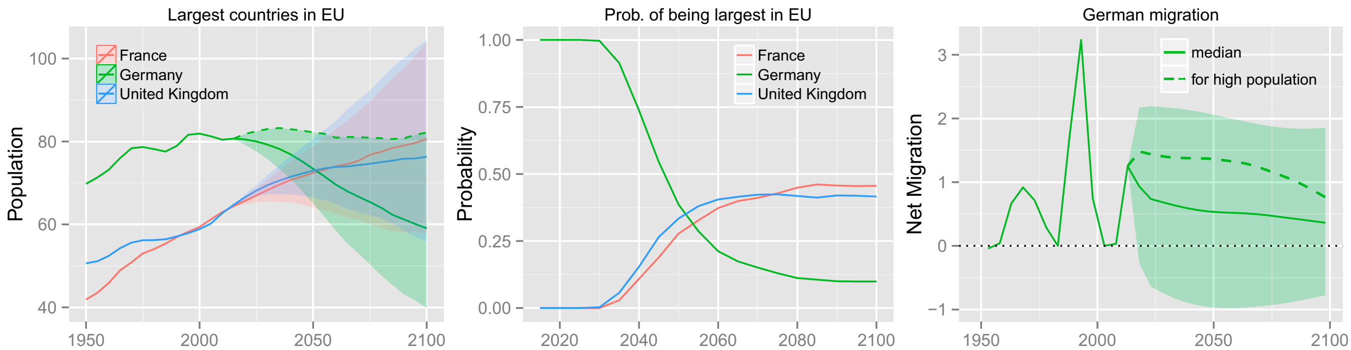

Fig. 3. (*Left*) Population (in millions) of France, Germany, and the United Kingdom, with median projection and 80% prediction interval. (*Center*) Projected probabilities of being the largest of the current European Union (EU) member states. (*Right*) Net migration (5-year count, millions of migrants) for Germany. From 2020 to 2100, median (solid line) and 80% prediction interval (shaded area). Dashed line is the median net migration among trajectories that match the upper bound of the 80% prediction interval for Germany's population shown as dashed line in (*Left*).

France and the United Kingdom are roughly 25% smaller, with 2015 populations of 64 million and 65 million, respectively (17). Our projections show that Germany's population is likely to decline in both the short term and long term, with substantial uncertainty about the rate of decline. This raises the question of how likely it is that Germany will retain its position as the most populous country in the European Union.

Fig. 3, *Left*, shows population projections with 80% prediction intervals for Germany, France, and the United Kingdom. The median projection has Germany's population shrinking by a quarter in the remainder of the century whereas France's and the United Kingdom's continue to grow. However, a wide variety of trajectories for all three countries is possible. Fig. 3, *Center*, shows our estimates of the probability over time that each of the three countries will be the largest of the current European Union member states. In the vast majority of trajectories, Germany remains the most populous European Union country until at least 2030. After 2030, its probability of primacy declines sharply. By 2060, France and the United Kingdom are both more likely than not to be more populous than Germany, and by 2075 Germany has only one chance in eight of being the most populous.

Fig. 3, *Right*, shows the likely level of future net migration if Germany attains the upper bound of the 80% interval, which corresponds to maintaining approximately constant population. The dashed line shows the median migration level corresponding to the upper limit of the 80% predictive interval for population, shown as a green dashed line in Fig. 3, *Left*. The solid line and the shaded area show the median and 80% predictive interval, respectively, produced by our migration model. In a typical scenario in which Germany's population remains roughly constant, it will have over a million migrants in each 5-year period. This figure is still substantially lower than the replacement migration figures given by Bijak et al. (24), who predict that 2.1 million in-migrants will be needed in 2047–2052 if Germany is to maintain constant population. Our figure is lower due to two main methodological differences. First, Bijak et al. (24) assume a fertility target of 1.5 children per woman in 2050, whereas our median projection for Germany is 1.62. Second, we report the conditional median of migration given high population; near the target population trajectory, it is likely that fertility and/or life expectancy will also be higher than average.

Migration is key to population dynamics in the European Union. In other regions, population momentum plays a larger role. For example, we project that India's population will almost certainly surpass that of China by 2025.

Out-of-Sample Validation Results. Out-of-sample validation for population and migration projections over the 2000–2015 time period is given in Table 2. The fully probabilistic model is labeled "All probabilistic." We compare it to models that replace one or more components with deterministic projections. We consider two alternative models. The first plugs in the true total fertility rate (TFR) and life expectancy at birth (e_0) in held-out time points. The second plugs in the true net migration (these are labeled "True TFR and e_0" and "True migration"). Neither of these is a feasible alternative for projection purposes, because they rely on knowing future values of fertility, mortality, or migration. They are included to give a sense of the relative importance of correctly projecting fertility, mortality, and migration. Two further alternatives use deterministic migration projections based on either assuming that net migration counts will persist indefinitely at current levels or fixing net migration rates at the posterior predictive medians from our migration model (labeled "persistent migration" and "median migration").

The fully probabilistic model is a dramatic improvement over both the persistent migration model and the median migration model. The latter two models produce population projection intervals that are far too narrow because they fail to account for uncertainty in migration. Undercoverage is less problematic in the model that conditions on knowing the true migration counts, as was used in ref. 16. The mean absolute relative error is markedly lower if we assume knowledge of net migration than if

Table 2. Out-of-sample validation of projections of population and migration, 2000–2015

			Coverage, %	
Model	MARE	SAPE	80% PI	95% PI
	Population			
All probabilistic	0.053	0.90	79	92
True TFR and e_0	0.048	0.83	78	92
True migration	0.017	0.98	77	92
Persistent migration	0.057	2.32	43	58
Median migration	0.059	2.45	43	53
	Migration			
	MAE	SAPE	80% PI	95% PI
All probabilistic	0.024	0.66	85	94
True TFR and e_0	0.024	0.65	85	94
Persistent migration	0.025	—	—	—
Median migration	0.025	—	—	—

Coverage refers to the proportion of the 2000–2015 observations that fell within their prediction interval (PI), in percent. All evaluation occurs on the 201 countries included in WPP 2015 (17). MAE, mean absolute error; MARE, mean absolute relative error; SAPE, standardized absolute prediction error.

we assume knowledge of fertility and mortality—1.7% versus 4.8%. This suggests that migration is a larger contributor to error in population projections than fertility and mortality combined.

Results for other held-out time periods are presented in *SI Appendix*, Tables S1 and S2.

Discussion

Alternative approaches would be to attempt to model both in- and out-migration for all countries, or to model the complete bilateral flow table. We chose to model net migration because it is the only reasonably reliable form of migration data for many countries. Estimates of past and current net migration can be produced with residual methods in countries where births, deaths, and population change can be estimated. In contrast, more detailed forms of migration data are challenging to estimate accurately, even in European countries where population registers exist (25).

A strength of the Bayesian framework is that it allows projections to incorporate expert judgment in a principled way. Although our migration model puts diffuse prior distributions on model parameters, expert judgment takes the form of thresholds on results, which can be viewed as a prior on model output. Details of these thresholds are given in *SI Appendix*.

Our predictions are conditional on the autoregressive models we use to project fertility, mortality, and migration. A key feature of our migration model is stationarity, which is a practical assumption that disallows dramatic, systematic growth of migration rates over the next century. Out-of-sample validation over short time periods suggests that we are not substantially understating uncertainty in the short run by failing to account for model uncertainty.

By modeling migration as an autoregressive process we have not explicitly included many factors known to influence migration. To include these factors would require forecasting them in the long term, which is difficult. The autoregressive model forecasts future migration in terms of current and past migration, which may be viewed as implicitly incorporating the history of these factors. Also, in light of the lack of a comprehensive theory of migration (26), an empirical approach may be desirable (14).

Our migration model does not specify that projected migration must respect current migration quotas or that current countries of net in-migration should remain net receivers. As a result, our migration projections are occasionally different from current migration quotas, recent trends in migration, or both. Although it is tempting to think that the present state of affairs will persist indefinitely into the future, dramatic shifts in migration can and do happen. We do not incorporate knowledge of migration quotas, because these quotas do change over time, as is happening with European Union quotas in the wake of Syria's refugee crisis. Furthermore, our migration model gives results consistent with the historical frequency with which countries switch between being net senders and net receivers of migrants (21).

One common trend in historical migration data is that a refugee out-migration is often followed by a large return migration. An ideal migration model would be able to replicate this phenomenon. Such refugee movements are hard to model and especially hard to predict, but finding a way to include them might improve our migration model.

Materials and Methods

Data. We used the estimates of age- and sex-specific vital rates and population counts in 5-year periods from 1950 to 2015 for all countries of the world, published by the UN (17). When estimating the Bayesian models for projecting fertility, mortality, and migration, we excluded the small countries with populations below 100,000. However, our method does generate projections for these countries.

Probabilistic Projections of Fertility and Mortality. Our methodology uses the same probabilistic models for fertility and mortality (27–32) that are used to produce the UN projections in WPP 2015 (17).

Probabilistic Projection of Net Migration. We project net migration for all countries with a Bayesian hierarchical model on net migration rates (21). We define the net migration rate, r_{ct}, for country c in time period t as a ratio with the numerator being the net number of migrants in country c over a 5-year period starting at time t and the denominator being the population of country c at time t, divided by 5 so that it is expressed in migrants per year per 1,000 population. This allows us to translate between migration counts and rates for all countries as long as initial populations are known.

We disaggregate total net migration counts to age- and sex-specific net migration counts by applying deterministically projected age schedules of migration. These age schedules are inferred from the deterministic fertility and mortality projections in WPP 2015 (17) using the bayesPop R package (33, 34). In the near term, projected schedules are typically similar to recently observed age and sex patterns of migration in a particular country or region.

Trajectories from our migration model are constrained so that global net migration counts sum to zero in each age and sex category. We achieve this balance with a postprocessing step on projected net migration. For each set of simulated trajectories, projected net migration counts generally will not sum to zero. While nonzero, the sums are typically small: on the order of 0.2% of the world's population or less. In principle, one could handle this excess simulated migration with a postprocessing step that redistributes this overflow to all countries in proportion to population. This turns out to be unsatisfactory. In typical trajectories, the six member countries of the GCC (Bahrain, Kuwait, Oman, Qatar, Saudi Arabia, and the United Arab Emirates) are projected to have large net migration counts, with especially high in-migration among males aged 20–39 years. As a result, projections that redistribute overflow migration in proportion to population too often result in projected emigration of young males from high-population countries as a way of counterbalancing GCC in-migration. To solve this problem, we reapportion overflow migration differently for the GCC countries. Details of this reapportionment are given in *SI Appendix*.

After this redistribution of all overflow migration, it is possible that some simulated migration counts will result in negative projected populations in some age categories. If this is the case, we first attempt to resample new net migration rates for all countries. If this procedure repeatedly produces negative projected populations, we instead reconfigure the age schedule of migration in the country or countries involved to redistribute out-migration to other age groups.

We found that it was necessary to apply thresholds to the output of long-term migration projections to ensure that trajectories do not result in total depopulation or unrealistically explosive growth. We used thresholds on net migration rates and counts to prevent them from exceeding historical maxima too dramatically. Similar thresholds are used to ensure that population density is not allowed to grow without bound. Details of these thresholds are given in *SI Appendix*.

Probabilistic Population Projection. To produce probabilistic population projections for all countries for 2015–2100, we independently took 1,000 samples each of trajectories of future TFR values and female and male life expectancies from their respective posterior predictive distributions. We converted these to age-specific fertility and mortality rates as described in ref. 32. We also independently drew 1,000 samples from the posterior distributions of the parameters in the migration model.

For each of the 1,000 combinations of fertility and mortality, we then projected the population forward one 5-year increment at a time from 2015 to 2100, using the standard cohort component model. After applying population changes from fertility and mortality, we projected the net migration rate for all countries forward by one 5-year increment and converted from rates to age- and sex-specific counts while ensuring zero global net migration in each category. Projected net migration counts were applied as population changes at the end of each time period. This provides a sample of 1,000 values of any future population quantity of interest, which we use to approximate its predictive distribution.

This procedure assumes that forecast errors in TFR, life expectancy, and migration within a country are independent of one another, as has been previously found appropriate (35). We also find very low empirical correlations between estimated forecast errors for fertility, mortality, and migration in our model, as shown in *SI Appendix*, Table S4.

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