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Rates and Probabilities in Economic Modelling

Transformation, Translation and Appropriate Application

Rachael L. Fleurence[1] and *Christopher S. Hollenbeak*[2]

1 United BioSource Corporation, Bethesda, Maryland, USA
2 Surgery and Health Evaluation Sciences, Penn State College of Medicine, Hershey, Pennsylvania, USA

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Abstract

Economic modelling is increasingly being used to evaluate the cost effectiveness of health technologies. One of the requirements for good practice in modelling is appropriate application of rates and probabilities. In spite of previous descriptions of appropriate use of rates and probabilities, confusions persist beyond a simple understanding of their definitions. The objective of this article is to provide a concise guide to understanding the issues surrounding the use of rates and probabilities reported in the literature in economic models, and an understanding of when and how to transform them appropriately. The article begins by defining rates and probabilities and shows the essential difference between the two measures. Appropriate conversions between rates and probabilities are discussed, and simple examples are provided to illustrate the techniques and pitfalls. How the transformed rates and probabilities may be used in economic models is then described and some recommendations are suggested.

Economic modelling occupies an increasingly important role in evaluating the cost effectiveness of health technologies. Indeed, a number of regulatory bodies, such as the National Institute for Health and Clinical Excellence in the UK, require such evaluations in order to provide guidance to health authorities. In the US, the provision of economic evidence is strongly encouraged in applications for inclusion in health insurance formularies. The Academy of Managed Care Pharmacy, for example, has issued guidance containing such requirements.[1]

The practice of economic modelling requires the inclusion of data that come from a number of different sources.[2,3] It is not realistic to expect to find the data in the literature in exactly the form required for an economic model. For example, it is unlikely that a clinical trial will provide epidemiological data within the exact timeline used in an economic model, or that the variety of possible studies used to populate the model will report the data within the same time frame.[4] Moreover, economic models often require transformations on parameters to account for risk factors or increased risk.

A number of guidelines assessing the quality of economic models have been published.[2,5,6] One of the requirements for good modelling is to demonstrate an understanding of the differences between rates and probabilities and their appropriate use in models. For example, the report of the International Society for Pharmacoeconomics and Outcomes Research Good Research Practices Task Force cites "the method for transforming interval probabilities from the literature or from a clinical trial into an instantaneous rate and then into a transition probability or event probability, corresponding to the time interval used in the model" as a criterion for good quality.[6] However, there is little published guidance on the difference between rates and probabilities and when and how it is appropriate to perform conversions in the context of an economic model. The seminal article by Miller and Homan[7] on the subject is technical and may not be immediately useful to the non-statistician, although it is highly recommended as an essential resource.

The objective of this article is to provide those engaged in applied economic modelling with a concise guide to understanding the issues surrounding the use of rates and probabilities reported in the literature, and when and how to transform them appropriately. We begin by defining rates and probabilities and showing the essential difference between the two measures. Appropriate conversions between rates and probabilities are discussed, and examples are provided to illustrate the techniques and pitfalls. How the transformed rates and probabilities may be used in economic models is then described and some recommendations are suggested.

1. Distinguishing Rates from Probabilities

Rates are an instantaneous or 'velocity' measure that range from zero to infinity. Rates describe the number of occurrences of an event for a given number of patients per unit of time. In contrast, probabilities describe the likelihood that an event will occur for a single individual in a given time period and range from 0 to 1. The essential difference between rates and probabilities relates to the role of time. In the calculation of rates, time is included directly in the denominator. So an incidence rate describes the number of new cases of a disease during a given time period over the total person-time of observation. The denominator is the sum of each individual's time at risk (i.e. the sum of the time until end of follow-up for those who do not experience an event and the time to event for those who experience an event). For example, the rate of hip fractures in a cohort will be obtained by dividing the number of occurrences of hip fractures by the corresponding person-time of the entire cohort *before* the hip fractures occurred. In contrast, the computation of a probability does not include time in the denominator. In the hip fracture example, an estimate of probability would be obtained by dividing the number of hip fractures by the total number of people at risk in the relevant time frame.

The following example illustrates the difference. A cohort of 1000 people is followed for 3 years. During the 3 years, 12 hip fractures occur (in different patients). Since 12 people of 1000 incurred a fracture over 3 years, the 3-year *probability* (or cumulative incidence) of a hip fracture is 12/1000 = 0.012. In contrast the hip fracture *rate* is 12/2976 = 0.004 fractures per person-year assuming that the 12 hip fractures occurred on average after 1 year. The total amount of person-time in the denominator (2976 person-years) is calculated as the sum of the person-time contributed by those who did not experience an event ($988 \times 3 = 2964$ person-years) and the person-time contributed by those who did experience an event (12 person-years). The total person-time is, therefore, $2964 + 12 = 2976$ person-years. Rates take into account the time spent at risk, which is reflected in removing person-time from the denominator once the event, such as the hip fracture, has occurred.[8] Note that another difference is that to calculate the 3-year probability it is necessary that everybody be followed for 3 years, unless an event occurs. For calculating the rate this is not required.

2. Converting Between Rates and Probabilities

Once rates or probabilities have been correctly identified, converting rates into probabilities and *vice versa*, according to the needs of the model, is straightforward. If an event occurs at a constant rate r per time unit t, then the probability that an event will occur during time t is given by equation 1 (note that the unit of time used in r and t must be the same):

$$p = 1 - e^{-rt} \tag{Eq. 1}$$

On the other hand, if we have a probability and we want to convert it to a rate, we use equation 2:

$$r = -\frac{1}{t}\ln(1-p) \tag{Eq. 2}$$

where p is the probability, r is the rate and t is the unit of time.

2.1 Using Rates and Probabilities in Economic Models

One of the difficulties encountered in applied economic modelling is correctly identifying the nature of the measure; that is, whether what is reported in the literature is a rate or a probability. In the development of the majority of models, the transformation of parameters into the relevant time frame will be necessary. However, these transformations require some thought into the nature of the data that are available, what is needed and how to perform the calculations.

Consider the following example, which illustrates what may happen when the application of rates and probabilities is confused in the application to a Markov model. The example is based on Miller and Homan,[7] but the notation has been simplified. Assume 100 well patients are followed for 3 years. Over the 3-year period, 70 patients experience an event (for example death). If we (incorrectly) assume that this information should be used to calculate a rate, then the average annual transition rate is calculated as r = 70 transitions/(100 patients × 3 years) or 0.233 transitions per patient-year. The 1-year annual transition probability is obtained using equation 1. The annual probability is $p = 1 - e^{-0.233} = 0.208$. Using this transition probability of 0.208 as the annual risk of mortality results in a total of 50 incident cases over 3 years instead of the 70 actual cases (see Miller and Homan[7] for further details).

If instead the data are (correctly) identified as the basis for calculating a simple 3-year cumulative incidence (3-year probability or risk) then p = 70/100 or 70%. Using equation 2, the 1-year probability can be determined directly from the 3-year 70% probability (equation 3):

$$r = -\frac{1}{3}\ln(1-0.7) = 0.4013 \tag{Eq. 3}$$

Translating this rate into a transition probability using Equation 1 yields $p = 1 - e^{-0.4013 \times 1} = 0.3306$ and 70 incident cases are modelled over the 3 years (see Miller and Homan[7] for full details).

An example of an economic model using rates that are then transformed into probabilities can be found in Myers et al.[9]

2.2 Properties of Rates

The preceding section showed why it is important to understand the data that are reported in the literature so that they can be correctly transformed for the relevant time frames used in the model. However, correctly identifying rates and probabilities is also important for the calculations that can be performed on model parameters. Rates possess convenient mathematical properties that probabilities do not. For example, we can add and subtract rates (for the same time interval) and we can divide and multiply a given rate by a factor reflecting risk factors. We can also divide rates by time or by the number of patients.[10]

For example, consider again our hypothetical data that presents 70 deaths in 100 patients over 3 years. Assume further that a separate meta-analysis reports a relative risk of 2 for experiencing the event (death) in a high-risk population. How do we compute the probability of death in the high-risk population?

We have already shown that the 3-year probability of death is 0.7 and that the annual rate of death was 0.4013. The annual rate for the high-risk population is then calculated as the product of the rate and the relative risk: 0.4013 × 2 = 0.8026. Finally, the annual transition probability is given by equation 1: $p = 1 - e^{-rt} = 1 - e^{-0.8026 \times 1} = 0.55$. The adjustment of the rate with the relative risk cannot be performed directly with probabilities.

3. Conclusion

An understanding of the difference between rates and probabilities and how to correctly perform necessary transformations and translation between them is fundamental to the development of valid economic models. Because data in the literature are rarely available for the time frames required in economic modelling, it is essential that researchers engaged in applied economic modelling understand these concepts and how to use them. Moreover, it cannot be assumed that data are correctly reported in the literature. For example, Miller and Homan[7] describe the different terms that can be found throughout the literature to describe rates and probabilities. Other terms used to describe rates are 'force', 'hazard', 'potential', 'person time incidence', 'incidence density' and 'instantaneous risk'. Probability is also referred to as 'risk', 'likelihood', 'cumulative incidence' and 'product limit' (Kaplan-Meier). There is a confusing array of terms used in the literature to describe rates and probabilities and this highlights the importance of correctly understanding the concepts that are being used.

The objective of this article was to provide a simple guide to understanding the nature of rates and probabilities and when and how to transform them appropriately for those developing economic decision-making models. However, the reader should be aware that the examples provided in this short piece are valid for an underlying two-state model where there is only a single transition from state 1 to state 2, and state 2 is an absorbing state. Multi-state models with more than two states or in a two-state model with a backwards transition from state 2 to state 1 require more complex transformations.[11] The interested reader who is modelling more complex transitions should consult Welton and Ades.[11]

A better understanding of rates versus probabilities will ensure the validity of the economic models that are being developed. This is fundamental when such tools are increasingly being used by decision makers to allocate scarce health resources.

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Correspondence and offprints: Dr *Rachael L. Fleurence*, United BioSource Corporation, Health Care Analytics Group, 7101 Wisconsin Avenue, Suite 600, Bethesda, MD 20814, USA.
E-mail: rachael.fleurence@unitedbiosource.com

